General Observation Structures for Petri Nets

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Abstract

Observation structures usually considered for Petri nets assume that the firing of transitions may be observed through static labels and that the marking of some places may be measurable. These observation structures are rather limited and we consider in this paper more general ones, correspondingly defining two new classes of nets: labeled Petri nets with outputs *and* adaptive labeled Petri nets*. Several examples of nets in these classes are considered. Furthermore we also show the possibility to convert a labeled Petri net into an adaptive Petri net or a labeled Petri net.*

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1. Introduction

Petri nets are a powerful discrete event model capable of describing communication systems, manufacturing systems, etc., in a graphical and mathematical way[6]. In most cases, especially when addressing a fault diagnosis or a state estimation problem, only observations of events are taken into account [1, 2, 4, 5]: the corresponding model, called *labeled Petri nets* (LPNs), assumes that the firing of a transition can be detected through a static label that denotes an observable event, while the empty label is associated to unobservable transitions.

In several applications, however, sensors may also provide state information. This has led several researchers [3, 7, 9, 10] to consider a more general model where it is assumed that the number of tokens in some places can be measured. In this case there are two types of observations: labels of transitions and components of the marking. We call this model *labeled Petri nets with measurable places* (LPNMPs). In [10] it was also shown that an LPNMP can always be converted into an equivalent labeled Petri net by a suitable re-definition of the transition labels, hence the two models have the same modeling power.

These observation structures are rather limited and in this paper we consider more general ones. Firstly, we generalize LPNMPs by defining a new model called *labeled Petri nets with outputs* (LPNOs), i.e., labeled Petri nets endowed with a (general) state sensor that provides an observation that is an arbitrary function of the current net marking. We explore the modeling power of this model by means of several examples and in particular we show that an LPNO cannot always be converted into a standard LPN.

Looking for nets with only event observations whose modeling power encompasses that of LPNOs, we define *adaptive labeled Petri nets* (ALPNs). In this new model, that generalize LPNs, only transition firings are observable but the labeling function that associates events to transition is adaptive, i.e., depends on the current marking. We show that it is always possible to convert an LPNO into an ALPN albeit with a possibly infinite alphabet.

The remainder of this paper is organized as follows. In Section II we recall some basic definitions for Petri nets. In Section III a formal definition of LPNOs is presented. Some examples are discussed in Section IV. In Section V ALPNs are defined and compared with LPNOs. Conclusions and future works are discussed in Section VI.

2. Background

In this section, we provide some basic definitions and notation for Petri nets that will be used in this paper. For details we refer to [8].

2.1. Petri Nets

A *Petri net* is a structure $N = (P, T, Pre, Post)$, where P is a set of m *places* represented by circles; T is a set of n *transitions* represented by bars; $Pre : P \times T \to \mathbb{N}$ and $Post : P \times T \to \mathbb{N}$ are the *pre-* and *post-incidence functions* that specify the arcs; and $C = Post - Pre$ is the incidence matrix.

A *marking* is a vector $M : P \to \mathbb{N}^m$ that assigns to each place of a Petri net a non-negative integer number of tokens, represented by black dots. We denote $M(p)$ the marking of place p. A *Petri net system* $\langle N, M_0 \rangle$ is a net N with an initial marking M_0.

A transition t is *enabled* at M iff $M \geq Pre(\cdot, t)$ and may fire yielding a new marking $M' = M_0 + C(\cdot, t)$. We write $M[\sigma\rangle$ to denote that the sequence of transitions $\sigma = t_{j1} \cdots t_{jk}$ is enabled at M, and we write $M[\sigma\rangle M'$ to denote that the firing of σ yields M'. The set of all sequences that can fire in a net system $\langle N, M_0 \rangle$ is denoted by $L(N, M_0)$.

A marking M is *reachable* in $\langle N, M_0 \rangle$ if there exists a firing sequence $\sigma \in L(N, M_0)$ such that $M_0[\sigma\rangle M$. The set of all markings reachable from M_0 defines the *reachability set* of $\langle N, M_0 \rangle$ and is denoted by $R(N, M_0)$.

If $M_0[\sigma\rangle M$, then an equation called the *state equation* can be written as

$$M = M_0 + C \cdot y \tag{1}$$

where y is the *firing vector* of σ.

2.2. Labeled Petri Nets

A *labeled Petri net* (LPN) is a 3-tuple $G = (N, M_0, \Sigma, \ell)$, where $\langle N, M_0 \rangle$ is a Petri net system, Σ is an *alphabet* (a set of labels), and $\ell : T \to \Sigma \cup \{\varepsilon\}$ is a *labeling function* that assigns to each transition $t \in T$ either a symbol from Σ or the empty string ε.

Given an observed word w of a labeled Petri net $G = (N, M_0, \Sigma, \ell)$: the set of w*-consistent firing sequences* is $\mathcal{S}(w) = \{\sigma \in L(N, M_0) \mid \ell(\sigma) = w\}$; the set of w*-consistent markings* is $\mathcal{C}(w) = \{M \in \mathbb{N}^m \mid \exists \sigma \in \mathcal{S}(w), M_0[\sigma\rangle M\}$.

2.3. Labeled Petri Nets with Measurable Places

Several authors [3, 7, 9, 10] have studied labeled Petri nets where some places are observable, i.e., their token contents are known [1]. In this paper we call this model *labeled Petri nets with measurable places* and define them in the following.

Definition 1 A labeled Petri net with measurable places (LPNMP) is $G_m = (N, M_0, \Sigma, \ell, V)$, where (N, M_0, Σ, ℓ) is an LPN, P_o is a set of observable places and $V \in \{0,1\}^{|P_o| \times |P|}$ is a *place sensor configuration*. Each row of V is a canonical basis vector $\overrightarrow{e_j}^T$ associated with an observable place $p_j \in P_o$.

Ru and Hadjicostis [10] have given an algorithm to transform an LPNMP into an equivalent labeled Petri net. Namely, the set of firing sequences consistent to an observation in the LPNMP is identical to $\mathcal{S}(w)$, where w is the corresponding observation of the obtained labeled Petri net.

3. Labeled Petri nets with outputs

In LPNMPs two types of observations are available: transition labels and components of the marking. However, rather than providing precise measurements sensors may only provide aggregate state information. For example, a cheap sensor may be able to detect if a buffer is empty or not while the exact number of items in the buffer remains unknown.

This motivates us to define a more general class of labeled Petri nets.

Definition 2 A *labeled Petri net with outputs* (LPNO) is $Q = (N, M_0, \Sigma, \ell, f)$, where (N, M_0, Σ, ℓ) is an LPN and $f: R(N, M_0) \to \mathbb{R}^k$ is an *observation function* associated with k state sensors.

For an LPNO, the observed output of the net is not only a sequence of labels but also an output of the observation function.

Definition 3 Let $\sigma = t_1 t_2 \cdots t_k$ be a firing sequence with $M_0[t_1\rangle M_1[t_2\rangle M_2 \cdots M_{k-1}[t_k\rangle M_k$. The *output function* $L_{obs} : T^* \to (\Sigma \times \mathbb{R}^k)^*$ associates to firing sequence σ the observation

$$s = L_{obs}(\sigma) = (\ell(t_1), \Delta f^1)(\ell(t_2), \Delta f^2) \cdots (\ell(t_k), \Delta f^k),$$

where $\Delta f^i = f(M_i) - f(M_{i-1})$, $i = 1, 2, \cdots, k$. If $\ell(t_i) = \varepsilon$ and $\Delta f^i = 0$, $(\ell(t_i), \Delta f^i)$ is null.

As $\mathcal{S}(w)$ and $\mathcal{C}(w)$ defined in labeled Petri nets, in LPNOs, $\mathcal{S}(s)$ and $\mathcal{C}(s)$ are used to denote the set of s-consistent firing sequences and the set of s-consistent markings, respectively.

Definition 4 Given an observation s of an LPNO, the set of *s-consistent firing sequences* is $\mathcal{S}(s) = \{\sigma \in L(N, M_0) \mid L_{obs}(\sigma) = s\}$; the set of *s-consistent markings* is $\mathcal{C}(s) = \{M \in \mathbb{N}^m \mid \exists \sigma \in \mathcal{S}(s), M_0[\sigma\rangle M\}$

Definition 5 A labeled Petri net $G = (N, M_0, \Sigma', \ell')$ is said to be *equivalent* to an LPNO $Q = (N, M_0, \Sigma, \ell, f)$ if for all sequences $\sigma \in L(N, M_0)$ that produce an observation $w = \ell(\sigma)$ in G and an observation s in Q holds $\mathcal{S}(w) = \mathcal{S}(s)$ and $\mathcal{C}(w) = \mathcal{C}(s)$.

4. Some modeling examples

In this section we present some LPNO models to discuss the connection between LPNOs and classical labeled Petri nets, and potential techniques for analyzing LPNOs.

Example 1 Let us consider the LPNO in Figure 1 from [10], instead of the place sensor configuration in [10], which is equivalent to

$$f(M) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot M$$

the observation function is

$$f(M) = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \cdot M$$

If a firing sequence of transitions is $\sigma = t_2 t_4 t_5$, then the corresponding observation will be $s = L_{obs}(\sigma) = (b, [-2\ 1]^T)(c, [2\ 1]^T)(\varepsilon, [-1$ $2]^T)$.

[1] In [7, 10] this model is called *partially observed Petri nets*.

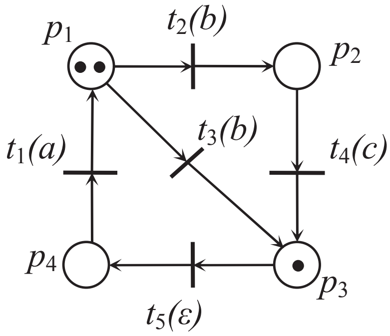

Figure 1. An LPNO with linear observation function.

Table 1. New labeling function for the net in Example 1.

Observation	$(a, [1\ 0]^T)$	$(b, [-2\ 1]^T)$	$(b, [0\ 2]^T)$	$(c, [2\ 1]^T)$	$(\varepsilon, [-1\ -2]^T)$
New label	a	b_1	b_2	c	ε_1

In fact, based on the state equation (1) and linear property of the observation function, the state observation is

$$\begin{aligned}\Delta f &= \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \cdot C \\ &= \begin{bmatrix} 1 & -2 & 0 & 2 & -1 \\ 0 & 1 & 2 & 1 & -2 \end{bmatrix}\end{aligned}$$

we can construct a mapping shown in Table 1 as same as the one in [10], where b_1, b_2 and ε_1 are new symbols in the alphabet of the LPN. With the new labeling function, this LPNO is converted into an equivalent labeled Petri net, i.e., given an observation s in the LPNO there will be a corresponding observation w in the labeled Petri net such that the set of consistent firing sequences and the set of consistent markings are the same. This shows that an LPNO whose observation function is a linear function of the marking can be converted into an LPN, thus generalizing the similar result of Ru and Hadjicostis [10] that only applies to LPNMPs.

For example, if the observation in the LPNO is $s = (b, [-2\ 1]^T)(b, [0\ 2]^T)(\varepsilon, [-1\ -2]^T)$, then the set of s-consistent firing sequences is $\mathcal{S}(s) = \{t_2 t_3 t_5\}$ and the set of s-consistent markings is $\mathcal{C}(s) = \{[0\ 1\ 1\ 1]^T\}$. While in the equivalent labeled Petri net, the corresponding observation is $w = b_1 b_2 \varepsilon_1$, the set of w-consistent firing sequences is $\mathcal{S}(w) = \{t_2 t_3 t_5\} = \mathcal{S}(s)$ and the set of w-consistent markings is $\mathcal{C}(w) = \{[0\ 1\ 1\ 1]^T\} = \mathcal{C}(s)$.

Incidentally, we note that the labeled Petri net equivalent to this LPNO is *free-labeled*. Therefore this net is structurally observable, i.e., $\forall M_0 \in \mathbb{N}^m$, given an observation the current marking can be exactly reconstructed from the knowledge of the net and of its initial marking. ◇

However, when more general observation functions are considered, such a conversion may not always be possible. The following example considers a net with a non-linear observation function.

Example 2 Consider the LPNO shown in Figure 2.

Let us first consider a nonlinear observation function $f(M) = M(p_1)^2$. The firing of transition t_1 and t_2 will increase tokens in p_1, while the firing of transition t_3 will decrease a token from p_1 and furthermore the number of possible observations Δf^i is infinite. However, one can readily check that $\Delta f^1 = f(M') - f(M)$ is always an even positive integer number, $\Delta f^2 = f(M'') - f(M)$ is an odd positive integer number and $\Delta f^3 = f(M''') - f(M)$ is an odd negative integer number, where $M[t_1\rangle M'$, $M[t_2\rangle M''$ and $M[t_3\rangle M'''$. Therefore, the effect of all these observations is equivalent to detecting the firing of t_1, t_2 and t_3: thus by relabeling the three transitions $\ell'(t_1) = a_1$, $\ell'(t_2) = a_2$ and $\ell'(t_3) = \varepsilon_1$ one obtains an equivalent LPN.

However, if the observation function of the LPNO in Figure 2 is $f(M) = min\{M(p_1), 1\}$, then this net cannot be converted into a labeled Petri net. In fact, the firing of transition t_3 is not observable from all markings such that $M(p_1) > 1$, while from marking $M = [1]$ the firing of t_3 is observable: this is clearly impossible for an LPN. ◇

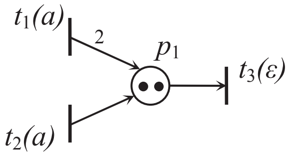

Figure 2. An LPNO that cannot be converted into an LPN.

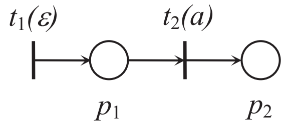

Figure 3. An LPNO that can be converted into an ALPN with an infinite alphabet.

5. Adaptive labeled Petri nets

Motivated by the fact that an LPN equivalent to a given LPNO may not exist, this section considers a more general class of nets with a marking dependent labeling function and discusses its relationship with the class of LPNOs.

Definition 6 An *adaptive labeled Petri net* (ALPN) is $\widetilde{G} = (N, M_0, \widetilde{\Sigma}, \widetilde{\ell})$, where $\langle N, M_0 \rangle$ is a Petri net system, $\widetilde{\Sigma}$ is an alphabet and $\widetilde{\ell} : R(N, M_0) \times T \to \widetilde{\Sigma} \cup \{\varepsilon\}$ is an *adaptive labeling function*.

An LPN can be seen as a special type of ALPN with a static labeling function, i.e., $\forall t \in T$ and $\forall M, M' \in R(N, M_0)$, $\widetilde{\ell}(M, t) = \widetilde{\ell}(M', t)$. Thus adaptive labeled Petri nets are a proper generalization of labeled Petri nets.

Proposition 1 Any LPNO can be converted into an equivalent ALPN [2], if infinite alphabets are allowed.

This proposition can be informally proved as follows. To each transition of an LPNO whose firing at marking M produces an observation $(\ell(t), \Delta f)$, where $M[t\rangle M'$ and $\Delta f = f(M') - f(M)$, we can associate in the equivalent ALPN a label $\widetilde{\ell}(M, t) = [\ell(t), \Delta f]$. Hence once the transition fires in the ALPN, the new label exactly describes the observation of the LPNO.

Next example shows the case of an LPNO that cannot be converted into an equivalent LPN but can be converted into an equivalent ALPN with a finite alphabet.

Example 3 Consider the LPNO in Figure 2 with observation function $f(M) = min\{M(p_1), 1\}$. In Example 2 it was shown that this net cannot be converted into an LPN. However, we can convert it into an ALPN if we let the labeling function depend on the marking as follows: $\forall M \in \{M \in \mathbb{N}^m | M(p_1) > 1\}$ then $\widetilde{\ell}(M, t_3) = \varepsilon$ while for $M' = [1]$, $\widetilde{\ell}(M', t_3) = \varepsilon_1$, where ε_1 is a new event in alphabet $\widetilde{\Sigma}$. ◇

Next example shows a case in which an ALPN equivalent to a given LPNO requires an alphabet $\widetilde{\Sigma}$ with an infinite cardinality. Such a condition is not acceptable if one assumes that the cardinality of an alphabet should be finite by definition.

Example 4 Let us consider the LPNO in Figure 3 with observation function $f(M) = M(p_1) \cdot M(p_2)$. Assume a firing sequence $\sigma = t_1^k t_2$ occurs in this net. The observation produced by $\sigma = t_1^k$ is the empty string and it holds: $\mathcal{S}((\varepsilon, 0)) = \{t_1^k \mid k \in \mathbb{N}\}$ while $\mathcal{C}((\varepsilon, 0)) = \{[k\ 0]^T \mid k \in \mathbb{N}\}$. However, as soon as t_2 fires (thus marking p_2) the exact number of tokens in p_1 will be exactly identified by the observed value of f. This means we have to assign to t_2 a different label for each marking in $\mathcal{C}((\varepsilon, 0))$, i.e., for all $M = [k\ 0]^T$, $k \in \mathbb{N}$ it should hold $\widetilde{\ell}(M, t_2) = a_k \in \widetilde{\Sigma}$ where a_k is a new event. Since the set $\mathcal{C}((\varepsilon, 0))$ is infinite, then also $\widetilde{\Sigma}$ is necessarily infinite. ◇

[2] The formal definition of equivalence between LPNOs and ALPNs can be given as in Definition 5.

6. Conclusions and future works

We have considered Petri net observation structures that are more general than those usually considered in the literature and we have correspondingly defined two new classes of nets: *labeled Petri nets with outputs* and *adaptive labeled Petri nets*. Several examples of nets in these classes have been considered. Furthermore we have also compared the modeling power of the two classes, showing that they are incomparable under the common assumption that the alphabet of events has finite cardinality.

In this paper, the process of converting an LPNO into an equivalent adaptive labeled Petri net (or labeled Petri net) is not discussed in details. For future works, we will concentrate on developing an effective algorithm to construct the equivalent adaptive labeled Petri nets with minimal new labels and characterizing the LPNOs that can be converted into an equivalent labeled Petri nets. Observability, fault diagnosis and diagnosability problems will also be further discussed.

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