

A Constraint Transformation Technique for Petri Nets with Certain Uncontrollable Structures

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Abstract

In this paper we study the problem of constraint transformation. We consider a special type of system in which the uncontrollable subnet is an assembly flow system, which is a subclass of backward-synchronization-backward-conflict-free Petri net. We propose an algorithm to transform a given inadmissible GMEC into an equivalent admissible OR-GMEC. The algorithm is based on a technique that adds new constraints obtained by composition of elementary ones.

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1 Introduction

Generalized Mutual Exclusion Constraints (GMECs) [1] represent an efficient control approach in Petri nets which has drawn lots of attentions in recent years. A single GMEC defines a set of legal markings, and any marking which is not legal should be prohibited by a controller. The GMEC approach has many advantages since a single GMEC can be easily implemented by one monitor place without much computations. Many type of forbidden marking problems in Petri nets, such as deadlock prevention [2], can be solved in the framework of GMECs.

When uncontrollable transitions are present in a Petri net model, as it is common in the supervisory control framework, such an implementation becomes difficult. A GMEC is said to be *uncontrollable* if the firing of some uncontrollable transitions will increase its token count thus possibly leading to a violation of the constraint. For a given uncontrollable GMEC we have to propose a more restrictive control policy which prohibits not only the forbidden markings but also some other weakly forbidden markings, from which the system may uncontrollably violate the control law.

Up to now, both *on-line* and *off-line* approaches have been proposed to solve this problem. In the on-line approaches, in each step an integer or linear programming problem [3] has to be solved. The off-line approaches seek a solution substituting a given uncontrollable GMEC by one or more controllable GMECs: this technique is also called *GMEC transformation*. The off-line approaches have their advantages since they provide a closed form solution and do not require exhaustive on-line computations. However, in [1] it has been proved that in some cases there do not exist a single GMEC which is equivalent to an given uncontrollable GMEC. [4] proposed a method to transform a given uncontrollable GMEC into a new controllable GMEC. Their approach is very efficient in computation but the solution is suboptimal, i.e., some legal markings may no longer be reachable. [5] studied a very similar problem and proposed an algorithm to estimate the maximal number of tokens a place may uncontrollably get. Furthermore, Luo and Wang [6–8] extensively studied the GMEC transformation problem in different subclass of Petri net systems, e.g., *forward-synchronization-forward-conflict-free* nets.

Recently, Luo *et al.* extended their approach to solve GMEC transformation problems with fairly arbitrary uncontrollable subnet structures [9]. However, we believe that some key results in [9] are not correct [10]. Therefore, although we believe that the GMEC transformation technique is an interesting and fruitful technique to explore, the general GMEC transformation problem still lacks a general solution.

In this paper, we focus on the Petri net models in which the uncontrollable subnet is an *assembly-flow system* (AF system, which will be defined in Section 3). An AF system is sequentially composed by several *assembly-workstations* (AWs), each of which contains a shared source place, several sequential work flows, an assembly transition, and a sink place. An AF system belongs to the Petri net subclass *backward-*

synchronization-backward-conflict-free nets (BSBCF nets), i.e., each transition has no more than one output arc and each place has no more than one input arc. AF system can model assembly work flows with a shared resource. The model includes both conflicts and synchronizations. We also assume that the initial uncontrollable GMEC to be transformed imposes an upper bound for the marking of the final sink place. This type of GMEC is commonly used to trim an uncontrollable supervisor [11] in the supervisory control framework .

To our knowledge, the GMEC transformation problem in AF systems and BSBCF nets has not been studied yet. Therefore the contribution of this paper is two-fold. First, the structure and properties of AW and AF uncontrollable structure are introduced and characterized. Secondly, we propose an approach called *GMEC composition* to obtain a maximally permissive solution which is a disjunction of GMECs called *OR-GMEC*. Such type of GMEC can be implemented by a Petri net controller [12, 13]. This GMEC composition technique can well handle the conflict-synchronization structure in GMEC transformation. We note that the AW and AF model are simple but contain the main features, such as conflict and synchronization, that characterize complex systems. We believe that these results represent a significant step towards the final goal of solving the GMEC transformation problem for general classes of Petri nets.

The paper is organized in six sections. Section 2 recalls the basic notions on Petri net and GMECs. Section 3 introduces the definition of assembly flow system and state the problem. In Section 4 an algorithm based on GMEC composition operation is proposed to transform a given GMEC into an equivalent admissible OR-GMEC if the uncontrollable subnet is an assembly workstation. Section 5 extends the approach in Section 4 to the systems in which the uncontrollable subnet is an assembly-flow system. Section 6 draws the conclusions.

2 Preliminaries

2.1 Petri Net

A Petri net is a four-tuple $N = (P, T, Pre, Post)$, where P is a set of m places represented by circles; n transitions represented by bars; $Pre : P \times T \rightarrow \mathbb{N}$ and $Post : P \times T \rightarrow \mathbb{N}$ are the *pre-* and *post-incidence functions* that specify the arcs in the net and are represented as matrices in $\mathbb{N}^{m \times n}$ (here $\mathbb{N} = \{0, 1, 2, \dots\}$). The *incidence matrix* of a net is defined by $C = Post - Pre \in \mathbb{Z}^{m \times n}$ (here $\mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$).

For a transition $t \in T$ we define its *set of input places* as $\bullet t = \{p \in P \mid Pre(p, t) > 0\}$ and its *set of output places* as $t \bullet = \{p \in P \mid Post(p, t) > 0\}$. The notion for $\bullet p$ and $p \bullet$ are analogously defined.

A *marking* is a vector $M : P \rightarrow \mathbb{N}$ that assigns to each place of a Petri net a non-negative integer number of tokens, represented by black dots and can also be represented as a m component vector. We denote by $M(p)$

the marking of place p . A *marked net* $\langle N, M_0 \rangle$ is a net N with an initial marking M_0 . We denote by $R(N, M_0)$ the set of all markings reachable from the initial one.

A transition t is *enabled* at M if $M \geq Pre(\cdot, t)$ and may fire reaching a new marking $M' = M_0 + C(\cdot, t)$. We write $M[\sigma]$ to denote that the sequence of transitions σ is enabled at M , and we write $M[\sigma]M'$ to denote that the firing of σ yields M' .

The transition set T can be partitioned into T_c and T_u which represent the controllable and uncontrollable transition set, respectively. A transition $t_u \in T_u$ is not controllable, i.e., it cannot be disabled by control places.

Given a net $N = (P, T, Pre, Post)$ we say that $\hat{N} = (\hat{P}, \hat{T}, \hat{Pre}, \hat{Post})$ is a subnet of N if $\hat{P} \subset P$, $\hat{T} \subset T$ and \hat{Pre} (resp., \hat{Post}) is the restriction of Pre (resp., $Post$) to $\hat{P} \times \hat{T}$. \hat{N} is said to be the *uncontrollable subset* of N if $\hat{T} = T_u$ and $\hat{P} = \{p \in P \mid (\bullet p \cup p \bullet) \cap T_u \neq \emptyset\}$.

In a net $N = (P, T, Pre, Post)$, a *path* is a sequence directed from a node x_1 to a node x_k is a sequence $\pi = x_1 x_2 \cdots x_k$ such that $x_i \in P \cup T$ for all $i = 1, \dots, k$, and $x_i \in \bullet x_{i+1}$ for all $i = 1, \dots, k-1$. A *maximal path* is a path π such that $|\bullet x_1| = |x_k \bullet| = 0$.

2.2 GMECs

A Generalized Mutual Exclusion Constraint (GMEC) is a pair (\mathbf{w}, k) where $\mathbf{w} \in \mathbb{Z}^m$ and $k \in \mathbb{N}$. A GMEC defines a set of legal markings:

$$\mathcal{L}_{(\mathbf{w}, k)} = \{M \in \mathbb{N}^m \mid \mathbf{w}^T \cdot M \leq k\}$$

OR-GMEC [13]: An *OR-GMEC* is a set of GMECs: $\xi = \{(\mathbf{w}_1, k_1), \dots, (\mathbf{w}_r, k_r)\}$. An OR-GMEC defines a set of legal markings:

$$\mathcal{L}_{OR}(\xi) = \{M \in \mathbb{N}^m \mid \exists i \in \{1, \dots, r\}, \mathbf{w}_i^T \cdot M \leq k_i\}$$

For sake of simplicity in the following we denote $\mathcal{L}_{OR}(\xi)$ by $\mathcal{L}(\xi)$.

3 Assembly Flow System and Problem Statement

We first gives the definition of *Assembly Flow System*.

Definition 1 An assembly workstation (AW) is an ordinary Petri net $N_A = (P_w \cup \{p_{st}\} \cup \{p_{ed}\}, T_w \cup \{t_a\}, Pre, Post)$ which satisfies: (1) in N_A all maximal paths start from p_{st} and end in $t_a p_{ed}$; (2) in N_A for any two maximal paths π_1, π_2 starting from p_{st} and ending in p_{ed} , if $\pi_1 \neq \pi_2$, then $\pi_1 \cap \pi_2 = \{p_{st}, p_{ed}, t_a\}$. \triangle

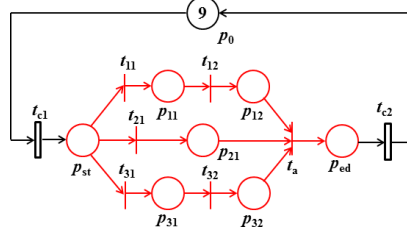


Figure 1: An example of AW.

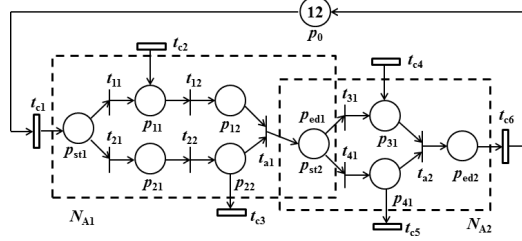


Figure 2: An example of AF.

Definition 2 An Assembly Flow System (AF system) is a Petri net N which is composed by q assembly workstations N_{A1}, \dots, N_{Aq} in which $N_{Ai} = (P_A, T_A, Pre_A, Post_A)$ is an AW and the following condition holds: (1) $P = \bigcup_i P_{Ai}, P_i \cap P_{i+1} = \{p_{i,ed}\} = \{p_{i+1,st}\}, \forall |i - j| > 1, P_i \cap P_j = \emptyset$; (2) $T = \bigcup_i T_{Ai}, \forall i \neq j, T_i \cap T_j = \emptyset$. \triangle

We give the illustration of an AW and an AF system as an example.

Example 1 In Figure 1, the controllable transition set is $T_c = \{t_{c1}, t_{c2}\}$ and the uncontrollable transition set is $T_u = \{t_{11}, t_{12}, t_{21}, t_{31}, t_{32}, t_a\}$. The subnet N (in red) is an assembly workstation. In Figure 2, the controllable transition set is $T_c = \{t_{c1}, t_{c2}, t_{c3}, t_{c4}, t_{c5}, t_{c6}\}$ and the uncontrollable transition set is $T_u = T \setminus T_c$. The subnet N (the union of two dashed boxes) is an assembly flow system composed by two assembly workstations N_{A1} and N_{A2} (in each dashed box, respectively). \triangle

From the example one can see that an AW contains a shared source place p_{st} followed by several sequential work flows π_i . All these work flows merge at an assembly transition $t_{i,a}$ which is followed by a sink place p_{ed} . From the definition, the AW and AF systems do not belong to the Petri net subclass *marked graph* nor *state machine* since each assembly transition $t_{i,a}$ has more than one inputs and each start place $p_{i,st}$ has more than one outputs. The AF systems belong to the Petri net subclass *backward-synchronization-backward-conflict-free nets*, i.e., each transition has no more than one output arc and each place has no more than one input arc. Note that an AF system contains a conflict at the starting place and a synchronization at the assembly transition.

For a given $\mathcal{L}_{(w,k)}$, due to the existence of uncontrollable transitions, there may exist some legal marking $M \in \mathcal{L}_{(w,k)}$ from which an illegal sequence of uncontrollable transitions $\sigma_u \in T_u^*$ may fire, i.e., $M[\sigma_u]M' \notin \mathcal{L}_{(w,k)}$

$\mathcal{L}_{(\mathbf{w},k)}$. Correspondingly we define the set of *admissible markings* $\mathcal{A}_{(\mathbf{w},k)}$ to characterize this situation:

$$\mathcal{A}_{(\mathbf{w},k)} = \{M \in \mathbb{N}^{|P|} \mid (\exists \sigma_u \in T_u^*) M[\sigma_u]M' \notin \mathcal{L}_{(\mathbf{w},k)}\} \quad (1)$$

We say that a (\mathbf{w},k) is not admissible if $\mathcal{L}_{(\mathbf{w},k)} \neq \mathcal{A}_{(\mathbf{w},k)}$. Since there is no way to prevent the firing of an uncontrollable sequence, all markings not in $\mathcal{A}_{(\mathbf{w},k)}$ should be avoided, that is, we need to determine an admissible ξ such that $\mathcal{L}(\xi) = \mathcal{A}_{(\mathbf{w},k)}$.

Finally we state the problem which we will study in the following of this paper.

Problem 1 Consider a Petri net N_0 and assume that the uncontrollable subnet of N_0 is an AF system N which is composed by q assembly workstations N_{A1}, \dots, N_{Aq} . Given a GMEC (\mathbf{w}_0, k_0) such that $\mathcal{L}_{(\mathbf{w}_0, k_0)} = \{M \mid M(p_{q,ed}) \leq k_0\}$, determine an equivalent admissible OR-GMEC ξ such that $\mathcal{L}(\xi) = \mathcal{A}_{(\mathbf{w},k)}$. \triangle

4 Composition of GMECs in Single Assembly Workstation

We start with the case in which there exists only one AW in N , i.e., $q = 1$. Since in an AW all maximal paths start in p_{st} and end at $t_a p_{ed}$. We use π_i to denote these paths, i.e., π_1, \dots, π_r where r is the number of maximal paths.

Definition 3 Given an assembly workstation N containing r maximal paths and given a legal marking set $\mathcal{L}_{(\mathbf{w},k)}$ in which (\mathbf{w},k) is a GMEC: $M(p_{ed}) \leq k_0$, the elementary constraint set of (\mathbf{w},k) is defined as $C_{(\mathbf{w},k)} = \{(\mathbf{w}_i, k_i), 1 \leq i \leq r\}$, in which each (\mathbf{w}_i, k_i) is computed by the following procedure:

$$\begin{cases} w_i(p) = 1, p \in \pi_i \\ w_i(p) = 0, \text{ else} \\ k_i = k_0 \end{cases} \quad (2)$$

\triangle

We use ξ_C to denote the disjunction of GMECs in $C_{(\mathbf{w},k)}$, i.e., $\mathcal{L}(\xi_C) = \{M \mid \exists (\mathbf{w},k) \in C_{(\mathbf{w},k)}, \mathbf{w}^T \cdot M \leq k\}$.

Proposition 1 Any $(\mathbf{w}_i, k_i) \in C_{(\mathbf{w},k)}$ is non-increasing, i.e., the firing of any transition in T_u will not increase the token count of (\mathbf{w}_i, k_i) .

Proof: Consider an arbitrary GMEC $(\mathbf{w}_i, k_i) \in C_{(\mathbf{w},k)}$. From the definition of $C_{(\mathbf{w},k)}$ a transition t in N must satisfy one of the following cases:

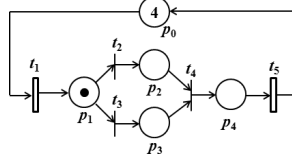


Figure 3: The Petri net for Example 2.

- (1) $t \in \pi_i$, then $\sum w_i(\bullet t) = \sum w_i(t^\bullet) = 1$;
- (2) $t \notin \pi_i, t \notin p_{st}^\bullet$, then $\sum w_i(\bullet t) = \sum w_i(t^\bullet) = 0$
- (3) $t \notin \pi_i, t \in p_{st}^\bullet$, then $\sum w_i(\bullet t) = 1, \sum w_i(t^\bullet) = 0$

Therefore the firing of any $t \in T_u$ does not increase the token count of $(\mathbf{w}_i, k_i) \in C_{(\mathbf{w},k)}$. \square \blacksquare From Proposition 1 we immediately have the following corollary.

Corollary 1 ξ_C is controllable.

The following theorem shows that any marking which satisfies ξ_C is admissible.

Theorem 1 Given an assembly workstation N and $\mathcal{L}_{(\mathbf{w},k)} = \{M | M(p_{ed}) \leq k_0\}$, it holds: $\mathcal{L}(\xi_C) \subseteq \mathcal{A}_{(\mathbf{w},k)}$.

Proof: For all $M \in \mathcal{L}(\xi_C)$, there must exist at least one (\mathbf{w}_i, k_i) in $C_{(\mathbf{w},k)}$ such that $\mathbf{w}_i^T \cdot M \leq k_i$. Suppose $\mathbf{w}_i^T \cdot M \leq k_i = k_0$. For all $M' \notin \mathcal{L}_{(\mathbf{w},k)}$, since $M'(p_{ed}) > k_0$, it must hold $\mathbf{w}_i^T \cdot M' > k_0$. According to Proposition 1, the firing of any transitions in N will not increase the token count of (\mathbf{w}_i, k_i) , therefore M would never reach $M' \notin \mathcal{L}_{(\mathbf{w},k)}$. \square \blacksquare

Unfortunately, the converse of Theorem 1 does not hold: typically $\mathcal{L}(\xi_C) \neq \mathcal{A}_{(\mathbf{w},k)}$. We prove this by means of an example.

Example 2 Consider the Petri net N_0 in Figure 3 in which $T_u = \{t_2, t_3, t_4\}$. Suppose the initial constraint is $(\mathbf{w}_0, k_0) = ([0, 0, 0, 1], 1)$, i.e., $M(p_4) \leq 1$. One can easily obtain $C_{(\mathbf{w}_0, k_0)} = \{([1, 1, 0, 1], 1), ([1, 0, 1, 1], 1)\}$. Therefore $M = [2, 0, 0, 0] \notin \mathcal{L}(\xi_C)$ since it violates both constraints. However, it is obvious that $M \in \mathcal{A}_{(\mathbf{w}_0, k_0)}$. \triangle

Example 2 shows that if one uses ξ_C to design a controller, all reachable markings of the closed-loop system are legal, but the controller is not always optimal, i.e., it is not maximally permissive. Notice that the two GMECs in ξ_C are exactly the two possible solutions obtained by Moody and Antsaklis' approach [4], this indicates that the disjunction of all the suboptimal GMECs in Moody and Antsaklis' approach is not always the optimal OR-GMEC solution for the system.

To solve this problem we introduce the *GMEC composition technique*.

Definition 4 Given an elementary constraint set: $C_{(\mathbf{w},k)} = \{(\mathbf{w}_1, k_1), \dots, (\mathbf{w}_r, k_r)\}$, let $X_i, i = 1, \dots, (2^r - 1)$ be the non-empty subset of $C_{(\mathbf{w},k)}$. For each constraints set X_i we use $s_{X_i} \in \mathbb{N}^r$ to denote the support vector of X_i : $(\mathbf{w}_j, k_j) \in X_i \Rightarrow s_{X_i}(j) = 1$, else $s_{X_i}(j) = 0$. \triangle

Definition 5 Given an assembly workstation N , a GMEC $(\mathbf{w}, k) : M(p_{ed}) \leq k$ and its elementary constraint set $C_{(\mathbf{w},k)}$, the composition GMEC set is defined as $D_{(\mathbf{w},k)} = \{(\mathbf{w}_i, k_i) | i = 1, \dots, (2^r - 1)\}$, in which for all $X_i, (\mathbf{w}_i, k_i)$ is computed by the following procedure:

$$\begin{cases} w_i(p_{st}) = 1 \\ w_i(p) = \sum_{j=1}^r s_{X_i}(j) \cdot w_j(p), p \neq p_{st} \\ k_i = (\sum_{j=1}^r s_{X_i}(j) \cdot (k_j + 1)) - 1 \end{cases} \quad (3)$$

We say that (\mathbf{w}_i, k_i) is composed by the j -th constraint for all $j : s_{X_i}(j) = 1$. \triangle

We let ξ_D be the disjunctive GMEC of $D_{(\mathbf{w},k)}$, i.e., $\mathcal{L}(\xi_D) = \{M | \exists (\mathbf{w}, k) \in D_{(\mathbf{w},k)}, \mathbf{w}^T \cdot M \leq k\}$. Obviously $\mathcal{L}(\xi_C) \subseteq \mathcal{L}(\xi_D)$.

Like the GMECs in $C_{(\mathbf{w},k)}$, the GMECs in $D_{(\mathbf{w},k)}$ are also non-increasing, which indicates ξ_D the disjunction of GMECs in $D_{(\mathbf{w},k)}$ is controllable.

Proposition 2 Any $(\mathbf{w}, k) \in D_{(\mathbf{w},k)}$ is non-increasing.

Proof: Consider an arbitrary GMEC (\mathbf{w}', k') in $D_{(\mathbf{w},k)}$ which is composed by several (\mathbf{w}_j, k_j) in C . Without loss of generality suppose (\mathbf{w}', k') is composed by the first \bar{r} constraints in $C_{(\mathbf{w},k)}$, i.e., $(\mathbf{w}_1, k_1), \dots, (\mathbf{w}_{\bar{r}}, k_{\bar{r}})$.

Since $\forall \pi_1, \pi_2$ in N , $(\pi_1 \neq \pi_2) \Rightarrow (\pi_1 \cap \pi_2 = \{p_{st}, t_a, p_{ed}\})$, then for all places $p \in \bigcup_1^{\bar{r}} \pi_i, p \neq p_{ed}, w'(p) = 1$, for all places $p \notin \bigcup_1^{\bar{r}} \pi_i, w'(p) = 0$, and $w'(p_{ed}) = \bar{r}$. Then, any transition t in N must satisfy one of the following cases:

- (1) $t \in \bigcup_1^{\bar{r}} \pi_i, t \neq t_a$, then $\sum w'(\bullet t) = \sum w'(t\bullet) = 1$;
- (2) $t = t_a$, then $\sum w'(\bullet t_a) = \sum w'(t_a\bullet) = \bar{r}$;
- (3) $t \notin \bigcup_1^{\bar{r}} \pi_i, t \notin p_{st}^\bullet$, then $\sum w'(\bullet t) = \sum w'(t\bullet) = 0$;
- (4) $t \notin \bigcup_1^{\bar{r}} \pi_i, t \in p_{st}^\bullet$, then $\sum w'(\bullet t) = 1, \sum w'(t\bullet) = 0$;

In each case the firing of t will not increase the token count of (\mathbf{w}', k') . Therefore any $(\mathbf{w}, k) \in D_{(\mathbf{w},k)}$ is non-increasing.. \square \blacksquare From Proposition 2 we immediately have the following corollary.

Corollary 2 *The firing of any $t \in \bigcup_1^{\bar{r}} \pi_i$ ($\bar{r} \leq r$) will not change the token count of $(\mathbf{w}, k) \in D_{(\mathbf{w}, k)}$ where (\mathbf{w}, k) is composed by $(\mathbf{w}_1, k_1), \dots, (\mathbf{w}_{\bar{r}}, k_{\bar{r}})$.*

Proof: Trivial, since t must be in one of the first two cases in the proof of Proposition 2. \square \blacksquare

Now we present the following theorem showing that a marking M is legal if and only if M satisfies ξ_D .

Theorem 2 *Given an assembly workstation N and given $(\mathbf{w}, k) : M(p_{ed}) \leq k_0$, it holds: $\mathcal{L}(\xi_D) = \mathcal{A}_{(\mathbf{w}, k)}$.*

Proof: We prove this theorem by proving $\mathcal{L}(\xi_D) \subseteq \mathcal{A}_{(\mathbf{w}, k)}$ and $\mathcal{L}(\xi_D) \supseteq \mathcal{A}_{(\mathbf{w}, k)}$.

$(\mathcal{L}(\xi_D) \subseteq \mathcal{A}_{(\mathbf{w}, k)})$: for any marking $M \in \mathcal{L}(\xi_D)$, there must exist at least one constraint (\mathbf{w}', k') in $D_{(\mathbf{w}, k)}$ such that $\mathbf{w}^T \cdot M \leq k'$. Without loss of generality, suppose (\mathbf{w}', k') is composed by the first \bar{r} constraints in $C_{(\mathbf{w}, k)}$. Notice that for any $M' \notin \mathcal{L}_{(\mathbf{w}, k)}$, $M'(p_{ed}) \geq k_0 + 1$. Since $w'(p_{ed}) = \bar{r}$, it must hold $\mathbf{w}'^T \cdot M' \geq \bar{r}(k_0 + 1) > [\bar{r} \cdot (k_0 + 1) - 1] = k'$. Since $\mathbf{w}^T \cdot M \leq k'$, according to Proposition 2, the firing of any transitions in N will not increase the token count of (\mathbf{w}', k') . Therefore M would never reach M' such that $\mathbf{w}'^T \cdot M' > k'$, which indicates $\mathcal{L}(\xi_D) \subseteq \mathcal{A}_{(\mathbf{w}, k)}$.

$(\mathcal{L}(\xi_D) \supseteq \mathcal{A}_{(\mathbf{w}, k)})$: Suppose M violates all constraints in $D_{(\mathbf{w}, k)}$. We prove that M can always evolve to a marking M' which is not in $\mathcal{L}_{(\mathbf{w}, k)}$ by firing only uncontrollable transitions.

For each $(\mathbf{w}_i, k_i) \in C_{(\mathbf{w}, k)}$, we construct a corresponding (\mathbf{w}'_i, k_i) such that $w'_i(p_{st}) = 0, \forall p \neq p_{st}, w'_i(p) = w_i(p)$, and we put all these GMEC in a set C' . Without loss of generality, we suppose M satisfies the first $\hat{r} \leq r$ constraints of C' . Then we do the following operation:

- Step-1: Let $t \in (p_{st}^\bullet \cap \pi_1)$ fire under M until the current marking M_1 satisfies: $\mathbf{w}'_1 \cdot M_1 = k_1 + 1$, i.e., (\mathbf{w}'_1, k_1) is just violated.
- Step-2: Let $t \in (p_{st}^\bullet \cap \pi_2)$ fire under M_1 until the current marking M_2 satisfies: $\mathbf{w}'_2 \cdot M_2 = k_2 + 1$, i.e., (\mathbf{w}'_2, k_2) is just violated.
- ...
- Step- \hat{r} : Let $t \in (p_{st}^\bullet \cap \pi_{\hat{r}})$ fire under $M_{\hat{r}-1}$ until the current marking $M_{\hat{r}}$ satisfies: $\mathbf{w}'_{\hat{r}} \cdot M_{\hat{r}} = k_{\hat{r}} + 1$, i.e., $(\mathbf{w}'_{\hat{r}}, k_{\hat{r}})$ is just violated.

We claim that this operation can always be carried out until step \hat{r} . Since each firing will modify the token counts of corresponding GMECs by one at most, if the \bar{r} -step ($\bar{r} < \hat{r}$) cannot be done, i.e., $M_{\bar{r}}(p_{st}) = 0$, the current marking $M_{\bar{r}}$ must satisfy:

$$\begin{cases} \sum_j M_{\bar{r}}(p_{ij}) + M_{\bar{r}}(p_{ed}) = k_i + 1, p_{ij} \in \pi_i, \forall i < \bar{r} \\ M_{\bar{r}}(p_{st}) + \sum_j M_{\bar{r}}(p_{\bar{r}j}) + M_{\bar{r}}(p_{ed}) < k_{\bar{r}} + 1, p_{\bar{r}j} \in \pi_{\bar{r}} \end{cases} \quad (4)$$

Therefore we have:

$$M_{\bar{r}}(p_{st}) + \sum_{i=1}^{\bar{r}} \sum_j M_{\bar{r}}(p_{ij}) + \bar{r} \cdot M_{\bar{r}}(p_{ed}) \leq \bar{r} \cdot (k_0 + 1) - 1 \quad (5)$$

which indicates $M_{\bar{r}}$ satisfies at least one $(\mathbf{w}'', k'') \in D_{(\mathbf{w}, k)}$ with the corresponding $\mathbf{s}_{(\mathbf{w}'', k'')} : s_{(\mathbf{w}'', k'')}(j) = 1, j \leq \bar{r}, s_{(\mathbf{w}'', k'')}(j) = 0, j > \bar{r}$. According to Corollary 2 the token count of (\mathbf{w}'', k'') should be the same under $M_{\bar{r}}$ and M , indicating M satisfies (\mathbf{w}'', k'') . This contradicts the fact that M violates all GMECs in $D_{(\mathbf{w}, k)}$. Therefore, this operation can be done completely and a marking M_r , which violates all (\mathbf{w}'_i, k_i) , is reached.

If $M_r(p_{ed}) > k_0, M_r \notin \mathcal{L}_{(\mathbf{w}, k)}$. Suppose $M_r(p_{ed}) = \bar{k} \leq k_0$ and all places in $P \setminus \{p_{st}, p_{ed}\}$ are empty (this can be done by repeatedly firing the transitions in $T \setminus \{t_{i1}, t_a\}$). Since for any $p_i \in \bullet t_a$ it holds $\sum_j M_r(p_{ij}) + M_r(p_{ed}) \geq k_0 + 1$, therefore there are at least $k_0 + 1 - \bar{k}$ tokens in each p_i . We let t_a fire $k_0 + 1 - \bar{k}$ times under M_r and then it would reach a marking M_B such that $M_B(p_{ed}) \geq k_0 + 1$, indicating $M_B \notin \mathcal{L}_{(\mathbf{w}, k)}$. Therefore $M \notin \mathcal{A}_{(\mathbf{w}, k)}$. Therefore $\mathcal{L}(\xi_D) \supseteq \mathcal{A}_{(\mathbf{w}, k)}$. \square \blacksquare

5 Assembly-Flow System

In this section we consider the case in which there are $q > 1$ AWs in an AF. We first give some extended definitions.

Definition 6 (Weakly non-increasing GMEC) *Given an assembly flow system N containing q assembly workstations N_{A1}, \dots, N_{Aq} , a GMEC (\mathbf{w}, k) is said to be weakly non-increasing with an index \bar{q} if the following two conditions hold: (1) the firing of $t_{\bar{q}-1, a}$ will increase the token count of (\mathbf{w}, k) ; (2) the firing of any other transition $t \neq t_{\bar{q}-1, a}$ will not increase the token count of (\mathbf{w}, k) . In particular, if there does not exist a $t_{\bar{q}-1, a}$ which satisfies the first condition, we say (\mathbf{w}, k) is weakly non-increasing with index 1. \triangle*

Example 3 *Consider the uncontrollable subnet in Figure 2. The following GMEC: $M(p_{ed1}) + M(p_{31}) + M(p_{ed2}) \leq 1$ is a weakly non-increasing GMEC with an index 2, because only the firing of $t_{1, a}$ in N_{A1} will increase its token count. \triangle*

We point out that the weakly non-increasing property is a basis of our approach (see Theorem 3). We will show, at the end of this section, that to solve Problem 1 in each iteration of Algorithm 1 the resulting GMECs

are always weakly non-increasing.

Then we extend the definition of elementary GMEC set $C_{(\mathbf{w},k)}$ and composition GMEC set $D_{(\mathbf{w},k)}$. We use $\pi_{j,i}$ to denote the path corresponding to the i -th work flow in N_{A_j} .

Definition 7 Given an assembly flow system N containing q assembly workstations (sequentially) N_{A_1}, \dots, N_{A_q} and given a GMEC (\mathbf{w}, k) which is weakly non-increasing with an index $\bar{q} + 1$. Suppose $N_{A_{\bar{q}}}$ contains r work flows. The elementary constraint set of (\mathbf{w}, k) is defined as $C_{(\mathbf{w},k)} = \{(\mathbf{w}_i, k_i), 1 \leq i \leq r\}$ is a set of GMEC, in which each (\mathbf{w}_i, k_i) is computed by the following procedure:

$$\begin{cases} w_i(p) = 1, p \in \pi_{\bar{q},i} \\ w_i(p) = w(p), p \in \pi_{\hat{q}}, \hat{q} \geq \bar{q} + 1 \\ w_i(p) = 0, \text{else} \\ k_i = k_0 \end{cases} \quad (6)$$

The composition GMEC set is defined as $D_{(\mathbf{w},k)} = \{(\mathbf{w}_i, k_i) | i = 1, \dots, (2^r - 1)\}$, in which for all $X_i, (\mathbf{w}_i, k_i)$ is computed by the following procedure:

$$\begin{cases} w_i(p_{\bar{q},st}) = 1 \\ w_i(p) = \sum_{j=1}^r s_{X_i}(j) \cdot w_j(p), p \neq p_{\bar{q},st} \\ k_i = (\sum_{j=1}^r s_{X_i}(j) \cdot (k_j + 1)) - 1 \end{cases} \quad (7)$$

△

The following proposition shows that all constraints in $C_{(\mathbf{w},k)}$ and $D_{(\mathbf{w},k)}$ are weakly non-increasing with a decreased index.

Proposition 3 For any GMEC (\mathbf{w}, k) which is weakly non-increasing with an index \bar{q} : (1) The firing of any $t \in T_{A_{\hat{q}}}, \hat{q} \geq (\bar{q} - 1)$ will not increase the token count of any GMEC in $D_{(\mathbf{w},k)}$; (2) The firing of any $t \in T_{A_{\bar{q}-1}}, t \in \bigcup_{j=1}^{\bar{r}} \pi_{\bar{q}-1,j}$ ($\bar{r} \leq r$) will not change the token count of $(\mathbf{w}', k') \in D_{(\mathbf{w},k)}$ where (\mathbf{w}', k') is composed by first \bar{r} GMECs in $C_{(\mathbf{w},k)}$.

Proof: For simplicity we do not present a formal proof. These properties can be deduced analogously to the proofs of Proposition 1, 2 and Corollary 2. □ ■

Corollary 3 If (\mathbf{w}, k) is a weakly non-increasing GMEC with an index \bar{q} , then each GMECs in $D_{(\mathbf{w},k)}$ is weakly non-increasing with an index $\bar{q} - 1$.

Proof: Straight forward from Proposition 3. □ ■

If the index $\bar{q} > 1$, the constraints in $C_{(\mathbf{w},k)}$ and $D_{(\mathbf{w},k)}$ are not controllable, i.e., $\mathcal{L}(\xi_D) \supset \mathcal{A}(\xi_D)$, because the firing of $t_{\bar{q}-1,a}$ will increase the token count of the GMECs in $C_{(\mathbf{w},k)}$ and $D_{(\mathbf{w},k)}$. This means that $M \in \mathcal{L}(\xi_D) \not\Rightarrow M \in \mathcal{A}_{(\mathbf{w},k)}$. However, the following theorem show that a marking M is legal if and only if $M \in \mathcal{A}(\xi_D)$. The proof is analogous to the proof of Theorem 1.

Theorem 3 *Given an assembly flow system N containing q AW N_{A1}, \dots, N_{Aq} and given a GMEC (\mathbf{w}, k) which is weakly non-increasing with index $\bar{q} > 1$, then it holds: $\mathcal{A}(\xi_D) = \mathcal{A}_{(\mathbf{w},k)}$.*

Proof: $\mathcal{A}(\xi_D) \subseteq \mathcal{A}_{(\mathbf{w},k)}$: suppose there exists a marking $M_0 \in \mathcal{A}(\xi_D)$. It indicates that for all M which is uncontrollably reachable from M_0 , M must satisfy at least one GMEC (\mathbf{w}', k') in ξ_D . Due to the same reasoning of Theorem 1, for any marking $M' \notin \mathcal{L}_{(\mathbf{w},k)}$, it must holds $\mathbf{w}'^T \cdot M > k'$. This means that M would never evolve to M' which violates (\mathbf{w}, k) .

$\mathcal{A}(\xi_D) \supseteq \mathcal{A}_{(\mathbf{w},k)}$: suppose there exists a marking $M_0 \notin \mathcal{A}(\xi_D)$, i.e., from M_0 by firing some uncontrollable transitions, a marking M which violates all constraints in ξ_D can be reached. We prove that M can always evolve to a marking $M' \notin \mathcal{L}_{(\mathbf{w},k)}$ by firing only uncontrollable transitions.

For each $(\mathbf{w}_i, k_i) \in C_{(\mathbf{w},k)}$, we construct a corresponding (\mathbf{w}'_i, k_i) such that $w'(p_{\bar{q}-1,st}) = 0$ and $\forall p \neq p_{\bar{q}-1,st}, w'(p) = w(p)$ and put all these GMECs in C' . Without loss of generality, suppose M satisfies the first \hat{r} constraints in C' . Then we do the following operation:

- Step-1: Let $t \in (p_{\bar{q}-1,st}^\bullet \cap \pi_{\bar{q}-1,1})$ fire under M until the current marking M_1 satisfies: $\mathbf{w}'_1 \cdot M_1 = k_1 + 1$;
- Step-2: Let $t \in (p_{\bar{q}-1,st}^\bullet \cap \pi_{\bar{q}-1,2})$ fire under M_1 until the current marking M_2 satisfies: $\mathbf{w}'_2 \cdot M_2 = k_2 + 1$;
- ...
- Step- r : Let $t \in (p_{\bar{q}-1,st}^\bullet \cap \pi_{\bar{q}-1,\hat{r}})$ fire under M_{r-1} until the current marking M_r satisfies: $\mathbf{w}'_{\hat{r}} \cdot M_{\hat{r}} = k_{\hat{r}} + 1$;

By the same reasoning in the proof of Theorem 1, this operation can always be carried out until the last step. Therefore we let $t_{\bar{q},a}$ fire $k + 1 - \bar{k}$ times and then it would reach a marking M_B such that $\mathbf{w}^T \cdot M_B \geq k + 1$ indicating $M_B \notin \mathcal{L}_{(\mathbf{w},k)}$. Therefore $M \notin \mathcal{A}$ which indicates $\mathcal{A}(\xi_D) \supseteq \mathcal{A}_{(\mathbf{w},k)}$. □ ■

Finally we present an iterative algorithm to solve Problem 1.

Algorithm 1 *Compute ξ*

INPUT: $N = N_{A1} + N_{A2} + \dots + N_{Aq}$, $(\mathbf{w}_0, k_0) \equiv (M(p_{q,ed}) \leq k)$

OUTPUT: An admissible OR-GMEC ξ , $\mathcal{L}(\xi) = \mathcal{A}_{(\mathbf{w}_0, k_0)}$

Step 1: Let counter index $u = q + 1$, let $\mathcal{D}_{q+1} = \{(\mathbf{w}_0, k_0)\}$;

Step 2: For each GMEC (\mathbf{w}', k') in \mathcal{D}_u , compute the corresponding $C_{u-1, (\mathbf{w}', k')}$ and then $D_{u-1, (\mathbf{w}', k')}$;

Step 3: Let $\mathcal{D}_{u-1} = \bigcup_{(\mathbf{w}', k') \in \mathcal{D}_u} D_{u-1, (\mathbf{w}', k')}$. Let $u = u - 1$. If $u > 1$, goto Step 2;

Step 4: Output ξ_1 as the disjunction of all GMECs in \mathcal{D}_1 . △

Algorithm 1 works in the following way. Suppose the AF system contains q AWs. For the given (\mathbf{w}_0, k_0) we first construct $\mathcal{D}_q = D_{(\mathbf{w}_0, k_0)}$. Then we iteratively compute $\mathcal{D}_{u-1} = \bigcup_{(\mathbf{w}', k') \in \mathcal{D}_u} D_{u-1, (\mathbf{w}', k')}$ until the count index is $u = 1$.

The correctness of Algorithm 1 is guaranteed by the following two theorems.

Theorem 4 In each iteration of Algorithm 1, $\mathcal{A}(\xi_{u-1}) = \mathcal{A}(\xi_u)$, where ξ_u is the disjunction of all constraints in \mathcal{D}_u .

Proof: ($\mathcal{A}(\xi_{u-1}) \subseteq \mathcal{A}(\xi_u)$) For any marking $M \in \mathcal{A}(\xi_{u-1})$, M must satisfy at least one GMEC in $\mathcal{D}_{(\mathbf{w}_i, k_i)}$. Therefore according to Theorem 3, M would never uncontrollably evolve to a marking which violates (\mathbf{w}_i, k_i) . This indicates $M \in \mathcal{A}(\xi_u)$. Therefore $\mathcal{A}(\xi_{u-1}) \subseteq \mathcal{A}(\xi_u)$.

($\mathcal{A}(\xi_{u-1}) \supseteq \mathcal{A}(\xi_u)$) For any marking $M_0 \notin \mathcal{A}(\xi_{u-1})$, M_0 would uncontrollably evolve to M' which violates all GMECs in $\mathcal{D}_{(\mathbf{w}_i, k_i)}$. Thus for each (\mathbf{w}_i, k_i) there exists $\sigma_i : M'[\sigma_i]M_i''$ which violates (\mathbf{w}_i, k_i) . Notice that σ_i can be constructed by doing the procedure described in the proof of Theorem 3, and all such sequences $\sigma_i \in T_{Au}^*$. Thus for all $p \in P_{Av}$, $v \geq u$, it holds $M_i''(p) = M_j''(p)$. Therefore we choose a \bar{M}_i'' such that for all $j \neq i$, $M_j''(p_{u,ed}) \leq \bar{M}_i''(p_{u,ed})$. Then \bar{M}_i'' can be reached from M by firing uncontrollable transitions and \bar{M}_i'' violates all (\mathbf{w}_i, k_i) . Therefore $\mathcal{A}(\xi_{u-1}) \supseteq \mathcal{A}(\xi_u)$. □ ■

Theorem 5 Given an AF system N which is composed by q assembly workstations N_{A1}, \dots, N_{Aq} and given a GMEC $(\mathbf{w}_0, k_0) : \{M | M(p_{q,ed}) \leq k_0\}$ as the input of Algorithm 1, let ξ_1 be the output of Algorithm 1. It holds: $\mathcal{L}(\xi_1) = \mathcal{A}_{(\mathbf{w}_0, k_0)}$.

Proof: Obviously (\mathbf{w}_0, k_0) in \mathcal{D}_{q+1} is a weakly non-increasing GMEC with an index $q + 1$. In each iteration each GMEC in D_u is substituted by $\bigcup D_{q, (\mathbf{w}, k)}$. According to Theorem 4, $\mathcal{A}(\xi_{u-1}) = \mathcal{A}(\xi_u)$. Since in each iteration each new generated GMECs are always weakly non-increasing, Theorem 3 and 4 can be applied repeatedly until $u = 1$. Therefore $\mathcal{A}(\xi_1) = \mathcal{A}_{(\mathbf{w}_0, k_0)}$. Since for all GMECs in D_1 are weakly non-increasing with an index 1, no uncontrollable transition in N will increase the token count of any GMECs in \mathcal{D}_1 , so we have $\mathcal{A}(\xi_1) = \mathcal{L}(\xi_1)$. This concludes $\mathcal{L}(\xi_1) = \mathcal{A}_{(\mathbf{w}_0, k_0)}$. □ ■

At the end of this section we give an example to illustrate Algorithm 1.

Example 4 Consider the net in Figure 2. Let $M = [M(p_{st1}), M(p_{11}), M(p_{12}), M(p_{21}), M(p_{22}), M(p_{st2}), M(p_{31}), M(p_{41}), M(p_{ed2})]$. Suppose in \mathcal{D}_3 the initial GMEC is $(\mathbf{w}_0, k_0) : M(p_{2,ed}) \leq 1$. In the first iteration, N_{A2} is analyzed, and two elementary GMEC $c_1 : M(p_{st2}) + M(p_{31}) + M(p_{ed2}) \leq 1$ and $c_2 : M(p_{st2}) + M(p_{41}) + M(p_{ed2}) \leq 1$ are put in C_2 and one composition GMEC $c_3 : M(p_{st2}) + M(p_{31}) + M(p_{41}) + 2M(p_{ed2}) \leq 3$ is put in D_2 . All these constraints are then put in \mathcal{D}_2 . In the second iteration, N_{A1} is analyzed, and each $c_i (i = 1, 2, 3)$ are replaced by two elementary GMECs c_{i1}, c_{i2} and one composition GMEC c_{i3} . All these GMECs are put in \mathcal{D}_1 . Then Algorithm 1 ends and output ξ the disjunction of nine GMECs in \mathcal{D}_1 (see below):

$$\begin{aligned}
(\mathbf{w}_{11}, k_{11}) &: ([1, 1, 1, 0, 0, 1, 1, 0, 1], 1) \\
(\mathbf{w}_{12}, k_{12}) &: ([1, 0, 0, 1, 1, 1, 1, 0, 1], 1) \\
(\mathbf{w}_{13}, k_{13}) &: ([1, 1, 1, 1, 1, 2, 2, 0, 2], 3) \\
(\mathbf{w}_{21}, k_{21}) &: ([1, 1, 1, 0, 0, 1, 0, 1, 1], 1) \\
(\mathbf{w}_{22}, k_{22}) &: ([1, 0, 0, 1, 1, 1, 0, 1, 1], 1) \\
(\mathbf{w}_{23}, k_{23}) &: ([1, 1, 1, 1, 1, 2, 0, 2, 1], 3) \\
(\mathbf{w}_{31}, k_{31}) &: ([1, 1, 1, 0, 0, 1, 1, 1, 2], 3) \\
(\mathbf{w}_{32}, k_{32}) &: ([1, 0, 0, 1, 1, 1, 1, 1, 2], 3) \\
(\mathbf{w}_{33}, k_{33}) &: ([1, 1, 1, 1, 1, 2, 2, 2, 4], 7)
\end{aligned} \tag{8}$$

△

6 Discussion and Conclusion

In this paper we studied the constraint transformation problem in Petri net systems which contain a special uncontrollable structure called assembly flow system. The composition GMECs are introduced to handle the conflict-synchronization structure. It is straight forward to extend this approach to systems in which the uncontrollable subnet has a *multi-conflict-multi-synchronization* structure, by expanding p_{st} and t_a into several sequential conflict places and synchronization transitions. An example is given in Figure 4. The uncontrollable subnet in Figure 4 is equivalent to the uncontrollable subnet in Figure 1.

We believe this approach can also be extended to uncontrollable subnets which have backward-conflict-free structure, i.e., each place has at most one input, which is a more general subclass of Petri nets. This will be the aim of our future work.

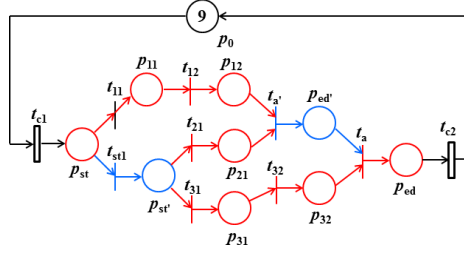


Figure 4: A Petri net with uncontrollable subnet AW equivalent to the AW subnet in Figure 1.

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