Optimal Petri Net Monitor Design

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Abstract

The classical partition of the event set into controllable and uncontrollable events from supervisory control theory is replaced by introducing the concept of control and observation cost of an event. This leads naturally to consider an optimal control problem for a given logical control specification. On the other hand, if we consider a timed model a performance optimization may be considered as well.

Here the case of generalized mutual exclusion constraint is considered for a Petri net plant. It has been shown that a constraint of this kind may be enforced via a monitor place. In this paper we propose an integer programming approach to synthesize the optimal monitor so as to minimize a given cost that represent a trade-off between the controller cost and cycle time of the closed loop net.

Keywords: Supervisory control, Petri nets, monitor places.

Published as:
F. Basile, P. Chiacchio, A. Giua, "Optimal Petri Net Monitor Design," in *Synthesis and Control of Discrete Event Systems*, B. Caillaud, X. Xie, Ph. Darondeau and L. Lavagno (Eds.), pp. 141-154, Kluwer, 2001.

1 Introduction

Supervisory control theory for discrete event systems (DESs) was initiated by [Ramadge and Wonham, 1989]. In their seminal work they represent both the plant — i.e., the system to be controlled — and the desired closed-loop behaviour, by regular languages. The specific problem addressed was to synthesize a controller, called *supervisor*, to achieve the largest subset of the desired language, disabling or enabling controllable events. The unwanted sequences may be related, for example, to safety requirements. Although regular languages have been an useful framework to start such DES control theory, they are limited in representing systems consisting of numerous interacting subsystems. For this reason, a control theory for DES modeled by Petri Net (PN) has been developed, extending general PN models with the concept of *controllable* transitions.

In the supervisory control PN theory it is assumed that the set of transitions T of a net is partitioned into two disjoint subsets: T_{uc}, the set of uncontrollable transitions, and T_c, the set of controllable transitions. Similarly T may also be partitioned into the set T_{uo} of unobservable transitions, and the set T_o of observable transitions. A controllable transition may be disabled by the supervisor, a controlling agent which ensures that the behaviour of the system be within a legal behaviour. When the controller is modeled by a PN structure, the disabling of transition t is possible if there is a pre-arc from a controller place to t. An uncontrollable transition represents an event which may not be prevented from occurring by a supervisor and thus we require that no arc goes from a controller place to it. Dually, when the controller is modeled by a PN structure, the controller observes a transition t only if the firing of t changes the marking of a controller place p. This happens only if the number of pre-arcs from p to t is different from the number of post-arcs from t to p. To rule out this possibility we will require neither a pre-arc nor a post-arc may exist between a controller place and an unobservable transition (in the monitor control structure we consider self-loops are not allowed).

Here we consider the problem of forbidden state specification represented by *generalized mutual exclusion constraint* (GMEC) of the form $(\boldsymbol{l}, k)$. Such a constraint limits the weighted sum of tokens in a subset of places (see [Giua et al., 1992], [Li and Wonham, 1994], [Moody et al., 1996b], [Krogh and Holloway, 1991]): the set of legal plant markings is $\{\boldsymbol{m} \mid \boldsymbol{l} \cdot \boldsymbol{m} \leq k\}$. It was shown in [Giua et al., 1992] and [Moody et al., 1996b] that it is possible to impose a GMEC by adding to a net a controller that takes the form of a single place called *monitor* with arcs going to and coming from the plant transitions. The monitor synthesis is very efficient from the computational point of view and it represents a compiled supervisor.

When the monitor has arcs going to uncontrollable (going to or coming from unobservable) transitions we say that the monitor and the corresponding GMEC are uncontrollable (unobservable). It has been shown [Moody et al., 1996a] that given a constraint $(\boldsymbol{l}, k)$, any constraint $(\boldsymbol{l}', k')$ where $\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l}$ — the elements of vector $\boldsymbol{r}_1$ and scalar r_2 are non negative integers — and k' is suitably chosen, is more restrictive than $(\boldsymbol{l}, k)$, i.e., $\{\boldsymbol{m} \mid \boldsymbol{l}' \cdot \boldsymbol{m} \leq k'\} \subseteq \{\boldsymbol{m} \mid \boldsymbol{l} \cdot \boldsymbol{m} \leq k\}$. Thus if $(\boldsymbol{l}, k)$ is not controllable (or not observable) we may look for a more restrictive but controllable and observable GMEC. Note that as the number of nonzero elements $\boldsymbol{r}_1$ increases the constraint becomes more restrictive.

In [Basile et al., 2000] we have considered a generalization of this approach in which two function $\boldsymbol{z}_c : T \to \mathbb{R}^+$ and $\boldsymbol{z}_o : T \to \mathbb{R}^+$ associate to each transition t its control and observation cost, respectively. As a particular case, if the cost functions only take value in the binary set $\{0, \infty\}$ we go back to the controllable/uncontrollable and observable/unobservable case. The problem in [Basile et al., 2000] was the following: given a GMEC $(\boldsymbol{l}, k)$, we want to find, among all monitors that enforce the constraint the one that has minimal cost. The set of the all monitors that enforce this constraint is clearly the set of all monitors corresponding to GMECs that are more restrictive than $(\boldsymbol{l}, k)$, and that can be written using Moody's parameterization. The cost corresponding to a monitor p_s is given by the sum over t of $\boldsymbol{c}^-(p_s,t)\boldsymbol{z}_c(t) + \boldsymbol{c}^+(p_s,t)\boldsymbol{z}_o(t)$, where $\boldsymbol{c}^-(p_s,t)$ counts the arcs from p_s to t and $\boldsymbol{c}^+(p_s,t)$ counts the arcs from t to p_s. This problem can be easily framed as a integer-linear programming problem.

In this paper we add a deterministic firing delay to each transition. In this framework a natural criterion for selecting the best among all monitors that satisfy Moody's parameterization may be the following: the optimal monitor is the one that minimizes the cycle time of the net, assuming a periodic execution of the net exists. We use the structural results of [Campos et al., 1992] to compute, solving an integer-linear programming problem, the monitor that minimizes a lower bound on the cycle time (for restricted classes of nets the actual cycle time is minimized).

Finally, we combine controller cost and cycle time cost into a single objective function. This is an interesting feature that allows us to use a single formalism, to solve at the same time logical problems (forbidden state avoidance) with performance criteria on the structure of the controller (the control and observation costs) and timing criteria on the closed loop system behaviour (the cycle time minimization).

2 Background

2.1 Petri nets

A place/transition (P/T) net is a structure $N = \langle P, T, \mathbf{Pre}, \mathbf{Post}\rangle$ where: P is a set of m *places* represented by circles; T is a set of n *transitions* represented by bars; $P \cap T = \emptyset$, $P \cup T \neq \emptyset$; $\mathbf{Pre}$ ($\mathbf{Post}$) is the $\mid P \mid \times \mid T \mid$ sized, natural valued, pre-(post-)incidence matrix. For instance, $\mathbf{Pre}(p,t) = w$ ($\mathbf{Post}(p,t) = w$) means that there is an arc from $p(t)$ to $t(p)$ with weight w. The incidence matrix $\boldsymbol{C}$ of the net is defined as $\boldsymbol{C} = \mathbf{Post} - \mathbf{Pre}$. For pre- and post-sets we use the conventional dot notation, e.g. ${}^\bullet t = \{p \in P \mid \mathbf{Pre}(p,t) \neq 0\}$. A pair $(p,t) \in P \times T$ is called a self-loop if p is both an input and output place of transition t. A *marking* is a $m \times 1$ vector $\boldsymbol{m} : P \rightarrow \mathbb{N}$ that assigns to each place of a P/T net a non-negative integer number of tokens. A P/T system or net system $\langle N, \boldsymbol{m}_0\rangle$ is a P/T net N with an initial marking $\boldsymbol{m}_0$. A transition $t \in T$ is enabled at a marking $\boldsymbol{m}$ iff $\boldsymbol{m} \geq \mathbf{Pre}(\cdot,t)$. If t is enabled, then it may fire yielding a new marking $\boldsymbol{m'} = \boldsymbol{m} + \mathbf{Post}(\cdot,t) - \mathbf{Pre}(\cdot,t) = \boldsymbol{m} + \mathbf{C}(\cdot,t)$. The notation $\boldsymbol{m}[t > \boldsymbol{m'}$ will mean that an enabled transition t may fire at $\boldsymbol{m}$ yielding $\boldsymbol{m'}$. A *firing sequence* from $\boldsymbol{m}_0$ is a (possibly empty) sequence of transitions $\sigma = t_1 ... t_k$ such that $\boldsymbol{m}_0[t_1 > \boldsymbol{m}_1[t_2 > \boldsymbol{m}_2..[t_k > \boldsymbol{m}_k$. A marking $\boldsymbol{m}$ is reachable in $\langle N, \boldsymbol{m}_0\rangle$ iff there exists a firing sequence σ such that $\boldsymbol{m}_0[\sigma > \boldsymbol{m}$. Given a net system $\langle N, \boldsymbol{m}_0\rangle$ the set of reachable markings is denoted $R(N, \boldsymbol{m}_0)$. The function $\boldsymbol{\sigma} : T \rightarrow \mathbb{N}$, where $\boldsymbol{\sigma}(t)$ represents the number of occurrences of t in σ, is called firing count vector of the fireable sequence σ. If $\boldsymbol{m}_0[\sigma > \boldsymbol{m}$, then we can write in vector form $\boldsymbol{m} = \boldsymbol{m}_0 + \boldsymbol{C}(\cdot,t) \cdot \boldsymbol{\sigma}$. This is known as the *state equation* of the system. Left annuller integer vectors of $\boldsymbol{C}$ are called P-semiflow, i.e. $\boldsymbol{y} : P \rightarrow \mathbb{N}, \boldsymbol{y} \neq \mathbf{0}$ such that $\boldsymbol{y}^T\boldsymbol{C} = \mathbf{0}$. Right annuller integer vectors of $\boldsymbol{C}$ are called T-semiflow, i.e. $\boldsymbol{x} : T \rightarrow \mathbb{N}, \boldsymbol{x} \neq \mathbf{0}$ such that $\boldsymbol{C}\boldsymbol{x} = \mathbf{0}$.

In this paper we consider a time delay associated with transitions. We call *deterministic time net* such model. In deterministic timed PN [Murata, 1989] we suppose that there is a delay of at least d_i units of time associated with the firing of transition t_i, $i = 1..n$. This means that when t_i is enabled, a number of $\mathbf{Pre}(p_j, t_i)$

tokens will be reserved in the place p_j for at least d_i units of time before their removal by firing t_i.

2.2 Generalized Mutual Exclusion Constraint

Assume we are given a set of legal markings $\mathcal{L} \subseteq \mathbb{N}^m$, and consider the basic control problem of designing a supervisor that restricts the reachability set of plant in closed loop to $\mathcal{L} \cap R(N, \boldsymbol{m}_0)$. Of particular interest are those PN state-based control problems where the set of legal markings $\mathcal{L}$ is expressed by a set of n_c linear inequality constraints called Generalized Mutual Exclusion Constraint (GMEC). A single GMEC is a couple $(\boldsymbol{l}, k)$ where $\boldsymbol{l} : P \to \mathbb{Z}$ is a $1 \times m$ weight vector and $k \in \mathbb{Z}$. The support of $\boldsymbol{l}$ is the set $Q_l = \{p \in P \mid \boldsymbol{l}(p) \neq 0\}$. Given the net system $\langle N, \boldsymbol{m}_0 \rangle$, a GMEC defines a set of markings that will be called *legal markings*: $\mathcal{M}(\boldsymbol{l}, \boldsymbol{k}) = \{\boldsymbol{m} \in \mathbb{N}^m \mid \boldsymbol{lm} \leq k\}$. The markings that are not legal are called *forbidden markings*. A controlling agent, called supervisor, must ensure the forbidden markings will be not reached. So the set of legal markings under control is $\mathcal{M}_c(\boldsymbol{l}, k) = \mathcal{M}(\boldsymbol{l}, k) \cap R(N, \boldsymbol{m}_0)$.

3 Monitor approach

If all transitions of a net are controllable and observable, it has been shown [Giua et al., 1992] that the Petri net controller that enforces $(\boldsymbol{l}, k)$ has the incidence matrix $\boldsymbol{c}_c \in \mathbb{Z}^{1\times n}$ given by

$$\boldsymbol{c}_c = -\boldsymbol{l}\boldsymbol{C}_p \tag{1}$$

where $\boldsymbol{C}_p$ is the incidence matrix of the plant and the initial marking of the controller $m_{c0} \in \mathbb{N}$ is given by

$$m_{c0} = k - \boldsymbol{l}\boldsymbol{m}_{p0} \tag{2}$$

where $\boldsymbol{m}_{p0} \in \mathbb{N}^{m\times 1}$ is the initial marking of the plant. The controller exists iff the initial marking is a legal marking, i.e.

$$k - \boldsymbol{l}\boldsymbol{m}_{p0} \geq \mathbf{0}. \tag{3}$$

Note that when an element of $\boldsymbol{c}_c$ is zero, there are no arcs at all connecting the given place and transition, i.e. there are no cancelling self-loop in the net controller structure. Thus, if we decompose $\boldsymbol{c}_c$ as follows

$$\boldsymbol{c}_c = \boldsymbol{c}_c^+ - \boldsymbol{c}_c^- \tag{4}$$

where $\boldsymbol{c}_c^+$ is obtained from $\boldsymbol{c}_c$ replacing each negative element with zero, while $\boldsymbol{c}_c^-$ is obtained from $\boldsymbol{c}_c$ replacing each positive element with zero and each negative element with its absolute value, we can say that $\boldsymbol{c}_c^+$ ($\boldsymbol{c}_c^-$) is the post-(pre-)incidence matrix of the monitor based control net.

The controller so constructed is maximally permissive, i.e. it prevents only transitions firings that yield forbidden markings. The control net has only one control place; no transition is added. Such control place is called *monitor place*. It is connected to the plant transitions as specified by the incidence matrix $\boldsymbol{c}_c$.

It has been showed by Moody et al. [Moody et al., 1996a] that it is possible to transform a control specification GMEC $(\boldsymbol{l}, k)$ into a more restrictive GMEC $(\boldsymbol{l}', k')$ as shown in the following proposition.

Proposition 1 ([Moody et al., 1996a]) *If we are able to find* $\boldsymbol{r}_1 \in \mathbb{N}^{1\times m}$, $r_2 \in \mathbb{N}$ *satisfying*

$$\begin{bmatrix} \boldsymbol{r}_1 & r_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_{p0} \\ \boldsymbol{l}\boldsymbol{m}_{p0} - (k+1) \end{bmatrix} \leq -1 \tag{5}$$

then the controller computed as

$$\boldsymbol{c}_c = -\boldsymbol{l}'\boldsymbol{C}_p \tag{6}$$

$$m_{c0} = k' - \boldsymbol{l}'\boldsymbol{m}_{p0} \tag{7}$$

where

$$\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l} \tag{8}$$

$$k' = r_2(k+1) - 1. \tag{9}$$

will be able to ensure that the closed-loop net system meet $\boldsymbol{l}\boldsymbol{m}_p \leq k$, and that the initial marking is a legal marking.

As consequence of proposition 1 we have that we can preserve the original constraint and a very efficient computation method for the controller (a simple matrices multiplication, as shown in (1)); at same time a number of freedom degrees represented by $\boldsymbol{r}_1$ and r_2 elements may be used to impose additional constraints. As shown in the following section here we want to use these freedom degrees to minimize the sum of the control and observation cost.

In the following the net formed by the plant net and a monitor will be called *closed loop net*.

4 Optimal monitor design

Adding time to transitions a possible criterion to select the suboptimal monitor could be the optimization of the cycle time of the closed loop net. We are interested in finding how fast each transition can initiate firing in a periodically operated timed Petri net, where a period Γ is defined as the time to complete a firing sequence leading back to the initial marking after firing each transition at least once. Γ is called *cycle time* (CT) of the net system. Thus, we are assuming that the net is consistent, i.e. there exists a T-semiflow containing all the transitions, that is a vector $\boldsymbol{x} : T \to \mathbb{N}$ such that $\boldsymbol{x} > \boldsymbol{0}$ and $\boldsymbol{C}\boldsymbol{x} = \boldsymbol{0}$.

Let us introduce the limit firing count vector per time unit $\overline{\boldsymbol{\sigma}} = \lim_{t\to\infty} \boldsymbol{\sigma}(t)/t$, and the mean time between two consecutive firings of a selected transition t_i, (mean cycle time of t_i), $\Gamma_i = 1/\overline{\boldsymbol{\sigma}}(t_i)$. In the case of strongly connected timed MGs (these nets have as unique minimal T-semiflow a vector of 1's) we have that $\Gamma_i = \Gamma^*$, $\forall i$, and the lower bound of the cycle time, that we denote by Γ_{min}, can be computed solving the following LPP [Campos et al., 1992]:

$$\Gamma_{min} = max_{\boldsymbol{y}} \quad \boldsymbol{y}^T \cdot \mathbf{Pre} \cdot \boldsymbol{d}$$
$$s.t. \begin{cases} \boldsymbol{y}^T \cdot \boldsymbol{C} = \boldsymbol{0} \\ \boldsymbol{y}^T \cdot \boldsymbol{m}_0 = 1 \\ \boldsymbol{y} \geq \boldsymbol{0} \end{cases} \tag{10}$$

where $\boldsymbol{d}(t_i)$ is the *time delay* of transition t_i. We recall that in the case of strongly connected MGs each minimal P-semiflow subsystem corresponds to an elementary circuit. In this system $\boldsymbol{y}$ is a P-semiflow, thus $\mathbf{Pre}^T\boldsymbol{y}$ is the characteristic vector (but for a scalar factor) of the transitions along the circuit and, finally, $\boldsymbol{y}^T \cdot \mathbf{Pre} \cdot \boldsymbol{d}$ is the sum of the time delay of all transitions along the circuit. Thus, an interpretation of the system (10) is that the cycle time can be computed looking at the slowest subsystem generated by the P-semiflows [Campos and Silva, 1992], considered in isolation with respect to delay nodes, where the CT of each subsystem can be computed making the summation of the time delays of all the transitions involved in it, and dividing by the tokens present in it. For a deterministic timed net system we have that $\Gamma = \Gamma_{min}$ when a transition fires as soon as it is enabled (earliest firing policy).

Let us introduce a class of nets with a unique consistent firing vector.

Definition 1 *A structurally bounded (i.e. bounded for any initial marking) net N is called mono-T-semiflow iff there is there exists a unique minimal T-semiflow that contains all transitions.*

For the class of mono-T-semiflow nets, we speak of mean cycle time of a certain transition because in order to complete a net system cycle each transition has to fire a different number of times. It was shown in [Campos et al., 1991] that for these nets, if $\boldsymbol{x}$ is the unique T-semiflow, then a lower bound of the cycle time of a transition t_i can be computed via the LPP (10) changing the objective function to $\boldsymbol{y}^T \cdot \mathbf{Pre} \cdot \boldsymbol{D}$, where for all $j = 1 \dots n$: $\boldsymbol{D}(t_j) = \boldsymbol{d}(t_j)\boldsymbol{x}(t_j)$. The mean cycle time of a transition t_i satisfies $\Gamma_i \geq max_{\boldsymbol{y}} \quad \boldsymbol{y}^T \cdot \mathbf{Pre} \cdot \boldsymbol{D}/\boldsymbol{x}(t_i)$. In this case the lower bound may not be reached under any firing policy.

In the following we only consider the case of open loop nets that are marked graphs. The addition of monitor to a MG — unless the monitor has only one input and output arc — leads to a closed loop net that is not a MG any more. However, the next proposition shows that the mono T-semiflow property is preserved.

Proposition 2 *Consider a PN where a monitor corresponding to the GMEC $(\boldsymbol{l}, k)$ has been added. The vector $\boldsymbol{x}$ is a T-semiflow of the plant net if and only if it is a T-semiflow of the closed loop net.*

Proof: The incidence matrix of the closed loop net is $\boldsymbol{C} = [\, \boldsymbol{C}_p \quad -\boldsymbol{l}\boldsymbol{C}_p \,]^T$, where $\boldsymbol{C}_p$ is the incidence matrix of the plant net. We have that $\boldsymbol{C}\boldsymbol{x} = \boldsymbol{0} \Longleftrightarrow \boldsymbol{C}_p\boldsymbol{x} = \boldsymbol{0} \bigwedge \boldsymbol{l}\boldsymbol{C}_p\boldsymbol{x} = \boldsymbol{0}$. Thus the "if" part is immediate. The "only if" follows by the fact that $\boldsymbol{C}_p\boldsymbol{x} = \boldsymbol{0} \Longrightarrow \boldsymbol{l}\boldsymbol{C}_p\boldsymbol{x} = \boldsymbol{0}$. ■

Corollary 1 *A strongly connected MG with the addition of monitors is a mono-T-semiflow net whose minimal T-semiflow is $\boldsymbol{x} = \mathbf{1}$.*

Proof: A strongly connected MG is structurally bounded and its unique minimal T-semiflow is $\boldsymbol{x} = \mathbf{1}$. The addition of monitors does not change these properties. ■

In [Moody et al., 1996b] it was shown that, when we have a monitor based controller, the closed loop net has all the P-invariants of the plant net and in addition only the invariant $[\, \boldsymbol{l} \quad 1 \,]$. Thus, if we optimize the choice of each transformed constraint so that the associated net subsystem results the faster in its family, we are optimizing the cycle time of the closed loop net by minimizing its lower bound because of (10) where the proper substitution have been made as explained above. On the other hand, optimizing the cycle time (or its lower bound in the general case) may require a big control cost. Thus, we consider the possibility of imposing a trade-off between cost of control and cycle time optimization. We remark that when the cycle time of the net system has no sense, we mean the cycle time of each transition and we note that if we optimize the cycle time of a transition t_i, also the cycle time of other transitions is optimized being scaled by a constant factor each other.

Proposition 3 *Let us consider the problem to impose a GMEC constraint $(\boldsymbol{l}, k)$ on the timed PN system $\langle N, \boldsymbol{m}_0 \rangle$ with uncontrollable transition set T_u and let $\boldsymbol{d}(t)$ be the time duration of the activity associated to* *transition t. The monitor that optimizes the cycle time (this is true if the closed–loop net is a MGs, in general the lower bound of cycle time will be optimized) of the closed loop net and the controller cost is the solution of the following ILP*

$$
\begin{aligned}
& min\ \Delta = \boldsymbol{c}_c^- \boldsymbol{z}_c + \boldsymbol{c}_c^+ \boldsymbol{z}_o + \beta \left[\boldsymbol{r}_1 + r_2 \boldsymbol{l} \quad 1 \right] \begin{bmatrix} \mathbf{Pre} \\ \boldsymbol{c}_c^- \end{bmatrix} \boldsymbol{D} \\
& s.t. \\
& (a) \quad \boldsymbol{r}_1 \boldsymbol{C} + r_2 \boldsymbol{l} \boldsymbol{C} = \boldsymbol{c}_c^- - \boldsymbol{c}_c^+ \\
& (b) \quad \boldsymbol{r}_1 \boldsymbol{m}_{p0} + r_2 (\boldsymbol{l} \boldsymbol{m}_{p0} - (k+1)) \leq -1 \\
& (c) \quad \boldsymbol{c}_c^- \geq \mathbf{0}_{1\times n} \\
& (d) \quad \boldsymbol{c}_c^+ \geq \mathbf{0}_{1\times n} \\
& (e) \quad \boldsymbol{r}_1 \geq \mathbf{0}_{1\times m} \\
& (f) \quad r_2 \geq 1
\end{aligned} \tag{11}
$$

with variables $\boldsymbol{r}_1 \in \mathbb{N}^{1\times m}$, $r_2 \in \mathbb{N}$, $\boldsymbol{c}_c^- \in \mathbb{N}^{1\times n}$, $\boldsymbol{c}_c^+ \in \mathbb{N}^{1\times n}$, $\beta \in \mathbb{N}$.

Proof: The equations (11-a,c,d,e,f) imposes that the incidence matrix of the controller is obtained from a Moody's parameterization: $\boldsymbol{l}'\boldsymbol{C} = \boldsymbol{c}_c = \boldsymbol{c}_c^- - \boldsymbol{c}_c^+$, with $\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l}$. Also, equation (11-a) extract the pre-incidence and post-incidence matrix of the control net $\boldsymbol{c}_c = \boldsymbol{c}_c^+ - \boldsymbol{c}_c^-$. The equation (11-b) imposes the initial marking condition verification ($\boldsymbol{l}'\boldsymbol{m}_{p0} \leq k'$). The objective function represents the cycle time of the net subsystem added to the net to force $(\boldsymbol{l}, k)$ by each parameterized monitor on the original net system. ■

We denote as $\mathcal{F}_1(\boldsymbol{y})$ the set of the natural valued vectors that are solutions solutions of (11-a,c,d,e,f), with $\boldsymbol{y} = [\boldsymbol{r}_1 \quad r_2 \quad \boldsymbol{c}_c^- \quad \boldsymbol{c}_c^+]^T$.

Proposition 4 *The optimal value Δ^* obtained from solving (11) has to be compared with cost of the controller that eventually is able impose the constraint $(\boldsymbol{l}, k)$ without increasing the cycle of the system. Thus, if we denote as Γ_p the cycle time (or its lower bound, in general) of the plant net, that may be computed by solving (10), the monitor that optimizes the controller cost without increasing the cycle time of the plant net is the solution of the following ILP*

$$
\begin{aligned}
& min\ \Delta' = \boldsymbol{c}_c^- \boldsymbol{z}_c + \boldsymbol{c}_c^+ \boldsymbol{z}_o \\
& s.t. \\
& (a) \quad \boldsymbol{r}_1 \boldsymbol{C} + r_2 \boldsymbol{l} \boldsymbol{C} = \boldsymbol{c}_c^- - \boldsymbol{c}_c^+ \\
& (b) \quad \boldsymbol{r}_1 \boldsymbol{m}_{p0} + r_2 (\boldsymbol{l} \boldsymbol{m}_{p0} - (k+1)) \leq -1 \\
& (c) \quad \boldsymbol{c}_c^- \geq \mathbf{0}_{1\times n} \\
& (d) \quad \boldsymbol{c}_c^+ \geq \mathbf{0}_{1\times n} \\
& (e) \quad \boldsymbol{r}_1 \geq \mathbf{0}_{1\times m} \\
& (f) \quad r_2 \geq 1 \\
& (g) \left[\boldsymbol{r}_1 + r_2 \boldsymbol{l} \quad 1 \right] \begin{bmatrix} \mathbf{Pre} \\ \boldsymbol{c}_c^- \end{bmatrix} \boldsymbol{D} \leq \Gamma_p
\end{aligned} \tag{12}
$$

The system (12) may not have solution. Thus, the optimal monitor will be the one relative to the minimum between Δ^* and Δ'^* if the system (12) has solution, otherwise the one relative to Δ^*.

Proof: The equations (11-a,c,d,e,f) imposes that the incidence matrix of the controller is obtained from a Moody's parameterization as shown in proposition 3. The equation (g) imposes that the P-semiflow subnet added by the monitor does not increase the plant net cycle time. ■

We denote as $\mathcal{F}_2(\boldsymbol{y})$ the set of the natural valued vectors that are solutions solutions of (12-a,b,c,d,e,f,g).

Along the line shown in [Basile et al., 2000], we can also consider the possibility of imposing a trade-off between cost of the control, cycle time optimization and the restrictions imposed by the monitor. We are looking for a GMEC $(\boldsymbol{l}', k)$, according to Moody's parameterization that we are adopting here, such that $\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l}$ and $k' = r_2(k+1) - 1$, with $\boldsymbol{r}_1 \in \mathbb{N}^{1\times m}$ and $r_2 \in \mathbb{N}$. Being $\boldsymbol{r}_1$ and r_2 natural valued, it is immediate to verify that $\mid \mathcal{M}(\boldsymbol{l}', k') \mid \leq \mid \mathcal{M}(\boldsymbol{l}, k) \mid$, and obviously $\mid \mathcal{M}_c(\boldsymbol{l}', k') \mid \leq \mid \mathcal{M}_c(\boldsymbol{l}, k) \mid$. Thus, if we add a weighted sum of the elements of $\boldsymbol{r}_1$ in the objective function, we can minimize this restriction on the plant, without taking into account the effect of r_2 parameter.

The following two ILPs can be derived:

$$\begin{aligned} &min\ \Delta_r \\ &s.t.\quad \boldsymbol{y} \in \mathcal{F}_1(\boldsymbol{y}) \end{aligned} \tag{13}$$

where $\Delta_r = \Delta + \boldsymbol{r}_1\boldsymbol{z}_r$,

$$\begin{aligned} &min\ \Delta'_r \\ &s.t.\quad \boldsymbol{y} \in \mathcal{F}_2(\boldsymbol{y}) \end{aligned} \tag{14}$$

where $\Delta'_r = \Delta' + \boldsymbol{r}_1\boldsymbol{z}_r$.

Example 1 *Let us to consider the net system in fig. 1. We have that*

$$\boldsymbol{C}_p = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{m}_{p0} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 2]$$

and consider the GMEC $(\boldsymbol{l}, k)$ with

$$\boldsymbol{l} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],\ k = 1$$

If we do not consider control and observation costs we obtain the monitor p_{c1} applying (1) and (2).

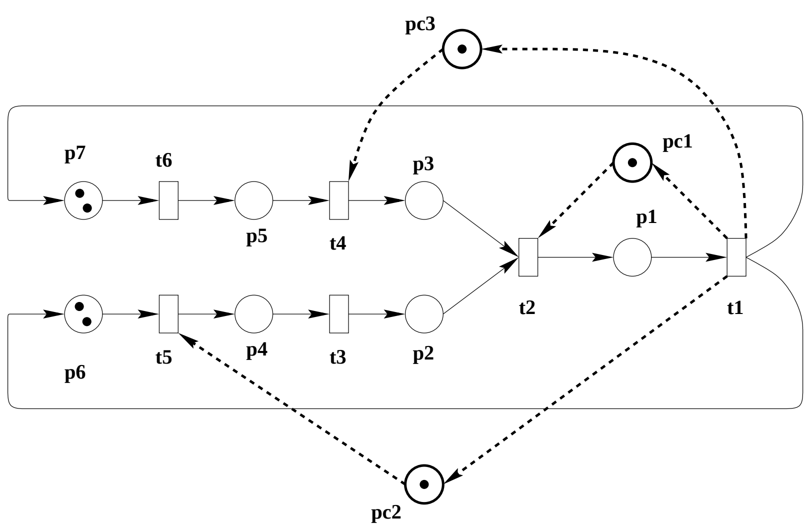

Figure 1: Net system in example 1.

Now let us to introduce the control and observation costs and the time delays for the transitions:

$$\boldsymbol{z}_c = [1 \quad 10 \quad 7 \quad 8 \quad 2 \quad 8],$$

$$\boldsymbol{z}_o = [1 \quad 3 \quad 4 \quad 4 \quad 3 \quad 2],$$

$$\boldsymbol{d} = [1 \quad 1 \quad 3 \quad 1 \quad 3 \quad 1].$$

If we adopt p_{c1} *we have* $\Delta = 13$.

From the ILP (11) it follows

$$\boldsymbol{l}'^* = [1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0],\ k'^* = 1$$

and the relative monitor place p_{c2} *with incidence matrix and initial marking*

$$\boldsymbol{c}_{c1}^* = [1 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0],\ m_{c01} = 1.$$

In this case $\Delta^* = 11$.

Introducing $\boldsymbol{z}_r = [2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2]$ *it follows by solving (13)* $\Delta_r^* = 13$ *and the optimal solution is* p_{c1} *as in the case we do not consider the controller cost.*

From (12) we obtain

$$\boldsymbol{l}''^* = [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0],\ k''^* = 1$$

and the relative monitor place p_{c3} *with incidence matrix and initial marking*

$$\boldsymbol{c}_{c2}^* = [1 \quad 0 \quad 0 \quad -1 \quad 0 \quad 0],\ m_{c02} = 1.$$

In this case $\Delta'^* = 9$.

On the other hand from (10) it follows that $\Gamma_p = 2$ *and thus solving the ILP (14) we obtain* $\Delta_r'^* = 11$ *and the optimal solution is* p_{c1} *again.*

5 Conclusions

In this paper we dealt with the control of timed Petri Net modeled plant. The concept of control and observation cost of a transition is introduced. We discussed the problem to enforce a mutual exclusion constraint so as to minimize the control and observation cost and the cycle time of the closed loop net as performance index. A monitor based controller form is chosen, because of its simplicity.

The novel contribution of this paper is in the introduction of control and observation costs for transitions of a PN modeled plant together with transition time delays. This makes possible to take into account the cost to detect or to enable an event occurrence that is not always negligible in the real plant together with cycle time of the closed loop net.

Two integer programming problems are formulated to synthesize the optimal monitor based controller: a first one by which a trade-off between controller cost and CT optimization is imposed and a second one by which only controller cost is considered but with constraint that the CT of the closed loop net is the same of the open loop one. The second, that represents a better solution, may have not solution.

Finally, this approach was extended to also take into account a cost associated to the restriction imposed by the monitor (in term of places in the support of the GMEC).

All the results are valid for the class of mono T-semiflow net. It is possible to extend this result to a general net system when it is possible to compute a T-semiflow covering all the transitions, but that satisfies additional conditions because of the presence of routing rates of the conflicting transitions necessary when a not unique minimal T-semiflow exists. Along this line we think there is open research.

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