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Brief Paper

Observer-controller design for cranes via Lyapunov equivalence[1]

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Abstract

We consider a linearized parameter-varying model of a planar crane and show how a controller can be designed, following the state-feedback stabilization technique for time-varying systems proposed by Wolovich. The resulting closed-loop system is equivalent, via a Lyapunov transformation, to a stable time-invariant system of assigned eigenvalues. We also show that an observer can be designed applying Wolovich procedure to the dual system of the plant. The proposed procedure leads to the computation of the desired time-varying gains for controller and observer in a parameterized form. The results of several simulations with data taken from a real container crane, are also shown. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Time-varying observers; Time-varying controllers; Mechanical cranes; Lyapunov equivalence

1. Introduction

The need for faster cargo handling, in particular in loading and unloading container ships whose service time is to be minimized, requires the control of the crane motion so that its dynamic performance is optimized.

The first software tools for swing control were developed by Brown-Boveri and Siemens, as reported by Ridout (1989). These tools operate in an open-loop mode: thus they are not able to compensate the effect of disturbances, such as the wind.

Several authors have considered different control optimization techniques. Auernig and Troger (1987) and Hippe (1970) have used minimal time control techniques; Sakawa and Shindo (1982) have used optimal control to minimize load swing. Since the swing of the load depends on the acceleration of the trolley, minimizing the cycle time and minimizing the load swing are partially conflicting requirements.

In a previous paper (Corriga et al., 1998), we used a linear parameter-varying model of the crane to implement a gain-scheduling controller. The varying parameter is the length of the rope that sustains the load. The set of frozen models — given by different constant values of the rope length — can be reduced to a single time-invariant model that does not depend on the value of the rope length using a suitable time scaling suggested by Marttinen and Virkkunen (1987). The time scaling relation was used to derive a control law for the time-varying system that implements an implicit gain-scheduling.

In this paper we consider the same linear parameter-varying model of the crane used by Corriga et al. (1998), and show how a controller can be designed for this system, following the approach discussed by Castia and Seatzu (1996), using a state-feedback stabilization technique for time-varying systems proposed by Wolovich (1968a). Wolovich's procedure computes a stabilizing state-feedback gain matrix $\mathbf{F}(t)$ that gives a closed-loop system equivalent, via a Lyapunov transformation, to a stable time-invariant system of assigned eigenvalues. Since Lyapunov equivalence preserves the state stability, the time-varying system is stable as well. The performance of the closed-loop system depends on the choice of the eigenvalues of the time-invariant equivalent system.

The state-feedback controller requires the knowledge of the system state (center of mass position and velocity, load displacement w.r.t. the vertical and its rate of change) and that of the time-varying parameter (rope length). In a first case we assume that only the trolley position and the rope length can be measured by appropriate sensors as discussed by several authors (Ridout,

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1989; Marttinen and Virkkunen, 1987; Virkkunen et al., 1990).

In a second case, we assume that the load angle can be measured as well. In both cases, we show how a time-varying observer can be designed to provide an estimate of the unknown state vector.

1.1. Comparison with previous works

The observer is designed applying Wolovich procedure to the dual system of the plant. As far as we know, this observer design technique has never been presented before, even if the duality between the observer and controller dates back to 1968.

It may be interesting to compare our approach with other time-varying observer design techniques which appeared in the literature. Nguyen and Lee (1985) proposed a technique for the construction of a full-order observer. Wolovich (1968b), Yüksel and Bongiorno (1971) and Shafai and Carroll (1986) have proposed minimal-order observer design techniques. In all these approaches, and in ours as well, the most burdensome step, as far as computational effort is concerned, consists in inverting a basis matrix $\tilde{\mathbf{Q}}_c(t)$. However, while in the other approaches it is also necessary to reduce the system to a canonical form by finding a suitable Lyapunov transformation and computing its inverse, in our approach the Lyapunov transformation exists but need not be computed. A comparison between all these observer design techniques has been carried out by Castia and Seatzu (1996).

There are several advantages in the approach we propose.

- We use the same framework for the design of the observer and the controller.
- The computation of the state-feedback gains $\mathbf{F}(t)$ and of the observer gains $\mathbf{G}(t)$, following Wolovich, does not require the explicit computation of the Lyapunov transformation and of its inverse. It is necessary, however, to compute the inverse of an upper triangular matrix $\bar{\mathbf{B}}(t)$, but in the case of single-input, single-output systems it reduces to a scalar. As the number of components of the output vector increases, minimal-order observer design may become computationally more efficient than the proposed observer design by duality.
- The proposed procedure leads to the computation of the desired gains $\mathbf{F}(t)$ and $\mathbf{G}(t)$ in a parameterized form, as a symbolic function of the desired closed-loop dynamics (i.e., the eigenvalues of the equivalent time-invariant systems), rope length, rope velocity, trolley and load mass. As these parameters vary, these gains need not be recomputed by reapplying the whole design procedure but can simply be obtained by function evaluation. Thus, the computationally difficult step of inverting the matrix $\tilde{\mathbf{Q}}_c(t)$ needs to be performed only once in the off-line part of our procedure.

1.2. Model assumptions

An important issue that requires some comments is the practical implementation of the methodology we describe and the validation of the model we have considered.

- The cranes usually considered in the literature are planar, i.e. it is assumed that the movement of the load lies within a plane. Cranes used to handle heavy loads, e.g. container cranes, are usually planar. Non-planar cranes are usually used to handle lighter loads and their control is of lesser interest in an industrial setting.

  Furthermore, it is easy to observe that a non-planar crane can be seen as a two-degree-of-freedom crane (like the one we consider) with an additional possible movement of the trolley in an orthogonal direction. In such a case the whole system has order eight and its dynamics can be described as two decoupled fourth-order systems: the first is the one considered in this paper and the second one has the same structure and is relative to the orthogonal movement. Thus, the technique we propose can be applied in a modular way to each of the two subsystems. In this paper, we only consider planar cranes.
- In this paper we make several assumptions to derive a linear time-variant model. In particular, we consider small angles, constant rope velocity and we consider the suspending rope as a rigid rod; the movement is assumed perfectly planar and no disturbance (such as the wind) acting on the load is taken into account. It is important to underline that the assumption that the rope behaves as a rigid rod is quite common in all works that appeared in the literature. Due to its own weight and the load weight, the rope of the container crane is stiff.

  Some of these assumptions however, like small angles, constant rope velocity and wind effect, will be removed during the simulations we present, where a more general model is considered.
- Our approach requires that the mass of the load be known to reconstruct the position of the center of gravity. This is a realistic assumption in many applications, such as the handling of ship containers, in which the information on the physical (initial and final) position and on the weight of each container is known before the loading/unloading operation is started. Thus, the position of the center of gravity and its derivative (that we have assumed as state variables) can also easily be computed. There exist industrial applications in which the value of the load mass is not known before hand. In this case, a strain gauge should be used during each crane operation to measure the

load weight on-line (Marttinen and Virkkunen, 1987; Virkkunen et al., 1990).

The paper is structured as follows. In Section 2 we give some background on time-varying systems. In Section 3 we recall Wolovich stabilization procedure for time-varying systems and show how it can be applied by duality to the design of an observer. In Section 4 we present the linear time-varying model of a planar crane, derived under the assumptions reported in the appendix. In Section 5 we apply the results of Section 3 to the design of a controller-observer for the crane. In Section 6 we discuss an application example and present the results of numerical simulations, showing that the proposed approach gives acceptable performance.

2. Background

In this section, we recall some properties of linear-time-varying systems. See Chen (1984) for a formal derivation.

We will consider a system of the form

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\,\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t),$$
$$y(t) = \mathbf{C}(t)\,\mathbf{x}(t), \tag{1}$$

where $\mathbf{x}(t)$ is the state vector with n components, $\mathbf{u}(t)$ is the input vector with m components, and $y(t)$ is the output vector with p components. $\mathbf{A}(t)$ $(n\times n)$, $\mathbf{B}(t)$ $(n\times m)$, and $\mathbf{C}(t)$ $(p\times n)$ are time-varying matrices. We assume that $m \le n$ and $p \le n$, that $\mathbf{B}(t)$ has rank m and $\mathbf{C}(t)$ has rank p for all $t \ge t_0$, and that the derivatives

$$\mathrm{d}^i\mathbf{A}(t)/\mathrm{d}t^i, \quad 0 \le i \le 2(n-1),$$
$$\mathrm{d}^i\mathbf{B}(t)/\mathrm{d}t^i, \quad 0 \le i \le 2n-1,$$
$$\mathrm{d}^i\mathbf{C}(t)/\mathrm{d}t^i, \quad 0 \le i \le 2n-1,$$

are defined and bounded for all $t \ge t_0$.

Definition 1. Consider the system (1) and let

$$\mathbf{B}_1(t) = \mathbf{B}(t),$$
$$\mathbf{B}_i(t) = \mathbf{A}(t)\,\mathbf{B}_{i-1}(t) - \dot{\mathbf{B}}_{i-1}(t), \quad 2 \le i \le n. \tag{2}$$

Let the $(n\times nm)$ controllability matrix be $\mathbf{Q}_c(t) = [\mathbf{B}_1(t)\ \ \mathbf{B}_2(t)\ \ \cdots\ \ \mathbf{B}_n(t)]$. The system (1) is said to be *instantaneously controllable* in $[t_1, t_2]$ if rank $\mathbf{Q}_c(t) = n$ for all $t \in [t_1, t_2]$.

If Eq. (1) is instantaneously controllable in t the *lexicographic basis* $\tilde{\mathbf{Q}}_c(t)$ of the controllability matrix can be univocally computed. Let $\mathbf{B}_i(t) = [\mathbf{b}_{i,1}(t) \cdots \mathbf{b}_{i,m}(t)]$ so that

$$\mathbf{Q}_c(t) = [\mathbf{b}_{1,1}(t)\quad \mathbf{b}_{1,2}(t)\cdots\quad \mathbf{b}_{1,m}(t)\quad \mathbf{b}_{2,1}(t)\cdots\quad \mathbf{b}_{n,m}(t)].$$

Construct a new matrix $\bar{\mathbf{Q}}_c(t)$ by removing from $\mathbf{Q}_c(t)$ (moving from left to right) all columns that are linearly dependent on the previous ones; let σ_k be the numbers of columns $\mathbf{b}_{\cdot,k}$ left. Reorder the columns with a lexicographic order (i.e., $\mathbf{b}_{i,j}$ should precede $\mathbf{b}_{k,j}$ if $i < k$, and $\mathbf{b}_{\cdot,i}$ should precede $b_{\cdot,j}$ if $i < j$). The $n\times n$ matrix thus constructed is the lexicographic basis $\tilde{\mathbf{Q}}_c(t)$.

Definition 2. Consider a time-varying system

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\,\mathbf{x}(t). \tag{3}$$

Let $\Lambda(t)$ be an $(n\times n)$ matrix. It is assumed that $\Lambda(t)$ and $\dot{\Lambda}(t)$ are nonsingular and continuous for all t. Let $\bar{\mathrm{x}}(t) = \Lambda(t)\mathbf{x}(t)$. Then the system

$$\dot{\bar{\mathbf{x}}}(t) = \bar{\mathbf{A}}(t)\,\bar{\mathbf{x}}(t), \tag{4}$$

where $\bar{\mathbf{A}}(t) = [\Lambda(t)\ \ \mathbf{A}(t) + \dot{\Lambda}(t)]\Lambda^{-1}(t)$ is said to be equivalent to Eq. (3) through the *equivalence transformation* $\Lambda(t)$.

Furthermore, if $\Psi(t)$ is a fundamental matrix of the first system, $\bar{\Psi}(t) = \Lambda(t)\Psi(t)$ is a fundamental matrix of the second system.

Unlike the time-invariant case, an equivalence transformation does not preserve state stability.

Definition 3 An equivalence transformation $\Lambda(t)$ is called a *Lyapunov transformation* if

1. $\Lambda(t)$ and $\dot{\Lambda}(t)$ are continuous and bounded on $[t_0, \infty)$;
2. there exists a constant l such that $|\det\ \Lambda(t)| > l > 0$ for all $t \ge t_0$.

If $\Lambda(t)$ is a Lyapunov transformation, so is $\Lambda^{-1}(t)$.

A Lyapunov transformation preserves the stability properties of a dynamical equation. In the following, we will use this property finding a time-invariant system (whose stability can be easily checked) that is Lyapunov equivalent to a given time-varying system.

3. Controller and observer design

In this section, we recall a stabilization procedure proposed by Wolovich (1968a) for time-varying systems. This procedure may also be extended, by duality, to the design of an observer. Some care must be taken in the choice of eigenvalues assignement for the dual system (they must have a positive real part) as will be seen.

Wolovich's (1968a) main result can be summarized as follows.

Theorem 4. *Consider the system* (1). *If for all* $t \in [t_0, \infty)$,

(a) *the system is instantaneously controllable*;
(b) *the lexicographic basis* $\tilde{\mathbf{Q}}_c(t)$ *of the controllability matrix* $\mathbf{Q}_c(t)$ *does not change*;

(c) *there exists a constant k such that* $|\det \tilde{\mathbf{Q}}_c(t)| \geq k > 0$ *for all* $t \geq t_0$;

then there exists a matrix $\mathbf{F}(t)$ *such that*

- $\mathbf{F}(t)$ *is defined and bounded on* $[t_0, \infty)$;
- *The feedback law* $\mathbf{u}(t) = \mathbf{F}(t)\mathbf{x}(t)$ *in Eq.* (1) *gives a closed-loop system* $\dot{\mathbf{x}}(t) = \bar{\mathbf{A}}(t)\mathbf{x}(t)$, *where* $\bar{\mathbf{A}}(t) = \mathbf{A}(t) + \mathbf{B}(t)\mathbf{F}(t)$ *is equivalent via a Lyapunov transformation to a constant matrix* $\mathbf{Y}$ *whose eigenvalues may be arbitrarily chosen. Thus,* $\bar{\mathbf{A}}(t)$ *is stable if and only if* $\mathbf{Y}$ *is stable.*

We will not report Wolovich's proof, but we will simply recall the steps involved in finding the required gain matrix $\mathbf{F}(t)$.

Algorithm 5. To compute a feedback gain matrix $\mathbf{F}(t)$ that stabilizes system (1):

1. Compute the lexicographic basis $\tilde{\mathbf{Q}}_c(t)$ of the controllability matrix. Let σ_k $(k = 1, \ldots, m)$ be the indexes as in Definition 1 and define $d_k = \sum_{i=1}^{k} \sigma_i$.
2. Compute its inverse $\tilde{\mathbf{Q}}_c^{-1}(t)$. Let $\mathbf{q}_{0,k}(t)$ be the d_kth row of $\tilde{\mathbf{Q}}_c^{-1}(t)$.
3. Compute the $(1 \times n)$ vectors

$$\mathbf{q}_{r,k}(t) = \mathbf{q}_{r-1,k}(t)\,\mathbf{A}(t) + \dot{\mathbf{q}}_{r-1,k}(t),$$
$$k = 1, \ldots, m,\; r = 1, \ldots, \sigma_k,$$

4. Compute the $(m \times m)$ matrix

$$\bar{\mathbf{B}}(t) = \begin{bmatrix} \mathbf{q}_{\sigma_1 - 1, 1}(t)\,\mathbf{B}(t) \\ \vdots \\ \mathbf{q}_{\sigma_m - 1, m}(t)\mathbf{B}(t) \end{bmatrix}.$$

This matrix is upper triangular, with 1's along the diagonal.

5. Let

$$\mathbf{P}_0(t) = \begin{bmatrix} \mathbf{q}_{0,1}(t) \\ \vdots \\ \mathbf{q}_{0,m}(t) \end{bmatrix}$$

and compute

$$\mathbf{P}_i(t) = \mathbf{P}_{i-1}(t)\,\mathbf{A}(t) + \dot{\mathbf{P}}_{i-1}(t), \quad i = 1, 2, \ldots, \eta = \max\{\sigma_k\}.$$

6. Choose n desired eigenvalues (in m groups of σ_k) for the closed-loop equivalent time-invariant system. Each group of eigenvalues is associated to a characteristic polynomial

$$p_k(s) = s^{\sigma_k} + a_{\sigma_k - 1, k} s^{\sigma_k - 1} + \cdots + a_{1,k} s + a_{0,k}.$$

Construct matrix $\mathbf{M}_i = \mathrm{diag}(-a_{i,1}, \ldots, -a_{i,m})$ for $i = 0, \ldots, \eta$. (If $i > \sigma_j$ then $a_{i,j} = 0$.)

7. The desired feedback gain matrix is

$$\mathbf{F}(t) = \bar{\mathbf{B}}^{-1}(t) \sum_{i=0}^{\eta} \mathbf{M}_i \mathbf{P}_i(t).$$

The use of a state-feedback law in Wolovich's approach requires that the state of the system be accessible. This is not always the case, thus it may be necessary to use in the control loop an observer to estimate the state vector.

Here we present an observer design technique that applies the results of Wolovich to the dual system.

Proposition 6. *If the two systems*

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\,\mathbf{x}(t), \tag{5}$$

$$\dot{\mathbf{y}}(t) = \mathbf{Y}\mathbf{y}(t) \tag{6}$$

are equivalent through the Lyapunov transformation $\Lambda(t)$, *then the two adjoint systems*

$$\dot{\mathbf{z}}(t) = -\mathbf{A}^{\mathrm{T}}(t)\,\mathbf{z}(t), \tag{7}$$

$$\dot{\mathbf{w}}(t) = -\mathbf{Y}^{\mathrm{T}}\mathbf{w}(t) \tag{8}$$

are equivalent through the Lyapunov transformation $\Gamma(t) = \Lambda^{-\mathrm{T}}(t)$.

Proof. By Definition 2, a fundamental matrix of Eq. (5) is $\Psi(t) = \Lambda^{-1}(t)\mathrm{e}^{Yt}$.

If $\Psi(t)$ is a fundamental matrix of system (5), then $\Phi(t) = \Psi^{-\mathrm{T}}(t)$ is a fundamental matrix of system (7) (Chen, 1984), i.e.,

$$\Phi(t) = \Psi^{-\mathrm{T}}(t) = \Lambda^{\mathrm{T}}(t)\,\mathrm{e}^{-Y^{\mathrm{T}}t}. \tag{9}$$

Let us consider the transformation $\Gamma(t) = \mathrm{e}^{-Y^{\mathrm{T}}t}\Phi^{-1}(t)$. Clearly, this is an equivalence transformation for Eqs. (7) and (8). By substituting Eq. (9) we can also write

$$\Gamma(t) = \mathrm{e}^{-Y^{\mathrm{T}}t}\Phi^{-1}(t) = \mathrm{e}^{-Y^{\mathrm{T}}t}\mathrm{e}^{Y^{\mathrm{T}}t}\Lambda^{-\mathrm{T}}(t) = \Lambda^{-\mathrm{T}}(t)$$

and this is a Lyapunov transformation by Definition 3. □

This proposition can also be extended to the complex field, by changing the transpose operator T into the complex conjugate transpose operator $*$.

The previous proposition may be used to construct a Luenberger observer for system (1). In fact, a Luenberger observer takes the form

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}(t)\,\hat{\mathbf{x}}(t) + \mathbf{B}(t)\mathbf{u}(t) + \mathbf{G}(t)\mathbf{C}(t)[\mathbf{x}(t) - \hat{\mathbf{x}}(t)] \tag{10}$$

and the dynamics of the error $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ is

$$\dot{\mathbf{e}}(t) = [\mathbf{A}(t) - \mathbf{G}(t)\,\mathbf{C}(t)]\mathbf{e}(t). \tag{11}$$

We want to choose $\mathbf{G}(t)$ such that Eq. (11) is Lyapunov equivalent to a system of the form (6) where $\mathbf{Y}$ has given stable eigenvalues.

By Proposition 6, $\mathbf{G}(t)$ is also the feedback gain that makes the state dynamics of the dual of system (1),

$$\dot{\mathbf{z}}(t) = -\mathbf{A}^{\mathrm{T}}(t)\mathbf{z}(t) + \mathbf{C}^{\mathrm{T}}(t)\mathbf{v}(t),$$

Lyapunov equivalent in closed loop to a system of the form (8) and can thus be computed with the procedure described in Algorithm 3 (note that in step 6 the desired eigenvalues must be chosen positive since $\mathbf{Y}$ and $-\mathbf{Y}^T$ have opposite eigenvalues).

4. Linear-time-varying model of a crane

We will consider a planar crane, whose model is shown in Fig. 1. The following notation is used:

- m_T, m_L are the mass of the trolley and that of the load, respectively;
- L is the length of the suspending rope;
- x_T, x_L are, respectively, the displacement of the trolley, and that of the load with respect to (w.r.t.) a fixed coordinate system;
- $x_C = (m_T x_T + m_L x_L)/(m_L + m_T)$ is the displacement of the center of gravity of the overall system w.r.t. a fixed coordinate system;
- φ is the angle between the suspending rope and the vertical taken as positive in the clockwise direction (see Fig. 7);
- $x_\varphi = x_T - x_L = L\sin\varphi$ is the displacement of the load w.r.t. the vertical;
- u is the control force, applied to the trolley;
- F_v is the wind force acting on the load; and
- g is the gravitation constant.

We take as measurable variable the trolley position x_T.

Considering the suspending rope as a rigid rod, under the assumptions made in Corriga et al. (1998) and reported in the appendix (namely, small angles and constant rope velocity), choosing the following state variables:

$$x_1(t) = x_\varphi(t), \qquad x_2(t) = x_C(t), \qquad x_3(t) = \dot{x}_\varphi(t), \qquad x_4(t) = \dot{x}_C(t), \tag{12}$$

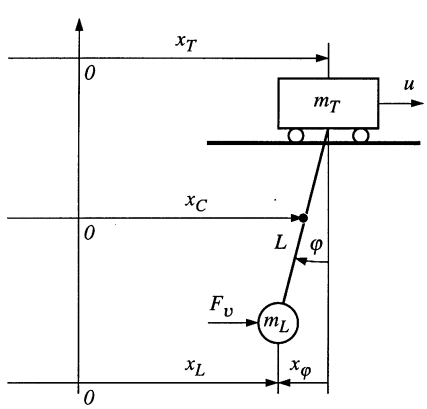

Fig. 1. Model of the crane.

and denoting

$$\omega(t) \equiv \omega(L(t)) = \left(\frac{g(m_T + m_L)}{m_T L(t)}\right)^{0.5}, \tag{13}$$

we get the following state variable equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{E}F_v(t), \qquad \gamma(t) = \mathbf{C}\mathbf{x}(t) \tag{14}$$

with

$$\mathbf{x}(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T$$

and

$$\mathbf{A}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega^2(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_T} \\ \frac{1}{m_T + m_L} \end{bmatrix};$$

$$\mathbf{C} = \begin{bmatrix} \frac{m_L}{m_T + m_L} & 1 & 0 & 0 \end{bmatrix}; \qquad \mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{m_L} \\ \frac{1}{m_T + m_L} \end{bmatrix}.$$

The model given by (14) is time-varying because ω is a function of $L(t)$.

We assume that $L(t) \in [L_{min}, L_{max}]$, with $L_{min} > 0$, and that $\dot{L}(t)$ has a constant value v in the time interval of interest, i.e.,

$$L(t) = L_0 + vt. \tag{15}$$

During the simulations (see Section 6) this assumption will be removed.

5. Controller-observer design for the crane

We show that system (14) satisfies the conditions of Theorem 4.

In fact, only the state matrix $\mathbf{A}(t)$ is time varying, and by the assumptions on $L(t)$ and $\dot{L}(t)$ in Eq. (15), $\mathbf{A}(t)$ is continuously differentiable.

Since $m = 1$, the lexicographic basis of the controllability matrix for system (14) is

$$\tilde{\mathbf{Q}}_c(t) = \mathbf{Q}_c(t) = \begin{bmatrix} 0 & \dfrac{1}{m_T} & 0 & \dfrac{-\omega^2(t)}{m_T} \\ 0 & \dfrac{1}{m_T + m_L} & 0 & 0 \\ \dfrac{1}{m_T} & 0 & \dfrac{-\omega^2(t)}{m_T} & \dfrac{-g(m_T + m_L)v}{m_T^2 L^2(t)} \\ \dfrac{1}{m_T + m_L} & 0 & 0 & 0 \end{bmatrix}$$

and

$$\det \tilde{\mathbf{Q}}_c(t) = \frac{g^2}{m_T^4 L^2(t)}.$$

Given the assumptions on $L(t)$, it is obvious that the system (14) satisfies the conditions of Theorem 3.

Similarly, if $m_L > 0$ it is possible to show that the dual of system (14) satisfies the conditions of Theorem 3, and thus an observer can be constructed. In fact, the lexicographic basis of the controllability matrix for the dual system is

$$\tilde{\mathbf{D}}_c(t) = \begin{bmatrix} \dfrac{m_L}{m_T + m_L} & 0 & \dfrac{-m_L\omega^2(t)}{m_T + m_L} & \dfrac{-m_L g v}{m_T L^2(t)} \\ 1 & 0 & 0 & 0 \\ 0 & \dfrac{-m_L}{m_T + m_L} & 0 & \dfrac{m_L\omega^2(t)}{m_T + m_L} \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

and

$$\det \tilde{\mathbf{D}}_c(t) = \frac{g^2 m_L^2}{m_T^2 L^2(t)}.$$

Note that if $m_L \ll m_T$ the matrix $\tilde{\mathbf{D}}_c(t)$ is very badly conditioned and the dual system becomes practically uncontrollable (note, on the contrary, that the controllability of the plant is not affected by the value of m_L).

One way to overcome this problem, is that of introducing a new sensor to measure the rope angle φ (or equivalently the load displacement x_φ). This gives a new output matrix

$$\mathbf{C}' = \begin{bmatrix} \dfrac{m_L}{m_T + m_L} & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

and the corresponding new matrix

$$\tilde{\mathbf{D}}'_c(t) = \begin{bmatrix} \dfrac{m_L}{m_T + m_L} & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -\dfrac{m_L}{m_T + m_L} & 0 & -1 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

is such that $\det \tilde{\mathbf{D}}'_c(t) = 1$, i.e., it is non-singular for all values of m_L.

Let $a(s) = s^4 + a_3 s^3 + \cdots + a_0$ denote the characteristic polynomial of the time-invariant system equivalent to the closed-loop plant. Applying the procedure described in Algorithm 5 we obtained the following symbolic expressions for the (1×4) controller gain matrix:

$$\mathbf{F}(t) = \begin{bmatrix} a_0 \dfrac{m_T^2 L(t)}{g(m_T + m_L)} + a_1 \dfrac{m_T^2 v}{g(m_T + m_L)} - a_2 m_T - a_3 \dfrac{2m_T v}{L(t)} + \dfrac{g(m_T + m_L)}{L(t)} \\ -a_0 \dfrac{m_T L(t)}{g} - a_1 \dfrac{m_T v}{g} \\ a_1 \dfrac{m_T^2 L(t)}{g(m_T + m_L)} + a_2 \dfrac{2m_T^2 v}{g(m_T + m_L)} - a_3 m_T - \dfrac{2m_T v}{L(t)} \\ -a_1 \dfrac{m_T L(t)}{g} - a_2 \dfrac{2m_T v}{g} \end{bmatrix}^T. \quad (16)$$

Let $b(s) = s^4 + b_3 s^3 + \cdots + b_0$ denote the characteristic polynomial of the time-invariant system equivalent to the observer with output matrix C. The observer gain is the (4×1) matrix:

$$\mathbf{G}(t) = \begin{bmatrix} b_1 \dfrac{m_T L(t)}{g m_L} + b_2 \dfrac{2m_T v}{g m_L} - b_3 \dfrac{m_T + m_L}{m_L} - \dfrac{2v(m_T + m_L)}{m_L L(t)} \\ -b_1 \dfrac{m_T L(t)}{g(m_T + m_L)} - b_2 \dfrac{2m_T v}{g(m_T + m_L)} \\ -b_0 \dfrac{m_T L(t)}{g m_L} - b_1 \dfrac{m_T v}{g m_L} + b_2 \dfrac{m_T + m_L}{m_L} + b_3 \dfrac{2v(m_T + m_L)}{m_L L(t)} - \dfrac{g(m_T + m_L)^2}{m_T m_L L(t)} \\ b_0 \dfrac{m_T L(t)}{(m_T + m_L)g} + b_1 \dfrac{m_T v}{g(m_T + m_L)} \end{bmatrix}. \quad (17)$$

Let $b'_1(s) = s^2 + b'_{1,1}s + b'_{0,1}$ and $b'_2(s) = s^2 + b'_{1,2}s + b'_{0,2}$ denote the two factors of the characteristic polynomial of the time-invariant system equivalent to the observer with output matrix $\mathbf{C}'$ (in fact, in this case $m = 2$). The observer gain with output matrix $\mathbf{C}'$ is the (4×2) matrix:

$$\mathbf{G}'(t) = \begin{bmatrix} 0 & -b'_{1,2} \\ -b'_{1,1} & b'_{1,2}\dfrac{m_L}{m_T + m_L} \\ 0 & b'_{0,2} - \dfrac{g(m_T + m_L)}{m_T L(t)} \\ b'_{0,1} & -b'_{0,2}\dfrac{m_L}{m_T + m_L} \end{bmatrix}. \tag{18}$$

Eqs. (16)–(18) give the desired control law in parameterized form. Here the parameters are: the trolley and load mass m_T and m_L, the hoisting velocity v, the rope length $L(t)$, the coefficients a_i of the desired characteristic polynomial of the time-invariant system equivalent to the closed-loop system, and the coefficients $b_i(b'_{i,j})$ of the desired characteristic polynomial of the time-invariant system equivalent to the observer dynamics.

The coefficients a_i and b_i $(b'_{i,j})$ are parameters that may be arbitrarily assigned by the designer to obtain an acceptable performance. To ensure stability, as we have discussed, the polynomial $a(s)$ must have all roots with negative real part, while the polynomial(s) $b(s)$ $(b'_1(s)$ and $b'_2(s))$ must have all roots with positive real part. Suitable values can only be found by trial and error procedure for a given crane. However, extensive simulations showed that they need not be changed as m_L changes from one operation to the other, i.e., they can be kept constant for a given crane regardless of the particular operation.

6. Simulation results

The above-described approach was applied to a container crane. The numerical values are taken from Sakawa and Shindo (1982) and are those of a container crane at the port of Kobe.

The trolley mass is $m_T = 6 \times 10^3$ kg. We assume the length of the suspending rope to be: $L(t) \in [L_{min}, L_{max}]$, where $L_{min} = 2$ m and $L_{max} = 10$ m. To deduce the controller and observer gain matrices we assumed that the rope length has a constant derivative $|\dot{L}(t)| = |v| = 1$ m/s. Clearly this is not true during a real movement. Therefore during numerical simulations, we have removed this assumption and we have imposed an acceleration of ± 2 m/s^2 at the beginning and at the end of the hoisting and lowering movement, while in the central part of the movement the velocity is constant and equal to ± 1 m/s.

During the simulations, we have also removed the assumption of linearity and we use the nonlinear model given in the appendix. The wind force acting on the load is taken into account as well.

For this crane, we were able to determine by trial and error two sets of eigenvalues for the controller and observer design, that give good performance for different load masses, and for both lifting and lowering movements. Furthermore, the same set of eigenvalues could be used for specifying the observer dynamic, regardless of the choice of output matrices $\mathbf{C}$ or $\mathbf{C}'$. By trial and error we have observed that in the controller case good performances are guaranteed if we choose a couple of real eigenvalues and a couple of complex conjugate eigenvalues with a high damping ratio: $\lambda_{1,2} = -1.125$, $\lambda_{3,4} = -5.25 \pm j0.075$. The chosen closed-loop eigenvalues for the observer design have a magnitude 2.5 times bigger: $\lambda'_{1,2} = 2.813$, $\lambda'_{3,4} = 13.125 \pm j0.188$.

In a first simulation, we considered a load mass $m_L = 42.5 \times 10^3$ kg. We assume that the only measurable variable is the trolley position x_T, i.e., we take $\mathbf{C}$ as the output matrix. The simulation was performed for a lifting movement from $L_0 = 10$ m to $L_f = 2$ m. The initial state of the crane was $x_\varphi(0) = 1.5$ m, $x_C(0) = -5$ m, $\dot{x}_\varphi(0) = \dot{x}_C(0) = 0.1$ m/s. The initial state of the observer can be chosen arbitrarily. In the following numerical simulations we assume that the only state variable of the observer initially different from zero is the centre of mass position, which is considered equal to the trolley position. So in the actual case the initial state of the observer was $\hat{x}_\varphi(0) = 0$ m, $\hat{x}_C(0) = -3.69$ m, $\dot{\hat{x}}_\varphi(0) = \dot{\hat{x}}_C(0) = 0$ m/s. We also take into account the wind effect and we assume that F_v acting on the load is a series of rectangular waves, alternatively positive and negative, whose magnitude is equal to 1000 N and whose period is 5 s. These values seem to be resonable considering the wind modellization reportedly taken from standard Engineering Handbooks. In Fig. 2, we show the plots of the variables of interest: here $e_\varphi(t) = x_\varphi(t) - \hat{x}_\varphi(t)$ and $e_C(t) = x_C(t) - \hat{x}_C(t)$.

In a second simulation, we considered the limit case of a movement with no load, i.e., $m_L = 10$ kg (this may represent the mass of the hook). We assume that the two measurable variables are the trolley position x_T and the first component of the state vector x_φ, i.e., we take $\mathbf{C}'$ as output matrix. The simulation was performed for a lowering movement from $L_0 = 2$ m to $L_f = 10$ m. The initial state of the crane was $x_\varphi(0) = 0.3$ m, $x_C(0) = -5$ m, $\dot{x}_\varphi(0) = \dot{x}_C(0) = 0.1$ m/s. The initial state of the observer was $\hat{x}_\varphi(0) = 0.3$ m, $\hat{x}_C(0) = -4.99$ m, $\dot{\hat{x}}_\varphi(0) = \dot{\hat{x}}_C(0) = 0$ m/s. In Fig. 3, we show the plots of the variables of interest.

The values of the control forces are resonable. We can claim this because the value of the trolley acceleration never exceeds 0.25 m/s^2.

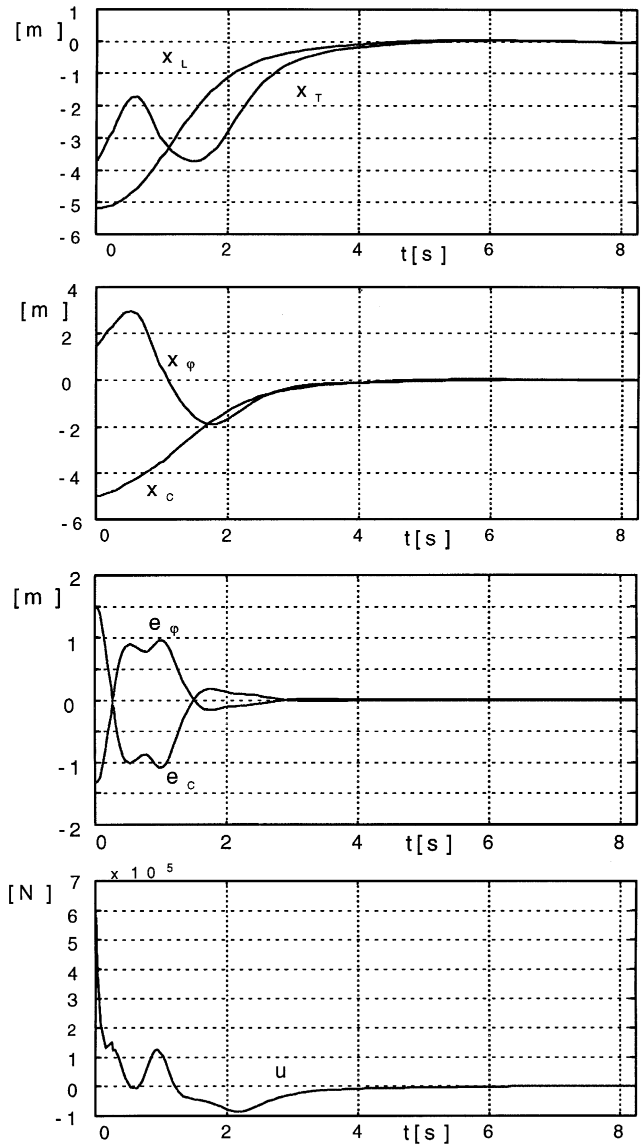

Fig. 2. Result of the first simulation: lifting movement for $m_L = 42.5 \times 10^3$ kg, and output matrix C.

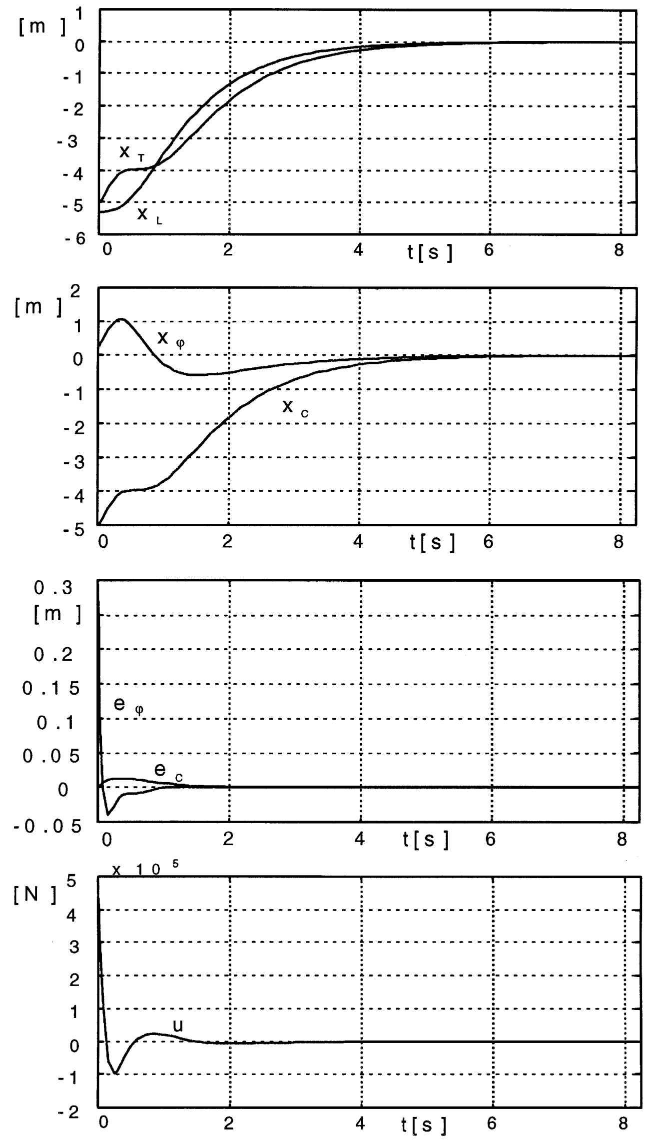

Fig. 3. Result of the second simulation: lowering movement for $m_L = 10$ kg, and output matrix C'.

7. Conclusions

We have discussed a general methodology for controlling mechanical cranes.

The non-linear crane model has been linearized and a parameter-varying model has been derived. A state-feedback controller for this time-varying system can always be constructed using the stabilization procedure of Wolovich.

The state-feedback controller requires the knowledge of the system's state. We have shown that it is possible to reconstruct the system's state from the measurement of the trolley position by means of a time-varying observer designed by application of Wolovich procedure to the dual system of the plant.

As the load mass goes to zero, the model becomes unobservable. In this case, we have shown how an observer can be constructed by the same procedure by using the additional measurement of the load angle.

We have given in a closed form the gains of the controller and of the observer as a symbolic function of the desired closed-loop dynamics (i.e., the eigenvalues of the equivalent time-invariant systems), rope length, rope velocity, trolley and load mass. As these parameters vary, these gains need not be recomputed by reapplying the whole design procedure but can simply be obtained by function evaluation.

By trial and error, we have been able to find suitable values of the design parameters (the eigenvalues of the equivalent time-invariant systems) that give good performance in the case of a container crane simulation.

Appendix. Model deduction

The dynamics of the system in Fig. 1 are described by the following equations (obtained by the translational equilibrium of the two masses):

$$m_T\ddot{x}_T = u - F\sin\varphi,$$
$$m_L\ddot{x}_L = F\sin\varphi + F_v, \tag{19}$$
$$m_L\ddot{y}_L = m_L g - F\cos\varphi,$$

where F is the force in the direction of the rope and F_v is the wind force whose magnitude and direction are assumed equal to that of the positive control force;

$$x_L = x_T - L\sin\varphi \tag{20}$$

is the displacement of the load in the horizontal direction w.r.t. to a fixed coordinate system;

$$y_L = L\cos\varphi \tag{21}$$

is the displacement of the load in the vertical direction w.r.t. to a fixed coordinate system.

With the coordinate trasformations

$$x_C = \frac{m_T x_T + m_L x_L}{\mathrm{m_L} + m_T}$$

and

$$x_\varphi = L\sin\varphi = x_T - x_L,$$

the first two equations of Eq. (19) can be rewritten as (we assume $m_L > 0$):

$$\ddot{x}_\varphi + \frac{F(\boldsymbol{\varphi},\mathbf{L})}{L}\left(\frac{1}{m_T} + \frac{1}{m_L}\right)x_\varphi = \frac{u}{m_T} - \frac{F_v}{m_L},$$
$$\ddot{x}_C = \frac{u + F_v}{m_T + m_L} \tag{22}$$

where the rope force $F(\boldsymbol{\varphi},\mathbf{L})$ is a function of $\boldsymbol{\varphi}$: $(\varphi, \dot{\varphi}, \ddot{\varphi})$ and $\mathbf{L}$: $(L, \dot{L}, \ddot{L})$ as can be determined by twice differentiating Eq. (21) and substituting into the third equation of Eq. (19):

$$m_L(\ddot{L}\cos\varphi - 2\dot{L}\dot{\varphi}\sin\varphi - L\dot{\varphi}^2\cos\varphi - L\ddot{\varphi}\sin\varphi)$$
$$= m_L g - F\cos\varphi. \tag{23}$$

Linearizing around the equilibrium point $\boldsymbol{\varphi}$: $(\varphi = 0, \dot{\varphi} = 0, \ddot{\varphi} = 0)$ is equivalent to setting

$$\sin\varphi = \varphi, \qquad \cos\varphi = 1, \qquad \dot{\varphi}\sin\varphi = 0,$$
$$\dot{\varphi}^2 = 0, \qquad \ddot{\varphi}\sin\varphi = 0,$$

and assuming $\ddot{L}(t) = 0$, Eq. (23) yields $F(\boldsymbol{\varphi}, \ddot{L}(t) = 0) = m_L g$, i.e., the force along the rope is equal to the weight of the load. Substituting this value of F into Eq. (22) we obtain the linearized model

$$\ddot{x}_\varphi + \frac{g\ (m_T + m_L)}{m_T L}\ x_\varphi = \frac{u}{m_T} - \frac{F_v}{m_L},$$
$$\ddot{x}_C = \frac{u + F_v}{m_T + m_L}. \tag{24}$$

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