Stationary behavior of manufacturing systems modeled by timed weighted marked graphs

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Abstract

Timed Petri nets have proven to be suitable for modelling and analyzing embedded systems, assembly lines, and streaming applications. In this paper, a sub class of timed Petri net called timed weighted marked graphs (TWMGs) are studied. Stationary throughput analysis of these TWMGs is an important step for checking stationary stationary throughput requirements of concurrent real-time applications and finding the best schedules. We proposed some properties of TWMGs which is useful for designers or engineers to study the stationary throughput and solve the scheduling problem for practical usage.

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I. Introduction

Performance control or performance evaluation of manufacturing systems or embedded systems pose difficult issues because their representation deals with continuous and discrete models. Timed Petri nets (PNs) are well known as efficient tools for modeling discrete event systems and analysis performance of concurrent systems [1].

Timed weighted marked graphs and timed marked graphs (TMGs) find wide applications in streaming applications and manufacturing systems [9], [6]. They can model embedded systems and capture the active entities of the process. Process and buffers are usually modeled by transitions and places, respectively. Tokens model data transferred from a process to another. When modelling manufacturing systems, transitions and tokens represent workshop operations and products, respectively. Synchronous Data-Flow Graphs (SDFGs) are a well known formalism to model embedded applications which is equivalent to TWMGs [5]. The transmission of datas and the storage of buffers may incur economical consequence. Thus, it becomes very important to minimize the number of buffers and maximize the stationary throughput of the system. In the framework of TWMGs, this problem is equivalent to the minimization of the total number of tokens and maximization of the stationary stationary throughput of transitions.

The initial distributed state problem for TWMGs are studied in [13] and [12] which consider a trade off between the stationary throughput and resources used. However, the proposed algorithms are non-polynomial and cannot ensure an optimal solution. The existence of a polynomial solution for the optimization problem of a TWMG is still an open problem. The capacity of any place is proportional to the space used to store data and the problem of minimizing place capacities in order to enforce liveness is studied in [11]. The existence and computation of an optimal periodic schedule of a TWMG is discussed in [6]. Stationary throughput analysis for TWMGs are studied in [8] and [10]. However, both of them fail to present efficient algorithms to compute the stationary throughput for TWMGs.

The stationary throughput of a TMG can be easily analyzed by studying each elementary circuit of the system: as an example, the stationary throughput of a TMG is equal to the cycle time of its slowest circuit. On the contrary, for a TWMG the analysis is more complex and it is not sufficient to study each single circuit. This is well known and there exists some examples of this type for TWMGs under single server semantics [7] (under single server semantics services in a transition are provided sequentially, i.e., there is no self-concurrency). The objective of this paper is to show some examples that pertain to TWMGs under infinite server semantics (the degree of self-concurrency of each transition is infinite). Under an infinite server semantics the number of concurrent servers is equal to the enabling degree of the transition. Note that infinite server semantics is more general than single server (or in general k server) semantics. In fact, single (resp., k) server semantics can be simulated by just adding to each transition a self-loop place with one (resp., k) tokens. We study the relationship between stationary throughput and resource distribution from the structural point of view. Some important properties are presented to show how the stationary throughput is influenced by different resource distributions. These results are useful to further study the initial state assignment problem which is very important for the design of many discrete event dynamic systems.

This paper is structured as follows. In the following section, we briefly recall some basic concepts and the main properties of TWMGs. In Section III, we discuss the stationary throughput of a TWMG under infinite server semantics and present some important properties concerning the analysis of stationary throughput of a TWMG. Conclusions are finally drawn in Section IV.

II. Background

A. Generalities

We assume that the reader is familiar with the structure, firing rules, and basic properties of PNs (see [4]). In this section, we will recall the formalism used in the paper. A *place/transition* net (P/T net) is a structure $N = (P, T, \boldsymbol{Pre}, \boldsymbol{Post})$, where P is a set of n places; T is a set of m transitions; $\boldsymbol{Pre} : P \times T \rightarrow \mathbb{N}$ and $\boldsymbol{Post} : P \times T \rightarrow \mathbb{N}$ are the pre- and post-incidence functions that specify the arcs; $\boldsymbol{C} = \boldsymbol{Post} - \boldsymbol{Pre}$ is the incidence matrix, where $\mathbb{N}$ is a set of non-negative integers.

A vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)^T \in \mathbb{N}^{|T|}$ such that $\boldsymbol{x} \neq \boldsymbol{0}$ and $\boldsymbol{C} \cdot \boldsymbol{x} = \boldsymbol{0}$ is a *T-semiflow*. A vector $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)^T \in \mathbb{N}^{|P|}$ such that $\boldsymbol{y} \neq \boldsymbol{0}$ and $\boldsymbol{y}^T \cdot \boldsymbol{C} = \boldsymbol{0}$ is a *P-semiflow*. The supports of a T-semiflow and a P-semiflow are defined by $\|\boldsymbol{x}\|$=$\{t_i \in T | x_i > 0\}$ and $\|\boldsymbol{y}\|$=$\{p_i \in P | y_i > 0\}$, respectively. A *minimal* T-semiflow (P-semiflow) is a T-semiflow $\|\boldsymbol{x}\|$ (P-semiflow $\|\boldsymbol{y}\|$) that is not a superset of the support of any other T-semiflow (P-semiflow), and its components are mutually prime.

A marking is a vector $\boldsymbol{M} : P \rightarrow \mathbb{N}$ that assigns to each place of a P/T net a non-negative integer of tokens; we denote the marking of place p as $M(p)$. A P/T system or net system $\langle N, \boldsymbol{M_0} \rangle$ is a net N with an initial marking $\boldsymbol{M_0}$.

A P/T net is said to be *ordinary* when all of its arc weights are equal to one. A *marked graph* (also called an event graph) is an ordinary Petri net such that each place has exactly one input and one output transition. A *weighted marked graph* (also called a weighted event graph) is a net that also satisfies this structural condition but may not be ordinary, i.e., the weight associated with each arc is a non-negative integer number.

A net is *strongly connected* or *cyclic* if there exists a directed path from any node in $P \cup T$ to every other node. Let us define an *elementary circuit* γ (or elementary cycle) of a net as a directed path that goes from one node back to the same node without passing twice on the same node.

Given a place p of a WMG, we denote $Post(p, t)$ the weight of its unique input arc and $Pre(p, t)$ the weight of its unique output arc. The gain $G(\gamma)$ [3] of every circuit γ of a TWMG is

$$G(\gamma) = \prod_{p \in \gamma} \frac{\nu(p_i)}{w(p_i)}.$$

A TWMG is live if every circuit has a gain not less than one. Intuitively, if $G(\gamma) < 1$ (resp. $G(\gamma) < 1$), the whole number of tokens in circuit γ will go to zero (resp. infinity). In the most of previous works, people tents to study the case when $G(\gamma) = 1$ and the TWMG will be bounded. Any TWMG which satisfies this condition is said to be neutral. It is well known that a neutral TWMG has a unique minimal T-semiflow $\boldsymbol{x}$ which contains all transitions in its support [3]. In this paper, we limit our study to strongly connected and neutral TWMGs.

B. Dynamic behavior

A deterministic timed P/T net is a pair $N^\delta = (N, \delta)$, where $N = (P, T, \boldsymbol{Pre}, \boldsymbol{Post})$ is a standard P/T net, and $\delta : T \to \mathbb{N}$, called firing delay, assigns a non-negative integer fixed firing duration to each transitions [2].

A transition t_i is enabled at $\boldsymbol{M}_j$ if $\boldsymbol{M}_j \geq \boldsymbol{Pre}(\cdot, t_i)$ and an enabled transition t may fire yielding a marking $\boldsymbol{M}'$ with

$$\boldsymbol{M}' = \boldsymbol{M}_j + \boldsymbol{C}(\cdot, t_i), \tag{1}$$

where $\boldsymbol{Pre}(\cdot, t_i)$ (resp. $\boldsymbol{C}(\cdot, t_i)$) denotes the column of the matrix $\boldsymbol{Pre}$ (resp. $\boldsymbol{C}$) associated with transition t_i.

The state of a TWMG is defined not only by the marking, as for P/T nets, but also by the clocks associated with transitions. The enabling degree of t enabled at a marking $\boldsymbol{M}_j$ denoted by $\alpha_i(j)$ is the biggest integer number k such that

$$\boldsymbol{M}_j \geq k \cdot \boldsymbol{Pre}(\cdot, t_i). \tag{2}$$

In this paper, we consider the so-called infinite server semantics [2], i.e., we assume that the degree of self-concurrency of each transition is infinite.

Under infinite server semantics, at each time instant τ_j the number of clocks o_i associated with a transition t_i is equal to its current enabling degree, i.e., $o_i = \{o_{i,1}, \ldots, o_{i,\alpha_i(j)}\}$; this number changes with the enabling degree, thus it can change each time the net evolves from one marking to another one, namely, each time that a transition fires. If transition t_i is not enabled at marking $\boldsymbol{M}_j$, it has no clock. Assuming that $o_i^* = \min\{o_{i,1}, \ldots, o_{i,\alpha_i(j)}\}$ and letting $o^* = \min_{i=1,\ldots,m}\{o_i^*\}$ be the minimum among the values of the clocks o_i^*. At the time instant $\tau_{j+1} = \tau_j + o^*$, transitions whose clocks are equal to o^* fire yielding a new marking as in Eq. (1).

C. Stationary throughput and resources used

The *stationary throughput* $\beta(\boldsymbol{M})$ of a TWMG system $\langle N, \boldsymbol{M}\rangle$ is the average frequency to fire once the minimal T-semiflow under the *as soon as possible* (ASAP) execution, i.e., transitions are fired as soon as possible.

The ASAP execution of a live and strongly connected TWMG with integer delays is ultimately repetitive following an execution pattern. The period of the pattern is τ and the number of firings of every transition within a period is f (the periodicity). The number of firings of transition t_i within the steady period is f_i.

Definition 1: Let $t_i \in T$ be an arbitrary transition of a TWMG with the minimal T-semiflow $\boldsymbol{x}$. The stationary throughput of the TWMG is

$$\beta(\boldsymbol{M}) = \frac{f_i}{x_i \cdot \tau}. \tag{3}$$

Note that the value of stationary throughput does not depend on the considered transition. It has been shown in [8] that the stationary throughput of a TWMG can be computed by transforming it into an equivalent timed marked graph (TMG). However, this transformation called expansion, cannot produce efficient algorithms in an industrial context.

The *resources used* $R(\boldsymbol{M})$ of a TWMG (or a TMG) system $\langle N, \boldsymbol{M}\rangle$ is represented by a linear combination of tokens which is an invariant linear criteria

$$R(\boldsymbol{M}) = \boldsymbol{y}^T \cdot \boldsymbol{M}, \tag{4}$$

where $\boldsymbol{y} = (y_1, \ldots, y_n)^T$ is a non-negative weight vector. In most of previous works, $\boldsymbol{y}$ is chosen as a sum of all minimal P-semiflows, i.e., $\boldsymbol{y} = \sum_{\gamma\in\Gamma} \boldsymbol{y}_\gamma$. This is due to the fact that the value of $\boldsymbol{y}^T \cdot \boldsymbol{M}$ does not change with the evolution of the system. In terms of manufacturing domain, this value corresponds to the fact that resources remain constant as the production process proceeds.

III. Properties of stationary throughput for TWMGs

This section is devoted to illustrate how the initial distribution of tokens affects the stationary throughput of a TWMG under infinite server semantics. We will show that some important results that hold in the case of TMGs may not hold for this class of nets.

A. Stationary throughput analysis

Let Γ represent the set of elementary circuits of a cyclic TWMG and define $\beta_\gamma(\boldsymbol{M}_0)$ as the stationary throughput of circuit γ. It is well known that for a TMG the cycle time of the net is equal to the minimal stationary throughput over all circuits, i.e.,

$$\beta(\boldsymbol{M}_0) = \min_{\gamma\in\Gamma} \beta_\gamma(\boldsymbol{M}_0). \tag{5}$$

Property 1: The stationary throughput of a cyclic TWMG system $\langle N, \boldsymbol{M_0}\rangle$ is lower than or equal to the minimal stationary throughput among all circuits, i.e.,

$$\beta(\boldsymbol{M}_0) \leq \min_{\gamma\in\Gamma} \beta_\gamma(\boldsymbol{M}_0). \tag{6}$$

It is obvious that the stationary throughput of a cyclic manufacturing system can not be faster than the slowest stationary throughput among all circuits and at most is equal to the slowest one. The following example shows that the stationary throughput of the system can smaller than that of the slowest circuit.

Example 1: Let us consider a cyclic painting process. Machine MA_1 takes one unit of raw material and produces six semi-finished products PR_1 which needs to be painted. Machine MA_2 takes four liters of raw pigment and produces three bags of paint PR_2 (the volume of each is 4/3 liters). Then, Machine MA_3 takes one bag of paint PR_2 and four items of semi-finished product PR_1 and executes the painting process. Finally, a batch transportation device removes six painted product from the workshop and brings one unit of raw material to machine MA_1 and two liters of raw pigment to machine MA_2, respectively.

This automated cyclic painting process is modelled by a net with four timed transitions: each transition corresponds to a different operation. The TWMG model is depicted by Fig. 1 and Table I shows the physical meaning of each transition.

There are two elementary circuits, $\gamma_1 = p_4 t_1 p_1 t_3 p_3 t_4$ and $\gamma_1 = p_5 t_2 p_2 t_3 p_3 t_4$, corresponding to the manufacturing process of PR_1 and PR_2, respectively. The minimal P-semiflow of γ_1 is $\boldsymbol{y}_1 = (1, 0, 1, 6, 0)^T$ and while the minimal

TABLE I

PHYSICAL MEANING OF EACH TRANSITION.

Transitions	t_1	t_2	t_3	t_4
Operations	MA_1	MA_2	MA_3	transport
Execution times	1	2	7	3

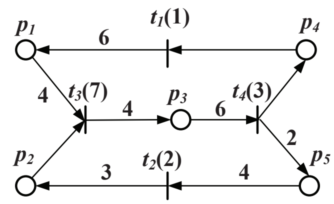

Fig. 1. TWMG model of Example 1.

P-semiflow of γ_2 is $\boldsymbol{y}_2 = (0, 4, 1, 0, 3)^T$. Thus, we consider a weight vector $\boldsymbol{y} = \boldsymbol{y}_1 + \boldsymbol{y}_2 = (1, 4, 2, 6, 3)^T$. The physical meaning of the weighted vector $\boldsymbol{y}$ is that six items of semi-finished product in p_1 are produced from one item of raw material in p_4 and three packaged paints in p_1 are produced from four items of raw dyestuff in p_5, while four items of painted products in p_3 are manufactured by using four items from p_1 and one item from p_2. Thus, the resources used ratio for each place is equal to $\boldsymbol{y}$.

Assuming the initial marking of the TWMG is $\boldsymbol{M}_0 = (2, 1, 22, 0, 0)^T$, the stationary throughput of the system is shown in Table **??**. The resources used for γ_1 and γ_2 are 24 and 26 and the stationary throughput of the two circuits are 0.182 and 0.167. Nevertheless, we find that the stationary throughput of the system is equal to 0.13 which is smaller than the minimal one of the two circuits, i.e.,

$$0.13 < \min\{0.182, 0.167\}$$

Property 2: The stationary throughput of two TWMG systems $\langle N, \boldsymbol{M}_0 \rangle$ and $\langle N, \boldsymbol{M}_1 \rangle$ with same net structure can be different even all the stationary throughput of their circuits are identical, i.e.,

$$\begin{aligned} \beta(\boldsymbol{M}_0) &\neq \beta(\boldsymbol{M}_1), \\ \beta_\gamma(\boldsymbol{M}_0) &= \beta_\gamma(\boldsymbol{M}_1), \ \forall \gamma \in \Gamma. \end{aligned} \tag{7}$$

We prove this property by showing the following example.

Example 2: Let us consider the TWMG model depicted in Fig. 1 and assume the initial marking $\boldsymbol{M}_1 = (0, 0, 24, 0, 0)^T$. Table II shows the stationary throughput analysis of marking $\boldsymbol{M}_1$. One may find that for marking $\boldsymbol{M}_1$ the stationary throughput of each circuit is identical with marking $\boldsymbol{M}_0$ while the stationary throughput of the system is greater than that of $\boldsymbol{M}_0$.

TABLE II

STATIONARY THROUGHPUT ANALYSIS OF THE PRODUCTION LINE.

Marking	β_{γ_1}	β_{γ_2}	β	R_{γ_1}	R_{γ_2}	R
$\boldsymbol{M}_0 = (2,1,22,0,0)^T$	0.182	0.167	0.13	24	26	50
$\boldsymbol{M}_1 = (0,0,24,0,0)^T$	0.182	0.167	0.167	24	24	48

B. Discussion

From Example 2, we find that the total number of resources used for marking $\boldsymbol{M}_1$ is 48 which is smaller than that of $\boldsymbol{M}_0$, while the stationary throughput of system is greater than that of $\boldsymbol{M}_0$. This has practical significance for the stationary throughput optimization problem which consists in finding an initial marking to maximize the stationary throughput of the system with a bounded resources. It means that marking $\boldsymbol{M}_1$ is better than $\boldsymbol{M}_0$ because it has a smaller resources used. In the following, we will further discuss this problem.

We study the TWMG systems $\langle N, \boldsymbol{M}_0\rangle$ and $\langle N, \boldsymbol{M}_1\rangle$ by analyzing the two circuits. Figs. 2 and Fig. 3 show the marking distribution of γ_1 and γ_1 associate to $\boldsymbol{M}_0$ and $\boldsymbol{M}_1$. We have $\boldsymbol{M}_0^{\gamma_1} = (0,0,24,0,0)$, $\boldsymbol{M}_0^{\gamma_2} = (0,0,24,0,0)$, $\boldsymbol{M}_1^{\gamma_1} = (2,0,22,0,0)$, and $\boldsymbol{M}_1^{\gamma_2} = (1,0,22,0,0)$.

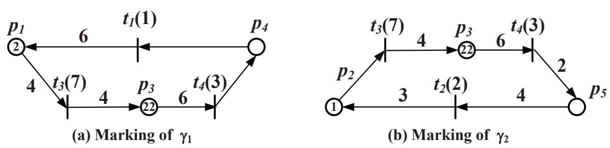

Fig. 2. The marking of each circuit under $\boldsymbol{M}_0$.

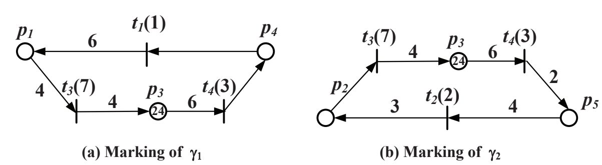

Fig. 3. The marking of each circuit under $\boldsymbol{M}_1$.

For $\boldsymbol{M}_0{}^{\gamma_1}$, we can firing transitions $t_4t_1t_3t_3$ in order and obtain a new marking $(0,0,24,0,0)^T$ which is identical to marking $\boldsymbol{M}_1^{\gamma_1}$, namely, $\boldsymbol{M}_1^{\gamma_1} \in R(N, \boldsymbol{M}_0^{\gamma_0})$. For $\boldsymbol{M}_0^{\gamma_2}$, transition t_3 can be fired which results in a new marking $(0,0,26,0,0)^T$ and this new marking has more tokens in p_3 than marking $\boldsymbol{M}_1^{\gamma_1}$. Thus, it seems that the stationary throughput of marking $\boldsymbol{M}_0$ should not be smaller than that of marking $\boldsymbol{M}_1$, which is contrary to the result shown in Table II. The fact is that for the TWMG system t_3 is not enabled because $\boldsymbol{M}_0(p_1) < Pre(p_1, t_1)$. Thus, two tokens in p_1 and one token in p_2 will be trapped, i.e., cannot be used for the system at marking $\boldsymbol{M}_0$, while no tokens are trapped at marking $M_1^{\gamma_1}$. This is mainly due to the synchronization of the two circuits. Each of them becomes mutually constrained and results in a lower stationary throughput of the system.

The existing solutions in [12] and [13] to solve the optimization problem of a TWMG are non-polynomial and the optimal solution for this problem is a challenging problem from a theoretical as well as from practical point of view. Properties 1 and 2 provide some practical significance to study the stationary throughput analysis and the optimization problem for a TWMG.

IV. Conclusion

In this paper, we discuss the stationary behavior of manufacturing systems in timed discrete event systems framework. We provide some results for the stationary throughput analysis of TWMGs under infinite server semantics and show that these results may be useful for the optimization problem for manufacturing systems.

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