Design of Optimal Petri Net Controllers for Disjunctive Generalized Mutual Exclusion Constraints

Ziyue Ma, Zhiwu Li, and Alessandro Giua

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Abstract

In this paper a type of specifications called OR-AND Generalized Mutual Exclusion Constraints (GMEC) for place/transition nets is defined. Such a specification consists of a disjunction of conjunction of several single GMECs, i.e., the requirement is that, at any given time, the controlled system should satisfy at least one set of conjunctive GMECs. We show that a bounded OR-AND GMEC can be enforced by a special control structure composed by a set of AND-GMEC monitor places plus a switcher that determines the current active ones. We also show that such a simple control structure can be modified to ensure maximal permissiveness. This approach can be used in the framework of supervisory control in Petri nets.

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Ziyue Ma is with School of Electro-Mechanical Engineering, Xidian University, Xi'an 710071, China (Email: maziyue@gmail.com).

Z. Li is with the School of Electro-Mechanical Engineering, Xidian University, Xian 710071, China, and with the Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia, and with the Institute of Systems Engineering, Macau University of Science and Technology, Taipa, Macau (e-mail: zhwli@xidian.edu.cn; systemscontrol@gmail.com).

A. Giua is with the School of Electro-Mechanical Engineering, Xidian University, Xian 710071, China, with the Aix Marseille Université, CNRS, ENSAM, Université de Toulon, LSIS UMR 7296, Marseille 13397, France, and with DIEE, University of Cagliari, Cagliari 09124, Italy (e-mail: alessandro.giua@lsis.org; giua@diee.unica.it).

1 Introduction

Generalized Mutual Exclusion Constraints [1] (GMECs) are a class of state specifications that can be efficiently enforced in a Petri net with controllable transitions by a simple control structure, called *monitor places*, which is maximally permissive. Since the monitor design does not require to enumerate the reachability set but can be solved by working on the net structure, the state explosion problem could be avoided and the controller design process is quite efficient. The GMEC approach has many advantages and has been used in a wide range of contexts [2–6]. Besides its efficient advantages, however, the modeling power of classical GMEC has its intrinsic restriction. In fact a single GMEC only considers a very special class of *legal markings* that belong to an n-dimensional half-space, where n is the number of places in the net. The legal marking set defined by a set of GMECs is given by the intersection of half-spaces and thus is always convex.

When Petri nets models are used in the *supervisory control theory* [7] for arbitrary language specifications [8, 9], the following problem arises. First, the set of legal markings in these types of problems are usually not convex. For instance, in the GMEC transformation problem [6], we need to propose a more restrictive control policy which prohibits not only the forbidden markings that violate the given GMEC, but also some other weakly forbidden markings, from which the system may uncontrollably violate the control law. A solution to such a problem has been provided by Moody and Antsaklis in [6, 10]: in these works they presented an efficient technique which determines a more restrictive admissible GMEC $(\mathbf{w}', k')$ from a given in admissible GMEC $(\mathbf{w}, k)$. However, the solution of Moody and Antsaklis is typically suboptimal, unless each uncontrollable transition has at most one input [11]. In more general classes of systems, although there does not exist a generalized solution yet, typically an inadmissible GMEC should be transformed into a set of disjunctive admissible GMECs, or even a disjunction of conjunctive admissible GMECs [12, 13]. Furthermore, in the forbidden marking problem such as deadlock prevention problems in S^3PR nets [14, 15], the legal marking set can also be non-convex. Therefore to obtain an optimal solution it would require an approach capable of handling non-convex sets of legal markings.

The control of disjunctive GMECs in Petri nets has been studied by Iordache and Antsaklis [5] and then extended to arbitrary logical structure of GMECs [16]. They designed a closed-loop controller to precisely keep track of the violation information, i.e., which GMECs are currently violated, to ensure that the plant would not violate all GMECs at the same time. Their approach works in a straightforward manner in classical models, but it still has some limitations. First, in their approach each GMEC must have a known upper and lower bound, and thus the approach is not applicable if the token count of a GMEC may go to positive or negative infinity. For example, if the support of a GMEC contains an unbounded place whose corresponding weight is negative, the GMEC may not have a lower bound: this is very common in the supervisory control framework (see Example 3.5). Second, the algorithms in [5, 16] are based on the *Petri net concurrent compo-*

sition, hence in the worst case the number of newly added control transitions in the closed-loop system would be exponential in the number of the disjunctions. In this sense, a more general method which can handle a larger class of systems and obtain the controllers with lower structural complexity is desirable.

In this paper we propose a different approach to construct a controller which realizes a given disjunction of conjunctive GMECs (this is called *OR-AND GMEC*). We introduce a *monitor-switcher* control structure which contains both places and transitions to enforce the disjunction of conjunctive GMECs. The monitor-switcher does not need to precisely record the violation information such that it has two advantages: (1) the controller does not require the lower bound of GMECs as in [5], therefore this method is more general; (2) the controller has a relative compact structure since the structural complexity of the monitor-switcher in the worst case is quadratic with respect to the number of disjunctions. This work also extends our previous approach in [17], in which a closed-loop controller is proposed for the disjunctive OR-GMEC. The contributions of this paper are summarized as follows:

- A two-stage design procedure for a controller capable of enforcing an OR-AND GMEC under the assumption that the GMECs are bounded (we refer to Section 3 for a formal definition of bounded OR-AND GMEC and for a discussion of its limitations). The monitor-switcher obtained using the first procedure is easy to be implemented and does not require the lower bound of GMECs as in [5]. We also show that a maximally permissive Petri net controller may not exist if the OR-AND GMEC is unbounded.
- We characterize the conditions under which the monitor-switcher is not maximally permissive by i-dentifying a special subset of transitions that may be over-restricted. We also show how the monitor-switcher could be modified, if necessary, to always obtain a maximally permissive controller. The modification procedures works on the net structure, thus the control problem can be efficiently solved. The final closed-loop controller has a quadratic structural complexity with respect to the number of disjunctions in the OR-AND GMEC.
- By using this approach a compiled Petri net controller to enforce a given OR-AND GMEC can be obtained. The fundamental advantage of a compiled controller is the possibility of constructing a model of a closed-loop system as a place/transition net that can be validated using existing techniques such as structural analysis.

The paper is organized in seven sections. Section II presents the Petri net formalism used in the paper. Section III introduces the OR-AND GMECs and some properties including boundedness are studied. Section IV presents the first stage of the monitor-switcher design. Section V analyzes the conditions under which the switcher is maximally permissive and shows how it can be modified to obtain a maximally permissive controller in the second stage of design. The complexity analysis of the resulting controller and the comparison with the work in literature are presented in Section VI, and conclusions are presented in Section VII.

2 Preliminaries

A Petri net is a four-tuple $N = (P, T, Pre, Post)$, where P is a set of m *places* represented by circles; T is a set of n *transitions* represented by bars; $Pre : P \times T \to \mathbb{N}$ and $Post : P \times T \to \mathbb{N}$ are the *pre-* and *post-incidence functions* that specify the arcs in the net and are represented as matrices in $\mathbb{N}^{m\times n}$ (here $\mathbb{N} = \{0, 1, 2, \ldots\}$).

The *incidence matrix* of a net is defined by $C = Post - Pre \in \mathbb{Z}^{m\times n}$ (here $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$). A net is said to be *self-loop free* if for all places $p \in P$ and for all transitions $t \in T$, $Pre(p,t) \cdot Post(p,t) = 0$ holds. For a self-loop free net, from the incidence matrix one may univocally determine the *Pre* and *Post* functions.

For a transition $t \in T$ we define *the set of its input places* as ${}^\bullet t = \{p \in P \mid Pre(p,t) > 0\}$ and *the set of its output places* as $t^\bullet = \{p \in P \mid Post(p,t) > 0\}$. This notation can also be applied to a set of transitions $\bar{T} \subseteq T$ by defining ${}^\bullet \bar{T} = \cup_{t\in\bar{T}} {}^\bullet t$ and $\bar{T}^\bullet = \cup_{t\in\bar{T}} t^\bullet$.

A *marking* is a vector $M : P \to \mathbb{N}$ that assigns to each place of a Petri net a non-negative integer number of tokens, represented by black dots. We denote by $M(p)$ the marking of place p. A *marked net* $\langle N, M_0\rangle$ is a net N with an initial marking M_0.

A transition t is (directly) *enabled* at M if $M \geq Pre(\cdot,t)$ and may fire reaching a new marking M' with $M' = M + C(\cdot,t)$. We write $M[\sigma\rangle$ to denote that the sequence of transitions $\sigma = t_{j1} \cdots t_{jk}$ is enabled at M, and we write $M[\sigma\rangle M'$ to denote that the firing of σ yields M'.

Given a sequence $\sigma \in T^*$, we associate to it an n-component vector $\mathbf{y} : T \to \mathbb{N}$, called the *firing vector (or Parikh vector)* of σ. Specifically, $y(t) = k$ if the transition t is contained k times in σ.

A marking M is *reachable* in $\langle N, M_0\rangle$ if there exists a firing sequence σ such that $M_0[\sigma\rangle M$. The set of all markings reachable from M_0 defines the *reachability set* of $\langle N, M_0\rangle$ and is denoted by $R(N, M_0)$. We denote by $PR(N, M_0)$ the *potentially reachable set*, i.e., the set of all markings $M \in \mathbb{N}^m$ for which there exists a vector $\mathbf{y} \in \mathbb{N}^n$ that satisfies the *state equation* $M = M_0 + C\cdot\mathbf{y}$, i.e., $PR(N, M_0) = \{M \in \mathbb{N}^m | \exists \mathbf{y} \in \mathbb{N}^n : M = M_0 + C\cdot\mathbf{y}\}$. We have that $R(N, M_0) \subseteq PR(N, M_0)$.

A place $p \in P$ of a marked net $\langle N, M_0\rangle$ is said to be *bounded* if there exists a nonnegative integer K such that for any marking $M \in R(N, M_0)$, $M(p) \leq K$ holds. A marked net is bounded if all its places are bounded.

Definition 2.1 *A Generalized Mutual Exclusion Constraint (GMEC) is a pair* $(\boldsymbol{w}, k)$ *that defines a set of legal markings:*

$$\mathscr{L}(\boldsymbol{w}, k) = \{M \in \mathbb{N}^m \mid \boldsymbol{w}^T \cdot M \leq k\}$$

where $\boldsymbol{w} \in \mathbb{Z}^m$ *and* $k \in \mathbb{N}$.

An AND-GMEC is a set of s GMECs denoted by a pair $(\boldsymbol{W},\boldsymbol{k})$ *where* $\boldsymbol{W} = [\boldsymbol{w}_1 \cdots \boldsymbol{w}_s] \in \mathbb{Z}^{m\times s}$ *and* $\boldsymbol{k} = [k_1 \cdots k_s]^T \in \mathbb{N}^s$. *In an AND-GMEC* $(\boldsymbol{W},\boldsymbol{k})$, *each* $(\boldsymbol{w}_i,k_i)$ *is a single GMEC. An AND-GMEC defines a set of legal markings*

$$\mathscr{L}_{AND}(\boldsymbol{W},\boldsymbol{k}) = \{M \in \mathbb{N}^m \mid \forall(\boldsymbol{w}_i,k_i) \in (\boldsymbol{W},\boldsymbol{k}), \boldsymbol{w}_i^T \cdot M \leq k_i\}$$

where $(\boldsymbol{w},k) \in (\boldsymbol{W},\boldsymbol{k})$ *indicates that there exists an entry i of* $\boldsymbol{W}$ *and* $\boldsymbol{k}$ *such that* $(\boldsymbol{w},k) = (\boldsymbol{w}_i,k_i)$. △

A single GMEC $(\mathbf{w},k)$ on a net system $\langle N,M_0\rangle$ with $N = (P,T,Pre,Post)$ (that is called a *plant net*) can be easily enforced through a control structure by adding to the net a loop free place q called *the monitor place* with incidence matrix $C(q,\cdot) = -\mathbf{w}^T \cdot C(\cdot,t)$ and initial marking $M(q) = k - \mathbf{w}^T \cdot M_0$. For the resulting *closed-loop net* $\langle N',M_0'\rangle$ with $N' = (P\cup\{q\},T,Pre',Post')$, the projection of its reachability set onto the set of places P of N satisfies $R(N',M_0')_{\uparrow P} \subseteq \mathscr{L}(\mathbf{w},k)$. An AND-GMEC can be enforced by a set of monitor places using the previous technique.

3 Definitions and Properties of OR-AND GMECs

In the classical GMEC condition, the logical relationship between constraints is **AND**, i.e., each legal marking must satisfy all the constraints in $(\mathbf{W},\mathbf{k})$. We consider the cases, however, in which the system is not required to satisfy all constraints but is only required to satisfy at least one GMEC from the given GMECs. This type of constraints is called the *OR-GMEC*.

Definition 3.1 *An OR-GMEC is a set of r GMECs denoted by a set* $W_{OR} = \{(\boldsymbol{w}_1,k_1), \ldots, (\boldsymbol{w}_r,k_r)\}$ *where each* $(\boldsymbol{w}_i,k_i)$ *is a single GMEC. An OR-GMEC defines a set of legal markings:*

$$\mathscr{L}_{OR}(W_{OR}) = \{M \in \mathbb{N}^m \mid \exists(\boldsymbol{w}_i,k_i) \in W_{OR}, \boldsymbol{w}_i^T \cdot M \leq k_i\}$$

△

Analogous to the OR-GMEC, if a system is required to satisfy at least one AND-GMEC from a set of AND-GMECs, this new type of constraints is called *OR-AND GMEC*. In particular, in a given OR-AND GMEC, if each AND-GMEC contains only one single GMEC, then the OR-AND GMEC is reduced to an OR-GMEC.

Definition 3.2 *Given a series of AND-GMEC* $(\boldsymbol{W}_1,\boldsymbol{k}_1), \ldots, (\boldsymbol{W}_r,\boldsymbol{k}_r)$, *the corresponding OR-AND GMEC is a set* $W_{OA} = \{(\boldsymbol{W}_1,\boldsymbol{k}_1), \ldots, (\boldsymbol{W}_r,\boldsymbol{k}_r)\}$ *with* $\boldsymbol{W}_i \in \mathbb{Z}^{m\times s_i}$ *and* $\boldsymbol{k}_i \in \mathbb{N}^{s_i}$ *for each* $1 \leq i \leq r$. *An OR-AND GMEC* *defines a set of legal markings:*

$$\mathscr{L}_{OA}(W_{OA}) = \{M \in \mathbb{N}^m \mid \exists(\boldsymbol{W}_i, \boldsymbol{k}_i) \in W_{OA},$$
$$\forall j : 1 \leq j \leq s_i, \boldsymbol{w}_{ij}^T \cdot M \leq k_{ij}\}$$

where $(\boldsymbol{w}_{ij}, k_{ij})$ *denotes the* j*-th single GMEC in* $(\boldsymbol{W}_i, \boldsymbol{k}_i)$*.* △

For a legal marking set $\mathscr{L}$, we can also define its corresponding bad marking set $\mathscr{B} = \mathbb{N}^m \setminus \mathscr{L}$. From the definitions of AND-GMEC, OR-GMEC, and OR-AND GMEC, we can easily derive the following proposition.

Proposition 3.3 $\mathscr{L}_{AND}$ *and* $\mathscr{B}_{OR}$ *are convex sets of integer vectors.*

Proof: $\mathscr{L}_{AND}$ is convex since a set of linear constraints defines a convex feasible set in the n-dimensional space. That $\mathscr{B}_{OR}$ is convex follows from the fact that $\mathscr{B}_{OR}$ defines the feasible set that for all GMEC in W_{OR}, $\mathbf{w}^T \cdot M > k$ holds, which is a set of conjunctive linear constraints. ■

Since in the classical Petri net model, markings are defined on the integer but not real space, the convexity of $\mathscr{L}_{AND}$ and $\mathscr{B}_{OR}$ refers to integer markings. For simplicity we call these sets "convex" if there is no confusion. We also note that typically neither $\mathscr{L}_{OA}$ nor $\mathscr{B}_{OA}$ is convex.

3.1 Boundedness of an OR-AND GMEC

Before presenting an algorithm to design an OR-AND GMEC controller, let us define the boundedness for an OR-AND GMEC.

Definition 3.4 *An OR-AND GMEC* $W_{OA} = \{(\boldsymbol{W}_1, \boldsymbol{k}_1), \ldots, (\boldsymbol{W}_r, \boldsymbol{k}_r)\}$ *is said to be bounded (with respect to* $\langle N, M_0 \rangle$*) if there exists an integer* $K < +\infty$ *such that for any constraint* $(\boldsymbol{w}_{ij}, k_{ij}) \in (\boldsymbol{W}_i, \boldsymbol{k}_i)$ *with* $1 \leq i \leq r$ *and for any marking* $M \in R(N, M_0) \cap \mathscr{L}_{OA}(W_{OA})$*,* $\boldsymbol{w}_{ij}^T \cdot M \leq K$ *holds.*

Example 3.5 *Consider the net in Figure 1 which represents an assembly workstation containing two input buffers* p_1 *and* p_3*, two workflows* p_2 *and* p_4*, one product stack* p_5 *and a stack counter* p_6*. Suppose that we want to enforce an OR-GMEC with* $r = 2$*:* $(M(p_2) + M(p_5) - M(p_6) \leq 0) \vee (M(p_4) + M(p_5) - M(p_6) \leq 0)$*, to ensure that the product stack* p_5 *will not exceed the threshold specified by the counter* p_6*. This OR-GMEC (that can also be considered as a particular case of OR-AND GMEC) is bounded according to Definition 3.4 with* $K = 5$*. Since* p_6 *is unbounded, this OR-GMEC does not have a lower bound and thus Iordache's approach in [5] cannot be applied.* △

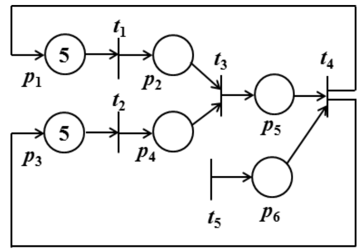

Figure 1: The supervisory control model for Example 3.5 with no lower bounds.

In the next section, to construct a control structure, one needs to find such a proper K. Typically one may have to explore the whole state space to precisely find the bound K, which is exhaustive in some cases. However, one may efficiently find a proper K by solving the state equation, as shown in the following proposition.

Proposition 3.6 *Given a net $\langle N, M_0 \rangle$, an OR-AND GMEC W_{OA} is bounded if for each $(\boldsymbol{w}_{ij}, k_{ij})$ and for each $(\boldsymbol{W}_x, \boldsymbol{k}_x)$ with $x \neq i$, the objective function of the following linear integer programming problem (IPP) is bounded:*

$$\begin{cases} K_{ij,x} = \max & \boldsymbol{w}_{ij}^T \cdot (M_0 + C\boldsymbol{y}) - k_{ij} \\ s.t. & M_0 + C\boldsymbol{y} \geq \boldsymbol{0} \\ & \mathbf{W}_x^T \cdot (M_0 + C\boldsymbol{y}) \leq \boldsymbol{k}_x. \end{cases} \tag{1}$$

Moreover, the bound K of W_{OA} in Definition 3.4 can be given as $K = \max_{i,j,x} K_{ij,x}$.

Proof: First, we observe that any marking that satisfies $M_0 + C\mathbf{y} \geq \mathbf{0}$ is in $PR(N, M_0)$. If each $K_{ij,x}$ is a finite integer (which implies that all IPPs are bounded), K must also be finite and $\mathbf{w}_{ij}^T \cdot M \leq K$ holds for any marking $M \in \bigcup_i (PR(N, M_0) \cap \mathscr{L}_{AND}(\mathbf{W}_i, \mathbf{k}_i))$. Since the following equalities hold:

$$\begin{aligned} &\bigcup_i (PR(N, M_0) \cap \mathscr{L}_{AND}(\mathbf{W}_i, \mathbf{k}_i)) \\ &\qquad = PR(N, M_0) \cap (\bigcup_i \mathscr{L}_{AND}(\mathbf{W}_i, \mathbf{k}_i)) \\ &\qquad = PR(N, M_0) \cap \mathscr{L}_{OA}(W_{OA}), \end{aligned}$$

for any marking $M \in PR(N, M_0) \cap \mathscr{L}_{OA}(W_{OA})$, $\mathbf{w}_{ij}^T \cdot M \leq K$ holds. By $PR(N, M_0) \supseteq R(N, M_0)$, for any marking $M \in R(N, M_0) \cap \mathscr{L}_{OA}(W_{OA})$, $\mathbf{w}_{ij}^T \cdot M \leq K$ holds, which concludes the proof. ■

Note that the parameter K found by the method in Proposition 3.6 may be larger than the actual reachable bound; however the behavior of the closed-loop net is not influenced by the selected value of K (see Section IV).

Two immediate consequences of Definition 3.4 are given as follows.

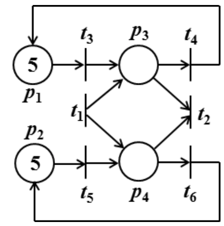

Figure 2: The supervisory control model for Example 3.9 containing an unbounded place with positive weight.

Proposition 3.7 *An OR-AND GMEC W_{OA} is bounded with respect to a plant net $\langle N, M_0 \rangle$ if for any constraint $(\boldsymbol{w}_{ij}, k_{ij})$, $w_{ij}(p) > 0$ implies that place p is bounded.*

Proof: Without loss of generality, for an arbitrarily chosen $(\mathbf{w}, k) \in \bigcup_i \bigcup_j \{(\mathbf{w}_{ij}, k_{ij})\}$ let us assume $w(p_v) > 0$ for $v \leq m' \leq m$ and $w(p_v) \leq 0$ for $v > m'$. Assume that all places p_v's are bounded for $v \leq m'$. Then for any marking $M \in R(N, M_0)$ and for any $v \leq m'$, $M(p_v) \leq c_v$ holds, where $c_v \in \mathbb{N}$ is the upper bound of p_v. Define a vector $\mathbf{w}' \in \mathbb{Z}^m$ where $w'(p_v) = w(p_v)$ for $v \leq m'$ and $w'(p_v) = 0$ for $v > m'$. Therefore for any marking $M \in R(N, M_0)$ and any $(\mathbf{w}_{ij}, k_{ij})$ we have:

$$\mathbf{w}_{ij}^T \cdot M \leq \mathbf{w}_{ij}'^T \cdot M \leq \sum_1^{m'} w_{ij}(p_v) \cdot c_j \leq K_{ij} \in \mathbb{N}. \tag{2}$$

Hence for any marking $M \in R(N, M_0)$, $\mathbf{w}_{ij}^T \cdot M \leq K$ holds where $K = \max_{i,j} K_{ij}$. Therefore for any marking $M \in R(N, M_0) \cap \mathscr{L}_{OA}(W_{OA})$, $\mathbf{w}_{ij}^T \cdot M \leq K$ holds and consequently W_{OA} is bounded according to Definition 3.4.
■

Corollary 3.8 *If $\langle N, M_0 \rangle$ is bounded, any OR-AND GMEC defined on its reachability set is bounded with respect to $\langle N, M_0 \rangle$.*

We point out that Proposition 3.7 and Corollary 3.8 only give a sufficient but not necessary condition for the boundedness of an OR-AND GMEC W_{OA}. If there exists an unbounded place p where for some $(\mathbf{w}_{ij}, k_{ij})$ the condition $w_{ij}(p) > 0$ holds, it does not imply that W_{OA} is unbounded: in the support of $(\mathbf{w}_{ij}, k_{ij})$ there may also exist some other unbounded places, say $\bar{p}$, with negative coefficient which prevents the token count of $(\mathbf{w}_{ij}, k_{ij})$ from going to positive infinity. In this sense we believe that the requirement for boundedness is not too restrictive since it applies not only to the class of bounded nets but to meaningful classes of unbounded nets as well.

Example 3.9 *Consider the net in Figure 2 which represents a workstation containing two input buffers p_1, p_2 and two workplaces p_3, p_4. There are three types of operations: assembling part A and part B (t_2), machining part A (t_4), and machining part B (t_6). Suppose that we want to enforce a GMEC: $(M(p_3) - M(p_4) \leq$*

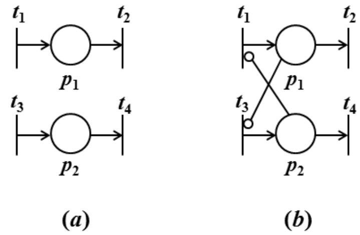

Figure 3: An unbounded OR-AND GMEC with unbounded places (a) and its inhibitor solution (b).

$2) \wedge (M(p_4) - M(p_3) \leq 2)$ *to balance the amounts of parts in the two workplaces. This AND-GMEC that can also be considered as a reduced OR-AND GMEC is bounded, although the support of the AND-GMEC contains two unbounded places with positive weights.* △

3.2 Requirement for Boundedness of OR-AND GMECs

In this subsection, we explain the importance of the boundedness of OR-AND GMECs. In Sections IV and V we will show that given a Petri net and a bounded OR-AND GMEC W_{OA}, the OR-AND GMEC can always be implemented by a Petri net structure which is maximally permissive. However, if W_{OA} is unbounded, it is not always possible to construct such a maximally permissive closed-loop system as a place/transition net. We show this by means of a simple example.

Example 3.10 *Consider the Petri net in Figure 3(a). The OR-AND GMEC that we want to implement is* $(M(p_1) \leq 0) \vee (M(p_2) \leq 0)$*, which is unbounded. One can intuitively verify that there does not exist a place/transition solution for this unbounded OR-AND GMEC. A solution presented by a* Petri net with inhibitor arcs[1] *is shown in Figure 3(b). However, it is well known that a Petri net with inhibitor arcs is equivalent to a Turing Machine and may not always be converted to a place/transition net.* △

We want to prove that it not possible to find a Petri net structure capable of enforcing the OR-AND GMEC discussed in Example 3.10 on the net in Figure 3(a). To show this in all generality, we prove an even stronger result. The formal definitions of *free-labeled Petri nets* and *Petri net languages* can be found in [18].

Proposition 3.11 *There does not exist a labeled Petri net* $\langle \hat{N}, \hat{M}_0 \rangle$ *with arbitrary labeling function that can generate the language of the free-labeled net with inhibitor arcs* $\langle N, M_0 \rangle$ *in Figure 3(b).*

Proof: We proof this by contradiction. Assume that such a net $\langle \hat{N}, \hat{M}_0 \rangle$ exists and let $\hat{T}$ be its set of transitions and $\ell : \hat{T} \to \{t_1, t_2, t_3, t_4\} \cup \{\varepsilon\}$ its labeling function, where ε denotes the empty string. We require

[1] An inhibitor arc from a place p to a transition t ends with a small circle and denotes that the transition is disabled when the place is marked. In Fig. 3(b) there are two inhibitor arcs: the arc from p_1 to t_3 and the arc from p_2 to t_1.

that

$$L_\ell(\hat{N},\hat{M}_0) \equiv \{\ell(\hat{\sigma}) \mid \hat{\sigma} \in L(\hat{N},\hat{M}_0)\} = L(N,M_0). \tag{3}$$

Since for all $i \in \mathbb{N}$, $t_1^i t_2^i t_3 \in L(N,M_0)$, thus we can construct an infinite sequence $\hat{M}_1,\hat{M}_2,\hat{M}_3,\ldots,$ of markings in $R(\hat{N},\hat{M}_0)$ such that for $i > 0$:

$$(\exists \hat{\sigma}_i, \hat{\sigma}_i')\ \hat{M}_0[\hat{\sigma}_i\rangle \hat{M}_i[\hat{\sigma}_i'\rangle \text{ with } \ell(\hat{\sigma}_i) = t_1^i, \ell(\hat{\sigma}_i') = t_2^i t_3,$$

i.e., marking $\hat{M}_i$ is reachable by a firing sequence that generates t_1^i and from such a marking a sequence that generates $t_2^i t_3$ is firable.

We now claim that there necessarily exist integers u, v with $0 < u < v$ such that $\hat{M}_u \leq \hat{M}_v$. This follows from a stronger result [19] that states that from an infinite sequence of elements of $\mathbb{N}^m$ one can extract an infinite non-decreasing subsequence.

Since $\hat{M}_u[\hat{\sigma}_u'\rangle$ and $\hat{M}_v \geq \hat{M}_u$, $\hat{M}_v[\hat{\sigma}_u'\rangle$ also holds, which implies $t_1^v t_2^u t_3 \in L_\ell(\hat{N},\hat{M}_0)$ with $v > u$. By $t_1^v t_2^u t_3 \notin L(N,M_0)$, this contradicts Eq. (3). ■

Proposition 3.11 shows that given an unbounded OR-AND GMEC control requirement there may be no place/transition solution. We also point out that many Petri nets with unbounded OR-AND GMECs can be reduced to the problem in Figure 3(a). Since the place/transition solution for an unbounded OR-AND GMEC does not always exist, this paper only focuses on bounded OR-AND GMECs.

4 Petri Net Controller Design for OR-AND GMECs

4.1 Monitor-Switcher Design for OR-AND GMECs

In the classical AND-GMEC controller design approaches, a controller consists of a set of monitor places P_S that are added to a plant net to determine a closed-loop net $\hat{N}$ with $\hat{P} = P \bigcup P_S$ and $\hat{T} = T$. For an OR-AND GMEC, however, it is not in general possible to build a controller that only consists of control places. Here we are looking for a control structure that contains both additional control places P_S and control transitions T_S, i.e., the closed-loop net shall have a set of places $\hat{P} = P \bigcup P_S$ and a set of transitions $\hat{T} = T \bigcup T_S$. Considering that the firing of a transition $t \in T_S$ should not change the marking of the plant net, we assume that T_S is such a set satisfying ${}^\bullet T_S \bigcup T_S^\bullet \subseteq P_S$. This motivates the following problem.

Problem 1 *Given a Petri net system $\langle N, M_0\rangle$ with $N = (P,T,Pre,Post)$ and an OR-AND GMEC $W_{OA} = \{(\boldsymbol{W}_1,\boldsymbol{k}_1),\ \ldots,\ (\boldsymbol{W}_r,\boldsymbol{k}_r)\}$, determine a closed-loop net $\langle \hat{N},\hat{M}_0\rangle$ with $\hat{N} = (P \bigcup P_S,\ T \bigcup T_S,\ \hat{Pre},\ \hat{Post})$ such*

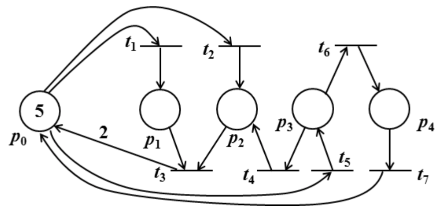

Figure 4: A plant net to be controlled by OR-AND GMEC $W_{OA} = \{(\mathbf{W}_1, \mathbf{k}_1), (\mathbf{W}_2, \mathbf{k}_2)\}$ in Example 4.1.

that it satisfies ${}^\bullet T_S \bigcup T_S^\bullet \subseteq P_S$, and the projection of the reachability set of the net $\hat{N}$ on the set of places P of N satisfies $R(\hat{N}, \hat{M}_0)_{\uparrow P} \subseteq \mathscr{L}_{OA}(W_{OA})$. To ensure the problem has a solution, it is natural for us to assume $M_0 \in \mathscr{L}_{OA}(W_{OA})$. △

The next algorithm determines a control structure, called a *monitor-switcher*, that is capable of enforcing a given bounded OR-AND GMEC.

Algorithm 1 ***Monitor-Switcher Design***

Input: *A plant net $\langle N, M_0\rangle$ and a bounded OR-AND GMEC $W_{OA} = \{(\boldsymbol{W}_i, \boldsymbol{k}_i), 1 \le i \le r\}$ with $M_0 \in \mathscr{L}_{OA}(W_{OA})$*

Output: *A closed-loop net $\langle \hat{N}, \hat{M}_0\rangle$ such that $R(\hat{N}, \hat{M}_0)_{\uparrow P} \subseteq \mathscr{L}_{OA}(W_{OA})$*

Step 1: *Select a sufficient large constant K that satisfies:*

$$K \ge K' = \max_{i=1,\ldots,r} \max_{j=1,\ldots,s_i} \max_{M \in R(N,M_0)} \{\boldsymbol{w}_{ij}^T \cdot M - k_{ij}\} \quad (4)$$

Step 2: *For each single constraint $(\mathbf{w}_{ij}, k_{ij})$ $(i = 1,\ldots,r, j = 1,\ldots,s_i)$, add a loop free place q_{ij} to the plant net $\langle N, M_0\rangle$ with $C(q_{ij}, t) = -\boldsymbol{w}_{ij}^T \cdot C(\cdot, t)$ and $M_0(q_{ij}) = k_{ij} - \boldsymbol{w}_{ij}^T \cdot M_0 + K$.*

Step 3: *Add r places $q'_1, \ldots, q'_r$. Add $r \times (r-1)$ transitions t_{ij} where $1 \le i, j \le r, i \ne j$. Let $Pre(q'_i, t_{ij}) = 1$ and $Post(q'_j, t_{ij}) = 1$. Let $Post(q_{i\cdot}, t_{ij}) = K$ and $Pre(q_{j\cdot}, t_{ij}) = K$.*

Step 4: *Pick an l which satisfies $M_0(q_{l\cdot}) \ge K$. Let $M(q'_l) = 1$ and $M(q'_i) = 0$ for $i \ne l$. Let $M(q_{l\cdot}) = M(q_{l\cdot}) - K$.*

Step 5: *Output the closed-loop net $\langle \hat{N}, \hat{M}_0\rangle$.* □

We briefly illustrate how this algorithm works before showing an example. In the first step a sufficiently large constant K is selected, which is larger than the maximal upper bound of all the single GMECs. This could be done by the method in Proposition 3.6. In Step 2 for each single GMEC (w_{ij}, k_{ij}) a standard GMEC monitor place q_{ij} is added. The initial tokens in each of these monitor places is the standard $k_{ij} - \mathbf{w}_{ij}^T \cdot M_0$ plus additional K tokens. Since K is greater than the upper bound of each single GMEC, all these AND-GMECs are considered as not active. In Step 3 the switcher structure is added and then in Step 4 we put a unique token in the switcher place corresponding to an AND-GMEC $(\mathbf{W}_l, \mathbf{k}_l)$ which M_0 satisfies. To ensure that $(\mathbf{W}_l, \mathbf{k}_l)$ is considered as active, K tokens are removed from the monitor place of each single GMEC of $(\mathbf{W}_l, \mathbf{k}_l)$. Algorithm 1 can always be applied since in Step 1, K always exists due to the boundedness of the

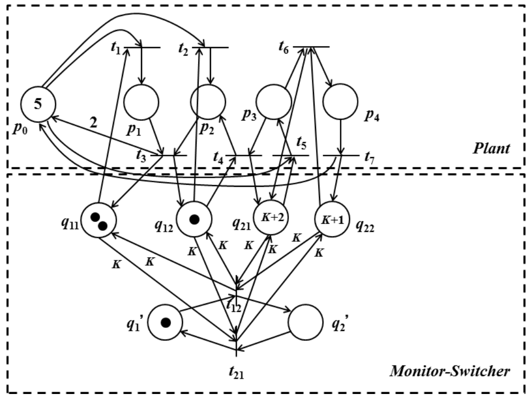

Figure 5: The monitor-switcher design for the net in Figure 4 in Example 4.1.

given OR-AND GMEC, and in Step 4, l can always be found since M_0 satisfies at least one AND-GMEC as assumed.

Example 4.1 [2] *In Figure 4 the OR-AND GMEC to be enforced is*

$$W_{OA} = \{(\boldsymbol{W}_1, \boldsymbol{k}_1), (\boldsymbol{W}_2, \boldsymbol{k}_2)\}$$

where two single GMECs $(\boldsymbol{w}_{11}, k_{11}) = ([0,1,0,0,0]^T, 2)$, $(\boldsymbol{w}_{12}, k_{12}) = ([0,0,1,0,0]^T, 1)$ *are in* $(\boldsymbol{W}_1, \boldsymbol{k}_1)$ *and two single GMECs* $(\boldsymbol{w}_{21}, k_{21}) = ([0,0,0,1,0]^T, 2)$, $(\boldsymbol{w}_{22}, k_{22}) = ([0,0,0,0,1]^T, 1)$ *are in* $(\boldsymbol{W}_2, \boldsymbol{k}_2)$. *In this net all places are 5-bounded, i.e.,* $\forall M \in R(N, M_0)$, $M(p_x) \leq 5$, $x = 0,1,2,3,4$. *By applying Algorithm 1, we obtain the closed-loop net in Figure 5 with the monitor-switcher structure. In the closed-loop net,* q_{ij} *is the monitor place associated to the* j*-th constraint in* $(\boldsymbol{W}_i, \boldsymbol{k}_i)$. *If the unique token in the switcher is in* q'_1, *monitor places* q_{11} *and* q_{12} *would have proper number of tokens such that* $(\boldsymbol{W}_1, \boldsymbol{k}_1)$ *is active, while monitor places* q_{21} *and* q_{22} *would have excessive number of tokens such that the corresponding AND-GMEC* $(\boldsymbol{W}_2, \boldsymbol{k}_2)$ *is inactive. It is also a similar case if the unique token in the switcher is in* q'_2. *Therefore the unique token in* q'_i *indicates that the i-th AND-GMEC* $(\boldsymbol{W}_i, \boldsymbol{k}_i)$ *is active, i.e., the current legal marking satisfies (at least)* $(\boldsymbol{W}_i, \boldsymbol{k}_i)$. $\triangle$

From the structure of the resulting net, we have the following result ensuring that only legal markings are reachable under control. We denote the marking of the closed-loop net $\hat{N}$ by $\hat{M}$ and the corresponding marking of the plant net N by M, i.e., $M = \hat{M}_{\uparrow P}$. Furthermore, we define $\hat{\mathbf{w}}^T \cdot \hat{M} = \mathbf{w}^T \cdot M$ for $\hat{M}_{\uparrow P} = M$, where $\mathbf{w} \in \mathbb{N}^{|P|}$ and $\hat{\mathbf{w}} \in \mathbb{N}^{|P \cup P_S|}$ is obtained by $\mathbf{w}$ assigning to all monitor and switcher places a weight 0, i.e., $\hat{\mathbf{w}}(i) = \mathbf{w}(i)$ if $p_i \in P$, else $\hat{\mathbf{w}}(i) = 0$. The next theorem shows that the monitor-switcher constructed by Algorithm 1 provides a candidate solution for Problem 1.

Theorem 4.2 *A net* $\langle \hat{N}, \hat{M}_0 \rangle$ *constructed by Algorithm 1 satisfies* $R(\hat{N}, \hat{M}_0)_{\uparrow P} \subseteq \mathscr{L}_{OA}(W_{OA})$.

[2]In Example 4.1 the number of disjunctions is $r = 2$. Another example is given in Figure 8 in Appendix to illustrate a switching mode with three disjunctions.

Proof: In Step 2 of the algorithm, the resulting net has the following invariants:

$$\hat{\mathbf{w}}_{ij}^T \cdot \hat{M} + \hat{M}(q_{ij}) = k_{ij} + K \tag{5}$$

After the monitor-switcher is added, the resulting net has the following invariants:

$$\begin{cases} \hat{\mathbf{w}}_{ij}^T \cdot \hat{M} + \hat{M}(q_{ij}) = k_{ij} + K - \hat{M}(q_i') \cdot K \\ \qquad \sum_i \hat{M}(q_i') = 1 \end{cases} \tag{6}$$

The last invariant ensures that for any marking $\hat{M} \in R(\hat{N}, \hat{M}_0)$ there exists a constraint x such that $\hat{M}(q_x') = 1$ and by Eq.(6) $\hat{M}$ also satisfies $\hat{\mathbf{w}}_{xj}^T \cdot \hat{M} + \hat{M}(q_{xj}) = k_{xj}$. Thus, $M = \hat{M}_{\uparrow P}$ satisfies at least the x-th AND-GMEC $(\mathbf{W}_x, \mathbf{k}_x)$ in W_{OA}. ■

One may notice that the number of reachable states of the closed-loop net is always the same regardless of K. Therefore, in some real systems the selection of K can be quite intuitive. In Example 4.1 we can let $K = 100,000$, and the resulting closed-loop net exactly has the same evolution as the closed-loop net with $K = K' = 5$. This algorithm cannot be applied for an unbounded OR-AND GMEC since a finite K that makes any constraint inactive does not exist.

4.2 Non-optimality

A desirable property of the obtained $\langle \hat{N}, \hat{M}_0 \rangle$ is *maximal permissiveness*, that is, if the firing of a transition t in the plant net is legal, it should also be firable in the closed-loop net. In the closed-loop net, however, the firing of t may depend on the previous firing of a control transition in the switcher. This motivates the following weaker definition of permissiveness.

Definition 4.3 *A closed-loop net $\langle \hat{N}, \hat{M}_0 \rangle$ (with respect to $\langle N, M_0 \rangle$) is said to be maximally permissive if for any marking $\hat{M} \in R(\hat{N}, \hat{M}_0)$ and for any marking $M = \hat{M}_{\uparrow P} \in \mathscr{L}$, the following condition[3] holds:*

$$(M[t\rangle_N M' \in \mathscr{L}) \Rightarrow (\exists \sigma_S \in T_S^* : \hat{M}[\sigma_S t\rangle_{\hat{N}}) \tag{7}$$

△

According to the condition in Definition 4.3, a closed-loop net is maximally permissive if, given any marking $\hat{M}$ such that $\hat{M}_{\uparrow P} \in \mathscr{L}$, the trajectory of $\hat{M}_{\uparrow P}[t\rangle_N$ is legal in the plant net, then in the closed-loop net, transition t must also be enabled after a proper evolution of the control transitions in T_S. Note that for

[3] Here we distinguish the enabling in the nets N and $\hat{N}$ using the notation $[\cdot\rangle_N$ and $[\cdot\rangle_{\hat{N}}$, respectively.

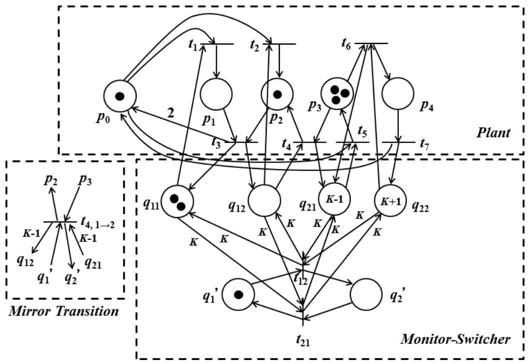

Figure 6: An example in which $\langle \hat{N}, \hat{M}_0 \rangle$ is not maximally permissive.

a monitor-switcher controlled net, we only need to check $|\sigma_S| \leq 1$ to verify maximal permissiveness, since the switcher can always unmark q'_x and mark q'_y in one step by firing t_{xy} (if this operation is legal). However, in some general types of control structures it may be necessary to fire more than one control transitions to enable a plant transition.

The following proposition shows that Algorithm 1 does not determine a maximally permissive net according to Definition 4.3.

Proposition 4.4 *The closed-loop net $\langle \hat{N}, \hat{M}_0 \rangle$ by Algorithm 1 is not always maximally permissive with respect to $\langle N, M_0 \rangle$.*

Proof: We prove this result by providing a counterexample. Consider again the closed-loop net $\langle N, M_0 \rangle$ in Figure 5. After firing $t_5 t_5 t_5 t_2$ it will reach a plant marking $M_a = [1,0,1,3,0]^T$, as shown in Figure 6 without transition $t_{4,1\rightarrow 2}$ in the dashed box. One can readily verify that both M_a (satisfying $(\mathbf{W}_1, \mathbf{k}_1)$) and $M_b = [1,0,2,2,0]^T$ (satisfying $(\mathbf{W}_2, \mathbf{k}_2)$) are legal markings, hence t_4 may legally fire at M_a. However, at M_a transition t_4 is blocked by q_{12}, and the switcher cannot shift from q'_1 to q'_2 since $M_a(q_{21}) = K - 1 < K$. Therefore M_b cannot be reached from M_a by firing a sequence (possibly empty) of transitions in T_S followed by t_4. ■

Remark 1 *We comment on the notion of maximal permissiveness defined above. In some areas of Petri net control, such as deadlock avoidance and prevention [14, 15, 20–23], a notion of maximal permissiveness based on reachable markings is typically used: it only requires that all markings that can be reached by legal trajectories are reachable in a closed-loop net, even if some of the sequences that lead to them may not be firable. However Definition 4.3 — which is defined on the firing sequences — implies that also all legal firing sequences must be firable in the closed-loop net. For instance, we have shown that in Proposition 4.4 from $[1,0,1,3,0]^T$ one cannot reach $[1,0,2,2,0]^T$ by firing $\sigma_S t_4$. However, $[1,0,2,2,0]^T$ can be legally reached from $[5,0,0,0,0]$ by firing $t_{12} t_2 t_2 t_5 t_5$. Hence Definition 4.3 is more strict.* △

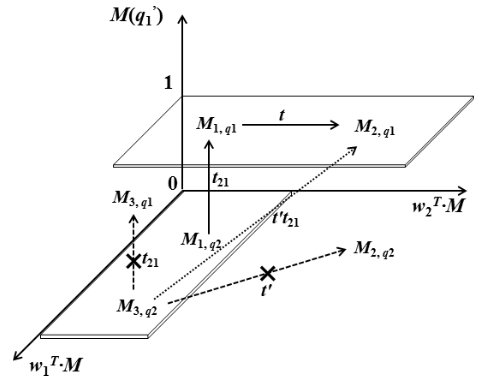

Figure 7: The illustration of the *unnecessarily blocking.*

To better illustrate this situation we introduce the concept of *weakly enabled* transitions.

Definition 4.5 *A transition t is said to be weakly enabled at M if there exists a firing sequence $\sigma_S \in T_S^*$: $\hat{M}[\sigma_S\rangle\hat{M}'$ and t is enabled at $\hat{M}'$.* △

Different from a directly enabled transition, a *weakly enabled* transition is not necessarily enabled at the current closed-loop marking but can be enabled after a proper firing sequence occurs in the switcher. From the definition it is obvious that a transition directly enabled at $\hat{M}$ is also weakly enabled, and in such a case σ_S is the empty string. Note that in the monitor-switcher controlled net, a transition is weakly enabled if and only if it is enabled after the firing of some single control transition t_{xy}, since the switcher can always unmark q'_x and mark q'_y in one step. But it is not always the case in more general types of controllers.

Figure 7 illustrates this situation. For better illustration, let us consider the reduced OR-AND GMEC, i.e., $|(\mathbf{W}_1,\mathbf{k}_1)| = |(\mathbf{W}_2,\mathbf{k}_2)| = 1$. The horizontal axes represent the token counts of $(\mathbf{w}_1,k_1) \in (\mathbf{W}_1,\mathbf{k}_1)$ and $(\mathbf{w}_2,k_2) \in (\mathbf{W}_2,\mathbf{k}_2)$ of the plant markings, and the vertical axis represents the token in the monitor place q'_1 of the monitor-switcher. In this illustration two markings M_{x,q_1} and M_{x,q_2} with the same projection share the same plant marking. Suppose that in the plant net without the controller, both $M_1[t\rangle M_2$ and $M_3[t'\rangle M_2$ are legal. After the monitor-switcher is added, $M_1[t\rangle M_2$ is not blocked: if the current marking is M_{1,q_1}, t is enabled; if the current marking is M_{1,q_2}, although t is not firable at M_{1,q_2}, after a proper evolution of the switcher, e.g., t_{21}, we can reach M_{1,q_1} at which t is enabled. Therefore t is weakly enabled at M_{1,q_2}. However, the monitor-switcher will block $M_3[t'\rangle M_2$, since M_3 only satisfies $(\mathbf{w}_2,k_2)$ and M_2 only satisfies $(\mathbf{w}_1,k_1)$. Thus, the possible full marking could only be M_{3,q_2}. As a result, both the firing of t' and the activation/deactivation of $(\mathbf{w}_1,k_1)/(\mathbf{w}_2,k_2)$ are not possible (two dashed arrows). Therefore t' is blocked at M_3 by the monitors. This is exactly the case happened in the example shown in Proposition 4.4. In the example the marking $M_a = [1,0,1,3,0]^T$ only satisfies $(\mathbf{W}_1,\mathbf{k}_1)$ (as M_3 in Figure 7) and $M_b = [1,0,2,2,0]^T$ only satisfies $(\mathbf{W}_2,\mathbf{k}_2)$ (as M_2 in Figure 7). Therefore neither t_{12} nor t_4 can fire under M_a, and consequently t_4 is not weakly enabled.

At the end of this section we point out that although the monitor-switcher that we synthesized up to now is not maximally permissive according to Definition 4.3, the evolution of the closed-loop system satisfies the condition: for any marking M_1 with $M_1[t\rangle M_2$, if M_1 satisfies $(\mathbf{W}_i, \mathbf{k}_i)$ then M_2 also satisfies $(\mathbf{W}_i, \mathbf{k}_i)$. In particular, if each $(\mathbf{W}_i, \mathbf{k}_i)$ contains only one single GMEC, the evolution of the closed-loop system would exactly satisfy the so-called *dynamic interpretation* of the disjunctive GMECs in [5]. Therefore the monitor-switcher controlled system in this paper can be considered satisfying the extended dynamic interpretation. However in this paper we only consider the optimality of the controller according to Definition 4.3. Since a maximally permissive closed-loop net is expected, the net $\langle \hat{N}, \hat{M}_0 \rangle$ has to be suitably modified, which is technically feasible as shown in the next section.

5 Modifying a Monitor-Switcher to Ensure Maximal Permissiveness

In the previous discussion we have seen that the legal firing represented by the dashed arc in Figure 7 is unnecessarily blocked. It is natural to find a way to create a shortcut for such blocking. In this section we will explore this approach to compile such a mechanism into the closed-loop Petri net. We first define the unnecessary blocking of a transition.

Definition 5.1 *A transition $t \in T$ is said to be unnecessarily blocked at $\hat{M}$ in a closed-loop net $\langle \hat{N}, \hat{M}_0 \rangle$ if $\hat{M}_{\uparrow P}[t\rangle_N M' \in \mathscr{L}$ and t is not weakly enabled at $\hat{M}$.* △

In the closed-loop net in Figure 6 we can see that t_1, t_2, t_3, t_5, t_6, and t_7 are not unnecessarily blocked under all closed-loop markings in $\langle \hat{N}, \hat{M}_0 \rangle$. However, t_4 is unnecessarily blocked since t_4 is not weakly enabled at all markings $\hat{M}$ with $\hat{M}_{\uparrow P} = [1,0,1,3,0]^T$.

Definition 5.2 *Given an OR-AND GMEC $W_{OA} = \{(\boldsymbol{W}_1, \boldsymbol{k}_1), \ldots, (\boldsymbol{W}_r, \boldsymbol{k}_r)\}$ and a plant marking M, the* satisfied GMEC index set *$S(M)$ is defined as:*

$$S(M) = \{i \in \{1, \ldots, r\} | \forall j : 1 \le j \le s_i, \boldsymbol{w}_{ij}^T \cdot M \le k_{ij}\}$$

△

Theorem 5.3 *In a closed-loop net $\hat{N}$ obtained by Algorithm 1 and corresponding W_{OA}, a transition t is unnecessarily blocked at the marking $\hat{M}$ with $\hat{M}_{\uparrow P} = M, M[t\rangle_N M'$ if and only if $S(M) \bigcap S(M') = \emptyset$, where $M, M' \in \mathbb{N}^{|P|}$.*

Proof: (If) Suppose that $S(M) \bigcap S(M') = \emptyset$. Since M is a legal marking satisfying some $(\mathbf{W}_i, \mathbf{k}_i)$'s, there exists an index $i \in S(M)$ with $\hat{M}(q_i') = 1$. Since $i \notin S(M')$, the firing of t would violate $(\mathbf{W}_i, \mathbf{k}_i)$. We conclude that there exists $j \in \{1,\ldots,s_i\}$ such that $\hat{M}(q_{ij}) < \hat{Pre}(q_{ij},t)$ is true. Since the index i is arbitrarily chosen, t is always blocked regardless the firing of $\sigma_S \in T_S$. Therefore t is not weakly enabled at $\hat{M}$.

(Only If) Suppose that $S(M)\bigcap S(M') \neq \emptyset$. We pick any index $x \in S(M)\bigcap S(M')$. If the unique marked place in the switcher is q'_x, t is not blocked (σ equals to the empty string) since q'_x does not restrict the firing of t and all the other $(\mathbf{W}_i,\mathbf{k}_i)$'s ($i \neq x$) are deactivated. If the unique marked place in the switcher is $q'_y \neq q'_x$, q'_x can be marked by firing t_{yx} such that t is weakly enabled. Therefore t is weakly enabled at $\hat{M}$ and is not unnecessarily blocked. ■

Note that in Theorem 5.3 we do not consider whether the marking M is reachable, since it is usually difficult to determine the reachability problem. However, an efficient trimming algorithm will be presented shortly without checking if such markings are reachable. Before this we use the following definitions to further characterize the situation of unnecessary blocking.

Definition 5.4 *The influence of a transition t on a GMEC $(\boldsymbol{w},k)$ is defined as $\eta_t(\boldsymbol{w}) = \boldsymbol{w}^T \cdot C(\cdot,t)$.* △

Definition 5.5 *Given an OR-AND GMEC $W_{OA} = \{(\boldsymbol{W}_1,\boldsymbol{k}_1),\ \ldots,\ (\boldsymbol{W}_r,\boldsymbol{k}_r)\}$, the increasable support of a transition t $\mathscr{Z}_t^+$ is defined as:*

$$\mathscr{Z}_t^+ = \{i|\exists j \in \{1,2,\ldots,s_i\}, \eta_t(w_{ij}) > 0\} \tag{8}$$

and the decreasable support of a transition t $\mathscr{Z}_t^-$ is defined as:

$$\mathscr{Z}_t^- = \{i|\exists j \in \{1,2,\ldots,s_i\}, \eta_t(w_{ij}) < 0\} \tag{9}$$

△

In short words, $i \in \mathscr{Z}_t^+$ means that the firing of t may increase the token count of some GMEC $(\mathbf{w}_{i\hat{j}},k_{i\hat{j}})$ in $(\mathbf{W}_i,\mathbf{k}_i)$, which implies the firing of t may make $(\mathbf{W}_i,\mathbf{k}_i)$ be violated. On the other hand, $i \in \mathscr{Z}_t^-$ means that the firing of t may decrease the token count of some GMEC $(\mathbf{w}_{i\hat{j}},k_{i\hat{j}})$ in $(\mathbf{W}_i,\mathbf{k}_i)$, which implies that if $(\mathbf{W}_i,\mathbf{k}_i)$ is violated, then the firing of t may make it be satisfied.

Definition 5.6 *A transition t_x is called a migrating transition if there exist two indices $i_1 \neq i_2$ such that $(i_1 \in \mathscr{Z}_{t_x}^+) \wedge (i_2 \in \mathscr{Z}_{t_x}^-)$.* △

Then we will show that the non-maximal permissiveness may be only due to the existence of migrating transitions.

Theorem 5.7 *If a transition t is unnecessarily blocked at the closed-loop marking $\hat{M}$ which corresponds to a plant marking $M = \hat{M}_{\uparrow P}$ with $M[t\rangle_N M'$, then t is a migrating transition with $S(M) \subseteq \mathscr{Z}_t^+$ and $S(M') \subseteq \mathscr{Z}_t^-$.*

Proof: Without loss of generality, let $S(M)$ and $S(M')$ be the index sets corresponding to the first r' and the following $(r'' - r')$ AND-GMECs, respectively. By Theorem 5.3, $S(M) \bigcap S(M')$ is empty. Hence M' necessarily violates all AND-GMECs in $S(M)$ while M necessarily violates all AND-GMECs in $S(M')$. As a result, for any $i \in S(M)$ there exists an index j $(1 \leq j \leq s_i)$ such that $\mathbf{w}_{ij}^T \cdot M' > k_{ij} > \mathbf{w}_{ij}^T \cdot M$, and for any $i \in S(M')$ there exists an index j $(1 \leq j \leq s_i)$ such that $\mathbf{w}_{ij}^T \cdot M' < k_{ij} < \mathbf{w}_{ij}^T \cdot M$. This implies $i \in \mathscr{Z}_t^+, i = 1, \ldots, r'$ and $i \in \mathscr{Z}_t^-, i = r'+1, \ldots, r''$. Therefore t is a migrating transition with $S(M) \subseteq \mathscr{Z}_t^+$ and $S(M') \subseteq \mathscr{Z}_t^-$. ■

Corollary 5.8 *If a migrating transition t_x is unnecessarily blocked at the closed-loop marking $\hat{M}$ which corresponds to a plant marking $M = \hat{M}_{\uparrow P}$ with $M[t_x\rangle M' \in \mathscr{L}$, then:*

$$\begin{cases} \forall i \in \mathscr{Z}_{t_x}^- \setminus \mathscr{Z}_{t_x}^+, \exists j, \boldsymbol{w}_{ij}^T \cdot M > k_{ij} \\ \forall i \in \mathscr{Z}_{t_x}^+ \setminus \mathscr{Z}_{t_x}^-, \exists j, \boldsymbol{w}_{ij}^T \cdot M' > k_{ij} \\ \forall i \in (\mathscr{Z}_{t_x}^+ \cap \mathscr{Z}_{t_x}^-), (\exists j_1, \boldsymbol{w}_{ij_1}^T \cdot M > k_{ij_1}) \\ \qquad\qquad\qquad\qquad \vee (\exists j_2, \boldsymbol{w}_{ij_2}^T \cdot M' > k_{ij_2}) \\ \forall i \notin (\mathscr{Z}_{t_x}^+ \cup \mathscr{Z}_{t_x}^-), \exists j, \boldsymbol{w}_{ij}^T \cdot M = \boldsymbol{w}_{ij}^T \cdot M' > k_{ij} \end{cases} \tag{10}$$

Proof: This result is straight forward from the conclusion of Theorems 5.3 and 5.7. If any condition above fails to hold, it will lead to a fact that $S(M) \cap S(M') \neq \emptyset$, which implies t_x is not unnecessarily blocked. This contradicts the statement that t_x is unnecessarily blocked at M. ■

From Theorems 5.3 and 5.7 we can clearly see that only migrating transitions should be treated to enhance permissiveness. Note that in some cases after the first stage of controller design, some of the migrating transitions are already maximally permissive, since those markings which satisfy the condition in Corollary 5.8 are actually not reachable in the origin plant net. However, to verify this there is no efficient method except to check the whole reachability graph. Therefore, a simpler way is to treat all migrating transitions by adding the so-called mirror transitions (their definition will be given shortly). If a migrating transition is already maximally permissive, the added transitions will not affect the evolution of the closed-loop system.

The following proposition shows under which condition a migrating transition should fire. It will be used to distinguish the unnecessary blocking and the normal blocking.

Proposition 5.9 *The firing of a migrating transition t does not lead to a violation of the OR-AND GMEC if the current plant net marking M satisfies the condition: $\exists i (1 \leq i \leq r), \forall j (1 \leq j \leq s_i), \boldsymbol{w}_{ij}^T \cdot M \leq k_{ij} - \eta_t(\boldsymbol{w}_{ij})$.*

Proof: Suppose that at the current marking M, $\exists i(1 \leq i \leq r), \forall j(1 \leq j \leq s_i), \mathbf{w}_{ij}^T \cdot M \leq k_{ij} - \eta_t(\mathbf{w}_{ij})$ holds. After the firing of t, the system will reach M'. Since $\eta_t(\mathbf{w}_{ij}) = \mathbf{w}_{ij}^T \cdot C(\cdot,t)$ and $M' = M + C(\cdot,t)$, then $\mathbf{w}_{ij}^T \cdot M' = \mathbf{w}_{ij}^T \cdot M + \eta_t(\mathbf{w}_{ij}) \leq k_{ij} - \eta_t(\mathbf{w}_{ij}) + \eta_t(\mathbf{w}_{ij}) = k_{ij}$. Therefore M' is a legal marking that satisfies at least $(\mathbf{W}_i, \mathbf{k}_i)$. ■

The migrating transitions in a Petri net can be easily found by incident matrix analysis. Once the potential non-maximally permissive migrating transitions are found, we propose to add a set of mirror transitions for each of these transitions thus constructing a new closed-loop net from $\langle \hat{N}, \hat{M}_0 \rangle$ obtained by Algorithm 1. Such a set of mirror transitions is described in the following definition.

Definition 5.10 *For a transition $t_x \in T$, its mirror transition set $\mathscr{T}(t_x)$ is the set of transitions constructed according to the following procedure:*

1. Let $\mathscr{T}(t_x) = \{t_x\}$.

2. For all $(i_1, i_2) \in \mathscr{Z}_{t_x}^+ \times \mathscr{Z}_{t_x}^-, i_1 \neq i_2$, add a transition $t_{x,i_1 \to i_2}$ to $\mathscr{T}(t_x)$ with:

$$\begin{aligned} &\forall p \in P \cup P_S \backslash \{q_{ij} | i = 1, \ldots, r, j = 1, \ldots, s_i\}, \\ &\quad \hat{Pre}(p, t_{x,i_1 \to i_2}) = \hat{Pre}(p, t_x) + \hat{Pre}(p, t_{i_1 i_2}), \\ &\quad \hat{Post}(p, t_{x,i_1 \to i_2}) = \hat{Post}(p, t_x) + \hat{Post}(p, t_{i_1 i_2}); \\ &\forall p \in \{q_{ij} | i = 1, \ldots, r, j = 1, \ldots, s_i\}, \\ &\quad \hat{Pre}(p, t_{x,i_1 \to i_2}) = \hat{Pre}(p, t_{i_1 i_2}) - \hat{Post}(p, t_x), \\ &\quad \hat{Post}(p, t_{x,i_1 \to i_2}) = \hat{Post}(p, t_{i_1 i_2}) - \hat{Pre}(p, t_x) \end{aligned} \tag{11}$$

3. Output $\mathscr{T}(t_x)$, end. △

Note that the definition of the mirror transition set can be applied to all transitions that are not necessarily migrating. However, for a non-migrating transition, its mirror transition set only contains itself such that we could actually ignore this procedure. By adding the mirror transition set for each migrating transition, from $\langle \hat{N}, \hat{M}_0 \rangle$ we obtain a new closed-loop net $\langle \tilde{N}, \tilde{M}_0 \rangle$, in which $\tilde{N} = (P \cup P_S, T \cup T_S \cup \mathscr{T}, \tilde{Pre}, \tilde{Post})$, $\mathscr{T} = \bigcup \mathscr{T}(t_x)$, t_x is a migrating transition, and $\tilde{Pre}$ and $\tilde{Post}$ denote the *pre* and *post* matrices of the modified net, respectively. The marking of $\langle \tilde{N}, \tilde{M}_0 \rangle$ is denoted as $\tilde{M}$. Since the places of $\langle \hat{N}, \hat{M}_0 \rangle$ and $\langle \tilde{N}, \tilde{M}_0 \rangle$ are the same, given a marking $\tilde{M}$ in $\tilde{N}$, the corresponding $\hat{M}$ in $\hat{N}$ are identical, i.e., $\tilde{M} = \hat{M}$. The GMEC $\hat{\mathbf{w}}$ and corresponding $\tilde{\mathbf{w}}$ are also identical, i.e., $\tilde{\mathbf{w}} = \hat{\mathbf{w}}$. From the definition of mirror transition set we have the following property.

Proposition 5.11 *For any mirror transition $t_{x,i_1 \to i_2}$ in $\mathscr{T}(t_x)$, we have:*

$$\tilde{C}(\cdot, t_{x,i_1 \to i_2}) = \hat{C}(\cdot, t_x) + \hat{C}(\cdot, t_{i_1 i_2}) \tag{12}$$

where $t_{i_1 i_2} \in T_S$.

From Definition 5.10 we now prove that the newly added mirror transition sets would enhance the maximal permissiveness of the net $\langle \hat{N}, \hat{M}_0 \rangle$.

Theorem 5.12 *If a transition t_x is unnecessarily blocked in $\hat{N}$ at the closed-loop marking $\hat{M}$ corresponding to a plant marking M with $M[t\rangle_N M'$, then in $\tilde{N}$ there necessarily exists a mirror transition $t_{x,i_1 \to i_2} \in \mathscr{T}(t_x)$ that is enabled at $\tilde{M}$. Moreover, the firing of a transition $t_{x,i_1 \to i_2} \in \mathscr{T}(t_x)$ in $\tilde{N}$ has the same impact on state evolution in the plant as the firing of sequence $\sigma = t_x t_{i_1 i_2}$.*

Proof: Suppose that the corresponding plant markings M and M' satisfy $(\mathbf{W}_{i_1}, \mathbf{k}_{i_1})$ and $(\mathbf{W}_{i_2}, \mathbf{k}_{i_2})$, respectively. To make t_x fire at $\tilde{M}$ we need to deactivate $(\mathbf{W}_{i_1}, \mathbf{k}_{i_1})$ and activate $(\mathbf{W}_{i_2}, \mathbf{k}_{i_2})$.

For any place $p \in \{q_{i_2 j} | 1 \leq j \leq s_{i_2}\}$, from Definition 5.10, the following equation holds:

$$\begin{aligned} \tilde{Pre}(q_{i_2 j}, t_{x,i_1 \to i_2}) &= \hat{Pre}(q_{i_2 j}, t_{i_1 i_2}) - \hat{Post}(q_{i_2 j}, t_x) \\ &= K - \eta_{t_x}(\tilde{\mathbf{w}}_{i_2 j}) \end{aligned} \tag{13}$$

Since $(\mathbf{W}_{i_2}, \mathbf{k}_{i_2})$ is not activated, for all j $(1 \leq j \leq s_{i_2})$ the following equation holds:

$$\tilde{M}(q_{i_2 j}) = k_{i_2 j} + K - \tilde{\mathbf{w}}_{i_2 j}^T \cdot \tilde{M} \tag{14}$$

According to Proposition 5.9, $\tilde{\mathbf{w}}_{i_2 j}^T \cdot \tilde{M} \leq k_{i_2 j} - \eta_{t_x}(\tilde{\mathbf{w}}_{i_2 j})$ holds. Therefore we have:

$$\tilde{M}(q_{i_2 j}) \geq K + \eta_t(\tilde{\mathbf{w}}_{i_2 j}) = \tilde{Pre}(q_{i_2 j}, t_{x,i_1 \to i_2}) \tag{15}$$

Therefore $t_{x,i_1 \to i_2}$ is not blocked by $q_{i_2 j}$ $(1 \leq j \leq s_i)$ at $\tilde{M}$. For the places $p \in P \cup P_S \backslash \{q_{i_2 j} | 1 \leq j \leq s_i\}$ which are the input places of $t_{x,i_1 \to i_2}$, $\tilde{M}(p) \geq \tilde{Pre}(p, t_{x,i_1 \to i_2})$ is obviously true. Therefore, $t_{x,i_1 \to i_2}$ is enabled at $\tilde{M}$.

By the condition in Eq. (12), it is clear that the firing of a transition $t_{x,i_1 \to i_2} \in \mathscr{T}(t_x)$ has the same impact on the state evolution of $\tilde{N}$ as the firing of sequence $\sigma = t_x t_{i_1 i_2}$ in $\hat{N}$. ■

Theorem 5.12 ensures that if in neither t_x nor $t_{i_1 i_2}$ is enabled but $\hat{M} + C_{\hat{N}} \sigma \geq \mathbf{0}$ in $\tilde{N}$, we can fire a mirror transition $t_{x,i_1 \to i_2}$ instead of firing t_x and $t_{i_1 i_2}$ sequentially, where $\sigma = t_x t_{i_1 i_2}$.

Since the firing of a transition $t_{x,i_1 \to i_2} \in \mathscr{T}(t_x)$ has the same impact on state evolution in the plant as the transition sequence $\sigma = t_x t_{i_1 i_2}$ in $\tilde{N}$, the definition of maximal permissiveness given in Definition 4.3 should be extended for the net $\langle \tilde{N}, \tilde{M}_0 \rangle$ as follows.

Definition 5.13 *A closed-loop net $\langle \tilde{N}, \tilde{M}_0 \rangle$ (with respect to $\langle N, M_0 \rangle$) is said to be maximally permissive if $\forall \tilde{M} \in R(\tilde{N}, \tilde{M}_0)$ the following condition holds:*

$$(\tilde{M}_{\uparrow P}[t\rangle_N M' \in \mathscr{L}) \Longrightarrow (\exists \sigma_S \in T_S^*, \exists \bar{t} \in \mathscr{T}(t) : \tilde{M}[\sigma_S \bar{t}\rangle_{\tilde{N}}) \tag{16}$$

△

Then we present an algorithm to construct the modified net.

Algorithm 2 ***Modified closed-loop net design***

Input: *A closed-loop net $\langle \hat{N}, \hat{M}_0 \rangle$ for W_{OA} obtained by Algorithm 1*

Output: *A modified closed-loop net $\langle \tilde{N}, \tilde{M}_0 \rangle$ that is maximally permissive*

Step 1: *Let $\tilde{N} = \hat{N}, \tilde{M}_0 = \hat{M}_0$.*

Step 2: *Let T_{mig} be the set of migrating transitions.*

Step 3: *Pick a transition t_x from T_{mig}. Construct its mirror transition set $\mathscr{T}(t_x)$. Let $\tilde{T} = \tilde{T} \cup \mathscr{T}(t_x)$.*

Step 4: *Remove t_x from T_{mig}.*

Step 5: *If $T_{mig} \neq \emptyset$ goto Step 2.*

Step 6: *Output the closed-loop net $\langle \tilde{N}, \tilde{M}_0 \rangle$.* □

Algorithm 2 can be illustrated in the following way. First we compute all migrating transitions which will be treated later. For each migrating transition t_x, according to Theorem 5.3, the only condition under which t_x is unnecessarily blocked at a marking $\hat{M}$, which corresponds to a plant marking $M = \hat{M}_{\uparrow P}$, is that the firing of t_x will make some satisfied $(\mathbf{W}_{i_1}, \mathbf{k}_{i_1}), i_1 \in \mathscr{Z}_{t_x}^+$, be violated while make some violated $(\mathbf{W}_{i_2}, \mathbf{k}_{i_2}), i_2 \in \mathscr{Z}_{t_x}^- (i_1 \neq i_2)$, be satisfied. Hence, before the firing of t_x, the unique token in the switcher must be in q'_{i_1}. Since the firing of t_x is legal, $\hat{M} + C_{\hat{N}}\mathbf{y} \geq 0$ necessarily holds, where $\mathbf{y}$ is the count vector of $\sigma = t_x t_{i_1 i_2}$. Therefore a mirror transition $t_{x, i_1 \to i_2}$ is added. The firing of $t_{x, i_1 \to i_2}$ implies the firing of t_x, the deactivation of $(\mathbf{W}_{i_1}, \mathbf{k}_{i_1})$, and the activation of $(\mathbf{W}_{i_2}, \mathbf{k}_{i_2})$ simultaneously. According to Definition 5.13 we can state the following result.

Theorem 5.14 *The closed loop-net $\langle \tilde{N}, \tilde{M}_0 \rangle$ by Algorithm 2 is maximally permissive with respect to $\langle N, M_0 \rangle$.*

Proof: According to Theorem 5.7, the non-maximal permissiveness may only arise due to migrating transitions. Suppose that a migrating transition t_x at the closed-loop marking $\hat{M}$ that corresponds to the plant marking M with $M[t\rangle_N M'$ is unnecessarily blocked by the monitors in $\hat{N}$. Since M satisfies $(\mathbf{W}_{i_1}, \mathbf{k}_{i_1}), i_1 \in \mathscr{Z}_{t_x}^+$, and M' satisfies $(\mathbf{W}_{i_2}, \mathbf{k}_{i_2}), i_2 \in \mathscr{Z}_{t_x}^-, i_1 \neq i_2$, according to Step 3 in Algorithm 2, a mirror transition of

$t_{x,i_1\to i_2}$ is added and enabled at $\tilde{M}$ due to Theorem 5.12. Since the only possibility that reduces the maximal permissiveness is eliminated, the closed-loop net $\langle \tilde{N}, \tilde{M}_0\rangle$ is maximally permissive. ■

Let us review Figure 7 in Section 4. Since t' is unnecessarily blocked at M_{3,q_2}, by applying Algorithm 2, a mirror transition $t'_{2\to 1}$ is added for t' while the firing of $t'_{2\to 1}$ has exactly the same influence as that of t' and t_{21} simultaneously. Transition $t'_{2\to 1}$ is enabled at M_{3,q_2} even if neither t' nor t_{21} is enabled.

Example 5.15 *In the net in Figure 6 without the dashed box, t_4 is the only migrating transition. Since $\mathscr{Z}^+_{t_4} = \{1\}$ and $\mathscr{Z}^-_{t_4} = \{2\}$, the firing of t_4 may activate $(\mathbf{W}_2,\boldsymbol{k}_2)$ while concurrently deactivate $(\mathbf{W}_1,\boldsymbol{k}_1)$. $|\mathscr{Z}^+_{t_4} \setminus \mathscr{Z}^-_{t_4}| = |\mathscr{Z}^-_{t_4} \setminus \mathscr{Z}^+_{t_4}| = 1$ indicates that only one additional transition should be added to $\mathscr{T}(t_4)$. Therefore by applying Algorithm 2, one transition $t_{4,1\to 2}$ is added with its incident matrix $\tilde{Pre}(\cdot,t_{4,1\to 2})$ and $\tilde{Post}(\cdot,t_{4,1\to 2})$ computed according to Definition 5.10. The resulting net $\langle \tilde{N}, \tilde{M}_0\rangle$ is shown in Figure 6 containing $t_{4,1\to 2}$ in the dashed box. This net is maximally permissive.* △

At the end of this section we point out that this approach can be extended to the more general cases where any legal markings must satisfy at least $\hat{r} \geq 1$ AND-GMECs. The two-stage algorithm in this paper can also be used after some modifications. However, in such a problem the complexity of the legal marking set is typically very high. To obtain a maximally permissive closed-loop net, for each migrating transition it is more complex to construct its mirror transition set. In the worst case it may grow exponentially with the increase of $\hat{r}$. To keep this paper simple we would not discuss this in detail.

6 Complexity Analysis and Comparison

In this section we first discuss the complexity of the two proposed algorithms. To convert an OR-AND GMEC $W_{OA} = \{(\mathbf{W}_1,\mathbf{k}_1),\ldots,(\mathbf{W}_r,\mathbf{k}_r)\}$ where $\mathbf{W}_i \in \mathbb{Z}^{m\times s_i}$ and $\mathbf{k}_i \in \mathbb{N}^{s_i}$, Algorithm 1 adds r places and $r\times(r-1)$ transitions as the switcher, in addition to $\sum_{i=1}^{r} s_i$ classical GMEC monitor places. By Algorithm 2, for each migrating transition t_x, $|\mathscr{Z}^+_{t_x} \setminus \mathscr{Z}^-_{t_x}| \times |\mathscr{Z}^-_{t_x} \setminus \mathscr{Z}^+_{t_x}|$ mirror transitions are added, which in the worst case means that $(r-1)\times(r-1)$ mirror transitions will be added. Assuming that all transitions in the initial plant net are migrating transitions, the total number of places and transitions added in the modified monitor-switcher structure is of order $O(r)$ and $O(nr^2)$, respectively, where n is the number of transitions in the plant net. Considering that the complexity is linear with respect to the size of the net (number of transitions) and quadratic with respect to the number of AND-GMECs in W_{OA}, while the enumeration of the reachability set is not required, we believe this approach is more efficient than the existing ones.

Let us briefly compare in terms of structural complexity the controllers obtained by our approach and by Iordache's approach [5, 16]. Both approaches consider a control structure that contains monitor places that enforce each AND-GMEC. However, they differ in the way of recognizing which AND-GMECs are violated during the evolution of the system.

In Iordache's approach, the controller precisely keeps track of the violation information, i.e., at each marking which AND-GMECs are violated and which are satisfied. To precisely keep track of this information, the weights of some control arcs must be calculated according to the upper and lower bounds of its corresponding GMECs. This mechanism prevents its token count from going to positive infinity or negative infinity. In our approach, however, we only need to keep track of one AND-GMEC (among possibly many) that is satisfied by marking the corresponding switcher place by a unique token. Since we do not need the entire satisfiction/violation information, only a constant K corresponding to the largest higher bound is introduced. Therefore the lower bound requirement could be removed and our approach is more general, e.g., it can handle the control problem in Example 3.5 while Iordache's approach cannot do so.

On the other hand, by loosing the necessity of keeping track of all violation information requirement, the structural complexity of the controller is also reduced. In Iordache's approach, for each transition t all the AND-GMECs which may be influenced by t (i.e., $\exists j, \mathbf{w}_{ij}^T \cdot C(\cdot,t) \neq 0$) must be considered. Therefore for a transition t which may influence r AND-GMECs, $2^r - 1$ duplicated transitions must be added. In our approach, however, since the switcher only need to specify one of the satisfied AND-GMECs and activate it, the structural complexity (mainly due to the mirror transitions) is quadratic with respect to r in the worst case, as discussed above. This greatly reduces the number of the newly added transitions. For example, suppose we want to enforce an OR-AND GMEC with $r = 10$ AND-GMECs and there is a migrating transition t_x which influences all the ten disjunctions. By Iordache's approach $2^{10} - 1 = 1023$ duplicated transitions will be introduced for t_x, while only $(10-1)^2 = 81$ mirror transitions will be introduced by our method in the worst case. Furthermore, only migrating transitions have to be treated in our approach, which would further reduce the structural complexity of the resulting controller.

7 Conclusion

This paper considered the OR-AND GMECs, i.e., a disjunction of conjunctions of single GMECs. We proposed a method to transform a bounded OR-AND GMEC into a Petri net control structure with maximal permissiveness. The obtained closed-loop net has a low structural complexity that is quadratic in the number of disjunctions in the OR-AND GMEC. This approach would be a supplement and extension to the classical GMEC approach and provides a framework that can be widely used in different systems where the legal marking set is not convex. It could be extended to the cases where the system must satisfy at least more than one constraints.

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Appendix

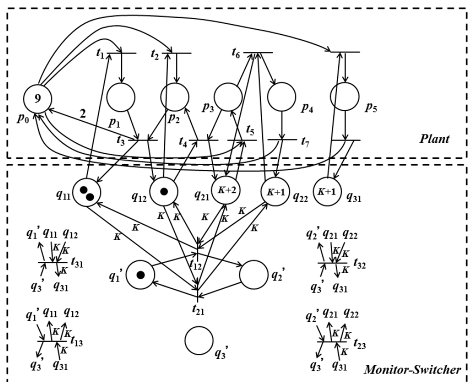

Figure 8: Another example to illustrate Algorithm 1 with $W_{OA} = \{(\mathbf{W}_1, \mathbf{k}_1), (\mathbf{W}_2, \mathbf{k}_2), (\mathbf{W}_3, \mathbf{k}_3)\}$ where two single GMECs $(\mathbf{w}_{11}, k_{11}) = ([0,1,0,0,0,0]^T, 2)$, $(\mathbf{w}_{12}, k_{12}) = ([0,0,1,0,0,0]^T, 1)$ are in $(\mathbf{W}_1, \mathbf{k}_1)$, two single GMECs $(\mathbf{w}_{21}, k_{21}) = ([0,0,0,1,0,0]^T, 2)$, $(\mathbf{w}_{22}, k_{22}) = ([0,0,0,0,1,0]^T, 1)$ are in $(\mathbf{W}_2, \mathbf{k}_2)$, and one single GMEC $(\mathbf{w}_{31}, k_{31}) = ([0,0,0,0,0,1]^T, 1)$ is in $(\mathbf{W}_3, \mathbf{k}_3)$.

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