SUPERVISORY CONTROL OF PETRI NETS BASED ON SUBOPTIMAL MONITOR PLACES

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Abstract

This paper deals with the problem of enforcing generalized mutual exclusion constraints (GMEC) on place/transition nets. An efficient control synthesis technique, that has been recently proposed in the literature, is to enforce GMEC constraints by introducing monitor places to create suitable place invariants. The method has been shown to be maximally permissive and to give a unique control structure in the case that the set of legal markings is controllable. This paper investigates on and formally shows that the class of controllers obtained by this technique may not have a supremal element for uncontrollable specifications.

1 Introduction

In the original approach of Ramadge and Wonham [7] to the supervisory control of discrete event systems (DESs), a DES G is a language generator whose behaviour (i.e., language) is denoted $L(G)$. Given a legal language L, the basic control problem is to design a supervisor that restricts the closed loop behaviour of the plant to $L \cap L(G)$. This is possible if and only if L is controllable (and prefix-closed). We recall that a language L is controllable with respect to a DES G with uncontrollable event set Σ_u if $\bar{L}\Sigma_u \cap L(G) \subseteq \bar{L}$. If L is not controllable, we can consider the class of controllable sublanguages of L, i.e., the set $\Omega(L) = \{K \subseteq L \mid K \text{ is controllable}\}$. For each language K in this class we may construct a supervisor, thus further restricting the closed loop behaviour of the plant to $K \cap L(G) \subset L \cap L(G)$. The class $\Omega(L)$ is not empty and closed under union, hence it admits a unique supremal element with respect to set inclusion. The element $L^{\uparrow} = \sup \Omega(L)$, called *supremal controllable sublanguage*, is the "optimal" solution to our control problem in the sense that it is the minimally restrictive solution.

A similar approach can also be taken when considering the state evolution of (rather than the traces of events generated by) a DES. This approach, that

we call *state-based*, is particularly attractive when Petri nets (PNs) are used to represent the plant and was taken by Holloway and Krogh [4] and Li and Wonham [5]. Let us consider a PN system $\langle N, \boldsymbol{\mu}_0 \rangle$ with m places, whose set of reachable markings is $R(N, \boldsymbol{\mu}_0) \subseteq \mathbb{N}^m$. Assume we are given a set of legal markings $\mathcal{L} \subseteq \mathbb{N}^m$, and consider the basic control problem of designing a supervisor that restricts the reachability set of plant in closed loop to $\mathcal{L} \cap R(N, \boldsymbol{\mu}_0)$. This is possible if and only if $\mathcal{L}$ is controllable (and reachable). If $\mathcal{L}$ is not controllable, we can consider the class of controllable subsets of $\mathcal{L}$, i.e., the class $\Omega(\mathcal{L}) = \{\mathcal{K} \subseteq \mathcal{L} \mid \mathcal{K} \text{ is controllable}\}$. For each set $\mathcal{K}$ in $\Omega(\mathcal{L})$ we may construct a supervisor, thus further restricting the reachability set of the plant in closed loop to $\mathcal{K} \cap R(N, \boldsymbol{\mu}_0) \subset \mathcal{L} \cap R(N, \boldsymbol{\mu}_0)$. The class $\Omega(\mathcal{L})$ is not empty and closed under union[1] hence it admits a unique supremal element with respect to set inclusion. The element $\mathcal{L}^{\uparrow} = \sup \Omega(\mathcal{L})$, called *supremal controllable subset*, is the "optimal" solution to this control problem.

Of particular interest are those PN state-based control problems where the set of legal markings $\mathcal{L}$ is expressed by a set of n_c linear inequality constraints called Generalized Mutual Exclusion Constraint (GMEC). In this case we write $\mathcal{L} = \mathcal{M}(\boldsymbol{L}, \boldsymbol{k}) \equiv \{\boldsymbol{\mu} \in \mathbb{N}^m \mid \boldsymbol{L}\boldsymbol{\mu} \leq \boldsymbol{k}\}$ to denote that $\mathcal{L}$ is expressed by the GMEC $(\boldsymbol{L}, \boldsymbol{k})$ with $\boldsymbol{L} \in \mathbb{Z}^{n_c \times m}, \boldsymbol{k} \in \mathbb{Z}^{n_c}$. Problems of this kind have been considered by several authors [2, 6, 5]. This special structure of the legal set has the advantage that if $\mathcal{L}$ is controllable then the supervisor for this class of problems takes the form of as many places, called *monitors*, as there are constraints. Thus if the matrix $\boldsymbol{L}$ has n_c rows, the supervisor will consist of n_c monitor places, each of which has arcs going to and coming from some transitions of the plant net. The DES plant and the controller are described by Petri nets in order to have an useful linear algebrique model for control analysis and synthesis. Morover the synthesis is not computation demanding since it involves only a matrix multiplication. Let us assume, however, that $\mathcal{L}$ is uncontrollable. The counterpart on the controller structure is that one of the monitors associated to this GMEC has arcs going to uncontrollable

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[1]This is true under the non-concurrency hypothesis. In the approach of Holloway and Krogh two transitions may fire concurrently and this is not true anymore [3].

transitions, i.e., it may be blocking an uncontrollable transition. Following the general approach outlined above, we have to compute the set $\mathcal{L}^\uparrow$, but unfortunately, as shown by Giua et al. [2], it may well be the case that this set cannot be expressed by a set of linear inequalities, i.e., the corresponding supervisor does not have a monitor-based structure. Li and Wonham [5] showed that if the plant net belongs to the special class of TS2 nets then $\mathcal{L}^\uparrow$ is guaranteed to be expressed by a set of n_c linear inequalities. Giua et al. [2] showed that if the plant net is safe then $\mathcal{L}^\uparrow$ is guaranteed to be expressed by a set of n'_c linear inequalities, where n'_c, however, may be very large (it may be of the same order of the cardinality of the reachability set).

This problem motivated Moody et al. [6] to consider as acceptable a further restriction of the reachability set. Given an uncontrollable legal marking set $\mathcal{L}$ epressed by n_c constraints, one may define the set $\Omega_{n_c}(\mathcal{L}) = \{\mathcal{K} \subseteq \mathcal{L} \mid \mathcal{K}$ is controllable, $\exists \boldsymbol{L}' \in \mathbb{Z}^{n_c \times m}, \boldsymbol{k}' \in \mathbb{Z}^{n_c} : \mathcal{K} = \mathcal{M}(\boldsymbol{L}', \boldsymbol{k}')\}$ of controllable and expressed by a set of n_c linear inequalities subsets of $\mathcal{L}$. In [6] a procedure was also given that leads to compute an element $\mathcal{K} \in \Omega_{n_c}(\mathcal{L})$, i.e., to compute a constraint $(\boldsymbol{L}', \boldsymbol{k}')$ with $\boldsymbol{L}' \in \mathbb{Z}^{n_c \times m}$, and its corresponding monitor structure, such that $\mathcal{K} = \mathcal{M}(\boldsymbol{L}', \boldsymbol{k}')$. We note that in this approach one restricts the reachability set of the plant in closed loop to be within $\mathcal{K} \subset \mathcal{L}^\uparrow$, i.e., one may prevent the closed loop system from reaching some perfectly legal marking. One gains, however, in simplicity because the controller takes a simple structure of n_c monitors.

In [1] it has been given an algorithm to construct a parameterization of all monitors corresponding to *supremal* elements of $\Omega_{n_c}(\mathcal{L})$ and because, since the elements of $\Omega_{n_c}(\mathcal{L})$ cannot be ordered by subset inclusions, the $\subseteq$ criterion of optimality is meaningless, two criteria of suboptimality have been proposed. In this paper we further pursue the investigation along these lines and we formally show that the class $\Omega_{n_c}(\mathcal{L})$ is not empty and not closed under union. Hence a supremal element exists but it is not necessarily unique.

2 Background

A place/transition (P/T) net is a structure $N = (P, T, I, O)$ where: P is a set of m *places* represented by circles; T is a set of n *transitions* represented by bar; $P \cap T = \emptyset$, $P \cup T \neq \emptyset$; $I : P \times T \to \mathbb{N}$ is the input function that specifies the arcs directed from places to transitions, with $\mathbb{N}$ is the set of non-negative integers; $O : P \times T \to \mathbb{N}$ is the output function that specifies the arcs directed from transitons to places. A *marking* is a $m \times 1$ vector $\boldsymbol{\mu} : P \to \mathbb{N}$ that assigns to each place of a P/T net a non-negative integer number of tokens, represented by black dots. A transition $t \in T$ is enabled at a marking $\boldsymbol{\mu}$ iff $\boldsymbol{\mu} \geq I(\cdot, t)$. If t is enabled, then t may fire yield-

ing a new marking $\boldsymbol{\mu}' = \boldsymbol{\mu} + O(\cdot, t) - I(\cdot, t)$. The notation $\boldsymbol{\mu}[t > \boldsymbol{\mu}'$ will denote that an enabled transition t may fire at $\boldsymbol{\mu}$ yielding $\boldsymbol{\mu}'$. $\mathbb{N}^m$ will denote the set of all possible markings that may defined on the net. A *firing sequence* from $\boldsymbol{\mu}_0$ is a (possibly empty) sequence of transitions $\sigma = t_1...t_k$ such that $\boldsymbol{\mu}_0[t_1 > \boldsymbol{\mu}_1[t_2 > \boldsymbol{\mu}_2..[t_k > \boldsymbol{\mu}_k$. A P/T system or net system $< N, \boldsymbol{\mu}_0 >$ is a P/T net N with an initial marking $\boldsymbol{\mu}_0$. A marking $\boldsymbol{\mu}$ is reachable in $< N, \boldsymbol{\mu}_0 >$ iff there exists a firing sequence σ such that $\boldsymbol{\mu}_0[\sigma > \boldsymbol{\mu}$. Given a net system $< N, \boldsymbol{\mu}_0 >$ the set of reachable markings (also called *reachability set* of the net) is denoted $R(N, \boldsymbol{\mu}_0)$.

A single generalized mutual exclusion constraint (GMEC) is a couple $(\boldsymbol{l}, k)$ where $\boldsymbol{l} : P \to \mathbb{Z}$ is a $1 \times m$ weight vector and $k \in \mathbb{Z}$. The support of $\boldsymbol{l}$ is the set $Q_l = \{p \in P \mid \boldsymbol{l}(p) \neq 0\}$. Given the net system $< N, \boldsymbol{\mu}_0 >$, a GMEC defines a set of markings that will be called *legal markings*: $\mathcal{M}(\boldsymbol{l}, k) = \{\boldsymbol{\mu} \in \mathbb{N}^m \mid \boldsymbol{l}\boldsymbol{\mu} \leq k\}$. The markings that are not legal are called *forbidden markings*. A set of GMEC $(\boldsymbol{L}, \boldsymbol{k})$, with $\boldsymbol{L} = [\boldsymbol{l}_1^T \boldsymbol{l}_2^T ... \boldsymbol{l}_{n_c}^T]^T$ and $\boldsymbol{k} = [k_1 k_2 ... k_{n_c}]^T$, will define the *legal markings* set $\mathcal{M}(\boldsymbol{L}, \boldsymbol{k}) = \{\boldsymbol{\mu} \in \mathbb{N}^m \mid \boldsymbol{L}\boldsymbol{\mu} \leq \boldsymbol{k}\}$. A controlling agent, called supervisor, must ensure the forbidden markings will be not reached. So the set of legal markings under control is $\mathcal{M}_c(\boldsymbol{L}, \boldsymbol{k}) = \mathcal{M}(\boldsymbol{L}, \boldsymbol{k}) \cap R(N, \boldsymbol{\mu}_0)$.

The set of the transitions T of a net N is now assumed to be partitioned into two disjoints subsets: T_u the set of the *uncontrollable transitions* and T_c the set of *controllable transitions*. The occurrence of a controllable transition may be disabled, while the occurrence of an uncontrollable transition cannot be disabled. In this case it is useful to consider the net N_u obtained from the net N eliminating the controllable transitions.

3 Suboptimal monitor places for uncontrollable specifications

Let us now consider the problem of restricting the reachability set of a PN within a set of legal markings $\mathcal{L}$.

Definition 1 *A set of legal markings $\mathcal{L} \subseteq \mathbb{N}^m$ is* controllable *with respect to a PN system $\langle N, \boldsymbol{\mu}_0 \rangle$ with uncontrollable subnet N_u if $\bigcup_{\boldsymbol{\mu} \in \mathcal{L} \cap R(N, \boldsymbol{\mu}_0)} R(N_u, \boldsymbol{\mu}) \subseteq \mathcal{L}$.*

According this definition, $\mathcal{L}$ is controllable if from any marking $\boldsymbol{\mu} \in \mathcal{L}$ no forbidden marking is reachable by firing a sequence containing only uncontrollable transitions, that cannot be disabled by a supervisor. If $\mathcal{L}$ is not controllable, we also must avoid reaching the set of markings $\mathcal{L}_{uf} = \{\boldsymbol{\mu} \in \mathcal{L} \mid \boldsymbol{\mu}[\sigma > \boldsymbol{\mu}', \boldsymbol{\mu}' \notin \mathcal{L}, \sigma \in T_u^*\}$. We can consider the class of controllable subsets of $\mathcal{L}$, i.e., the class $\Omega(\mathcal{L}) = \{\mathcal{K} \subseteq \mathcal{L} \mid \mathcal{K}$ is controllable$\}$. The class $\Omega(\mathcal{L})$ is not empty and closed under union, hence it admits a unique supremal element with respect to set inclu-

sion. The element $\mathcal{L}^{\uparrow} = \sup \Omega(\mathcal{L}) = \mathcal{L} \setminus \mathcal{L}_{uf}$, called *supremal controllable subset*, is the "optimal" solution to the control problem of restricting the reachability set of plant to legal markings.

We consider in the remaining part of this paper legal sets given by GMEC, i.e., $\mathcal{L}$ is expressed by a set of n_c linear inequality constraints and can be written as $\mathcal{L} = \mathcal{M}(\boldsymbol{L}, \boldsymbol{k}) \equiv \{\boldsymbol{\mu} \in \mathbb{N}^m \mid \boldsymbol{L}\boldsymbol{\mu} \leq \boldsymbol{k}\}$. If $\mathcal{L}$ is controllable and the initial marking is legal — i.e., $\boldsymbol{\mu}_0 \in \mathcal{L}$ — the optimal controller consists of n_c monitor places, whose $n_c \times m$ incidence matrix is given by [6] $\boldsymbol{D}_c = -\boldsymbol{L}\boldsymbol{D}_p$, where $\boldsymbol{D}_p$ is the $m \times n$ incidence matrix of the plant net. If $\mathcal{L}$ is not controllable, as discussed in the introduction, $\mathcal{L}^{\uparrow}$ may not be expressed by a set of n_c linear inequality constraints. In this case, one may define the set $\Omega_{n_c}(\mathcal{L}) = \{\mathcal{K} \subseteq \mathcal{L} \mid \mathcal{K}$ is controllable, $\exists \boldsymbol{L}' \in \mathbb{Z}^{n_c \times m}, \boldsymbol{k}' \in \mathbb{Z}^{n_c} : \mathcal{K} = \mathcal{M}(\boldsymbol{L}', \boldsymbol{k}')\}$ of *controllable* and *expressed by a set of n_c linear inequalities* subsets of $\mathcal{L}$.

Theorem *Consider a plant represented by a PN system* $\langle N, \boldsymbol{\mu}_0 \rangle$. *Let* $\mathcal{L} = \mathcal{M}(\boldsymbol{L}, \boldsymbol{k}) \equiv \{\boldsymbol{\mu} \in \mathbb{N}^m \mid \boldsymbol{L}\boldsymbol{\mu} \leq k\}$ *be an uncontrollable set with* $\boldsymbol{L} \in \mathbb{Z}^{n_c \times m}$ *and* $\boldsymbol{k} \in \mathbb{Z}^{n_c}$. *The class* $\Omega_{n_c}(\mathcal{L})$ *of* controllable *and* expressed by a set of n_c linear inequalities *subsets of* $\mathcal{L}$ *is:*
a) not empty;
b) not closed under union.

Proof:
a) Let us consider the set $\mathcal{K} = \emptyset \subset \mathcal{L}$. By definition 1, $\mathcal{K}$ is controllable. It can also be expressed by a set of linear inequalities: take any constraint set with no feasible solution. E.g., if we let $\boldsymbol{L}' = \{0\}^{n_c \times m}$ and $\boldsymbol{k}' = \{-1\}^{n_c}$, clearly $\mathcal{K} = \mathcal{M}(\boldsymbol{L}', \boldsymbol{k}')$. This shows that $\emptyset \in \Omega_{n_c}(\mathcal{L})$.
b) We show this giving a simple counterexample.
Consider the net in fig. 1 with $T_u = \{t_2\}$. Let $\mathcal{L} = \{\boldsymbol{\mu} \in \mathbb{N}^3 \mid \mu(p_1) \leq 1\}$. This set is not controllable, because the corresponding monitor requires an arc going to the uncontrollable transition t_2.
Consider the sets: $\mathcal{K}_1 = \{\boldsymbol{\mu} \in \mathbb{N}^3 \mid \mu(p_1) + \mu(p_2) \leq 1\}$ and $\mathcal{K}_2 = \{\boldsymbol{\mu} \in \mathbb{N}^3 \mid \mu(p_1) + \mu(p_3) \leq 1\}$. Clearly, $\mathcal{K}_1, \mathcal{K}_2 \in \Omega_{n_c}(\mathcal{L})$.
We will show that the set $\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2$ is not convex, hence it cannot be expressed by a set of linear inequalities. In fact, if we consider the markings $\boldsymbol{\mu}_1 = [1\ 0\ 2]^T \in \mathcal{K}_1 \subset \mathcal{K}$ and $\boldsymbol{\mu}_2 = [1\ 2\ 0]^T \in \mathcal{K}_2 \subset \mathcal{K}$, we have that the marking $\boldsymbol{\mu} = \frac{\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2}{2} = [1\ 1\ 1]^T$ does not belong to $\mathcal{K}$. ∎
Note that the part a) of the previous theorem shows that $\Omega_{n_c}(\mathcal{L})$ is not empty because it contains the empty set. However, if the supremal element of $\Omega_{n_c}(\mathcal{L})$ is the set $\mathcal{K} = \emptyset$, the (monitor-based) control problem has no solution, because the required condition that $\boldsymbol{\mu}_0 \in \mathcal{K}$ is clearly not satisfied.

Corollary *Consider a plant represented by a PN system* $\langle N, \boldsymbol{\mu}_0 \rangle$. *Let* $\mathcal{L} = \mathcal{M}(\boldsymbol{L}, \boldsymbol{k}) \equiv \{\boldsymbol{\mu} \in \mathbb{N}^m \mid \boldsymbol{L}\boldsymbol{\mu} \leq k\}$ *be an uncontrollable set with* $\boldsymbol{L} \in \mathbb{Z}^{n_c \times m}$

and $\boldsymbol{k} \in \mathbb{Z}^{n_c}$. *The element* $\sup \Omega_{n_c}(\mathcal{L})$ *exists but it is not necessary unique.*
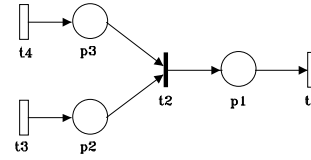

Figure 1: A P/T net with uncontrollable transition t_2.

4 Conclusions

This paper has shown that the class of controllable and expressed by a set of linear inequalities subsets of a set of markings that satisfies a given GMEC may not admit a unique supremal controllable element. Because this class of constraint can be enforced by a monitor places, there is not an optimal monitor based structure for a given uncontrollable GMEC. However this class of controller is not computation demanding and can be modelled as Petri net, and so a suboptimal monitor structure may be so acceptable. Further work on the choice of the suboptimal monitor structure is being undertaken.

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