Decentralized Stabilization of Heterogeneous Linear Multi-Agent Systems

Mauro Franceschelli, Andrea Gasparri, Alessandro Giua, and Giovanni Ulivi

Abstract

In this paper the formation stabilization problem for a system of heterogeneous agents is considered. Agents are characterized by different linear dynamics, and assumed to be able to collaborate by exchanging information if they are within their range of communication. A sufficient algebraic condition for the stability of the formation based on a generalization of the Gerschgorin circle theorem for block matrices is proposed. Furthermore, conditions under which the formation remains stable under switching topology are investigated. Simulation results are given to corroborate the theoretical results.

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I. Introduction

In the last decades systems theory has been driven toward the study of the systems of systems, i.e complex high-order dynamical systems arising from the interconnection of small order dynamical systems. While in the most general case the system of systems is an arbitrary interconnection between dynamical systems, in the framework of multi-agent systems the scope is restricted to the study of a set of homogenous dynamical systems, the agents, arbitrarily interconnected with some defined interaction rule. In such model the uncertainty lies in the interconnection topology which is arbitrary and possibly unknown. In this paper we are interested in investigating the stability properties of linear Multi-Agent systems dropping the assumption on the homogeneity of the network. Such assumption increases the generality of the results by allowing the agents to have different dynamics, eventually coming from model uncertainties.

In the Robotics field, multi-agent systems have been widely used by the research community as an effective (simulation) framework to investigate formation control techniques which could be applied to (real) robotics devices. In this framework, a multi-agent robotic system is commonly modeled with a graph, where each vertex describes the kinematics of the related robotic agent (simply "agent" from now on), while a link models the constrained interaction among agents. The formation control is a fundamental issue to properly achieve cooperation in a multi-robot system. Indeed, the capability to acquire a formation and maintain it over time while the multi-robot system is moving is fundamental in order to execute a variety of tasks, e.g., robotic soccer, surveillance, object transportation and so on. Moving in formation introduces several interesting advantages: higher robustness and efficiency while providing redundancy, as well as higher flexibility and reconfiguration capabilities. Formation control does not restrict itself only to ground mobile robots [**?**], [**?**]. Among the others, it has been successfully applied to aircrafts [**?**], [**?**], in particular unmanned aerial vehicles (UAVs) [**?**], underwater vehicles [**?**], [**?**], as well as satellites [**?**], [**?**]. For a comprehensive overview of the formation control problem the reader is referred to [**?**].

In this paper, the problem of formation stabilization for a set of heterogenous agents, i.e., the stabilization of their relative position [**?**], [**?**], is addressed. In particular, agents are assumed to have different linear dynamics and be able to collaborate by exchanging information if they are within their range of communication. A sufficient algebraic condition for the stability of the formation is proposed. In addition, conditions under which the formation remains stable under switching topology are investigated. The idea is to use a result on the generalizations of the Gerschgorin circle theorem for block matrices, which can be found in [?], to define a set of rules which can be applied locally by each agent to build a control law in a completely distributed way so that the global stability of the formation is guaranteed. In the past years, this result on the generalization of the Gerschgorin circle theorem for block matrices has been applied in several contexts. Among the others, in the analysis of interconnected systems for the elimination of fixed modes [?], and for the stabilization of large scale systems [?]. Such concept might be useful in multi-robot coordination applications because it allows each robot to estimate the position of the system eigenvalues only by looking at its dynamics and its local interconnection. This theory is developed for linear systems but still it is of great practical relevance for the local stabilization of equilibrium points corresponding to the desired formation where a linear approximation of the nonlinear vehicle dynamics is feasible.

A. Paper content:

- In Section II some basic notions of graph theory to model the network topology of a multi-agent system along with an overview of the Gerschgorin circle theorem are given.
- In Section III a formalization of the stabilization problem for a heterogeneous multi-agent system is given.
- In Section IV conditions for the stabilization of a multi-agent system with fixed topology are given.
- In Section V conditions for the stabilization of a multi-agent system under switching topology are given.
- In Section VI simulations to corroborate the theoretical results are shown.
- Finally, in Section VII conclusions are drawn and future work is discussed.

B. Assumptions:

In the rest of the paper, the following assumptions will be taken into account for the multi-agent system:

- Each agent i has its own linear dynamics described by the matrix A_i (the system is heterogeneous),
- A way to share a common (global) reference frame among agents must be available, for instance using the algorithms given in [**?**],
- Collaboration from agents i to j is achieved by exchanging data according to the interconnection matrix P_{ij},
- Collaboration does not necessarily need to be symmetric, that is it can be $P_{ij} \neq P_{ji}$,
- Communication does not necessarily need to be bidirectional.

II. Theoretical Background

A. *Concepts of Graph Theory*

In the paper the notion of graph as a model of the network topology is used. A graph $\mathcal{G} = \{V, E\}$ is a set of vertices (or agents) $V = \{1, \dots, n\}$ connected by a set of edges (or links) $E \subseteq \{V \times V\}$. A graph is said to be undirected if $(i,j) \in E \iff (j,i) \in E$. A couple of nodes i,j are said to be connected by a *path* if there exists a sequence of links that can be traveled uninterruptedly from i to j. An undirected graph is said to be connected if there exists a path between any couple of node $i, j \in V$. In the following we will refer to $\mathcal{N}_i$ as the neighborhood of agent i, namely the set of indices of the agents directly connected through an edge with agent i.

In the proposed network model, an interaction between agent i and agent j may occur only if agent i can directly communicate with agent j and viceversa. Since each agent is modeled with a limited sensing radius ρ_i, the generic couple of agents i and j with positions p_i and p_j may communicate if and only if $\|p_i - p_j\|_2 \leq \min\{\rho_i, \rho_j\}$. As a result, the interaction topology is modeled by a time-varying proximity graph. Given a set of agents positions $P = \{p_0, p_1, \dots, p_n\}$ with $p_i \in \mathbb{R}^d$ we define a time-varying proximity graph $\mathcal{G}(t) = \{V, E(t)\}$, where V is the set of vertices $V = \{1, \dots, n\}$ that represent the agents, and $E(t)$ is the time-varying set of edges that encodes the interaction topology at time t:

$$
\begin{aligned}
E(t) \;\; = \;\; & \Big\{(i,j) : \|p_i(t) - p_j(t)\| \leq \min\{\rho_i, \rho_j\}, \\
& \forall\, i,j \in V \quad i \neq j\Big\}.
\end{aligned}
$$

B. Gerschgorin circle theorem

The Gershgorin circle theorem can be used to provide a bound for the spectrum of a square matrix [?]. Let A be a complex $n \times n$ matrix with entries a_{ij}, and let R_i be defined as the sum of the absolute values of the off-diagonal entries in the i-th row, i.e., $R_i = \sum_{j \neq i} |a_{ij}|$. Let the Gershgorin disc D_i associated with the i-th row be defines as:

$$D_i = \{z \in \mathbb{C} : |z - a_{ii}| \leq R_i\}$$

and let $D = \bigcup_{i=1}^{n} D_i$, be the union of the Gershgorin discs $D_i,\ i = 1, \ldots, n$. The Gershgorin circle theorem states that every eigenvalue of the complex matrix A lies within the union of the Gershgorin discs D_i, that is:

$$\lambda_i \in D,\ \forall\ \lambda_i\ \in\ \sigma(A), \tag{1}$$

where $\sigma(\cdot)$ is the set of eigenvalues of a matrix.

A generalizations of the Gerschgorin circle theorem for block matrices can be found in [?]. In particular, let A be a generic block matrix of the form:

$$A = \left[\begin{array}{c:c:c:c} A_{11} & A_{12} & \ldots & A_{1N} \\ \hdashline A_{21} & A_{22} & \ldots & A_{2N} \\ \hdashline \vdots & & & \vdots \\ \hdashline A_{N1} & A_{N2} & \ldots & A_{NN} \end{array}\right].$$

In particular, for such a partitioned matrix A, each eigenvalue λ of A satisfies (Th. 2 [?]):

$$\left(\|(A_{ii} - \lambda I_i)^{-1}\|\right)^{-1} \leq \sum_{k=0, k \neq i}^{N} \|A_{ik}\|, \tag{2}$$

for at least one i, with $1 \leq i \leq N$, where the norm $\|\cdot\|$ is defined as:

$$\|A\| = \sup_{x \in \Omega, x \neq 0} \left(\frac{\|Ax\|}{\|x\|}\right). \tag{3}$$

and the quantity appearing on the right of the above inequality is defined as follows:

$$\left(\|A^{-1}\|\right)^{-1} = \inf_{x\in\Omega, x\neq 0}\left(\frac{\|Ax\|}{\|x\|}\right), \tag{4}$$

whenever A is not singular. Note that, in case the matrix A is singular the quantity $\left(\|A^{-1}\|\right)^{-1}$ can be defined by continuity to be zero ([**?**]).

Now by defining the Gerschgorin set S_i as the set of all complex numbers z for which the following holds:

$$\left(\|(A_{ii} - zI_i)^{-1}\|\right)^{-1} \leq \sum_{k=0, k\neq i}^{N} \|A_{ik}\|, \tag{5}$$

it is obvious that each set S_i always contains the eigenvalues of A_{ii} independently to the magnitude of the right side of the equation. Moreover, it can be defined the union of these sets:

$$S = \bigcup_{i=1}^{N} S_i,$$

and according to the condition (2) it can be stated that all the eigenvalues of the matrix A lie on the union S previously defined.

III. Problem Description

The problem we are going to address is inspired by the works [**?**], [**?**]. In these works the authors address the formation stabilization problem for a team of agents with identical dynamics. Differently in this paper the multi-agent system is supposed to be heterogeneous. In particular, the problem formulation can be stated as follows.

Let us consider a set of N agents described by the graph $\mathcal{G} = \{V, E\}$ with $N = |V|$, whose linear dynamics are denoted as:

$$\dot{x}_i = A_i\, x_i + B_i\, u_i, \tag{6}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state vector of the i-th agent, while $A_i \in \mathbb{R}^{n_i \times n_i}$ and $B_i \in \mathbb{R}^{n_i \times q_i}$ describe respectively the dynamic matrix and the input matrix of the i-th agent. Let us consider an interconnection for a couple of agents i, j with $(i, j) \in E$ of the form:

$$z_{ij} = P_{ij}\, x_j, \tag{7}$$

where $P_{ij} \in \mathbb{R}^{n_i \times n_j}$ is the interconnection matrix. We are willing to find a distributed control law $\mathbf{K}$ of the form:

$$u_i = \sum_{j \in \mathcal{N}_i} \hat{K}_{ij}\, x_j + K_{x_i}\, x_i, \tag{8}$$

where $\hat{K}_{ij} = k_{ij}\, P_{ij}$ with $k_{ij} \in \mathbb{R}$, so that the whole formation is stabilized. Note that, according to the definition given in [**?**], [**?**], with the term *stabilization of a formation*, the stabilization of the relative position for a set of multi-agent system is meant.

For sake of clarity, let us rewrite the whole problem formulation in a matrix form for a system composed of three agents:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} A_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix},$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} K_{x_1} & \hat{K}_{12} & \hat{K}_{13} \\ \hat{K}_{21} & K_{x_2} & \hat{K}_{23} \\ \hat{K}_{31} & \hat{K}_{32} & K_{x_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

where a generic $\hat{K}_{ij}$ might be zero if there is no communication from robot i to robot j.

Finally, by substituting the second equation into the first one we obtain the following matrix form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} A_1 + B_1 K_{x_1} & B_1 \hat{K}_{12} & B_1 \hat{K}_{13} \\ B_2 \hat{K}_{21} & A_2 + B_2 K_{x_2} & B_2 \hat{K}_{23} \\ B_3 \hat{K}_{31} & B_3 \hat{K}_{32} & A_3 + B_3 K_{x_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{9}$$

IV. MULTI-AGENT SYSTEM WITH STATIC TOPOLOGY

For such a scenario described in Section III, it might be of interest to investigate conditions under which the overall formation is stable. To this end, let us assume that the interconnection matrices P_{ij}s are given and they cannot be modified. This assumption might reflect the fact that this kind of interaction is generally constrained by the sensing capabilities of the agents. According to this scenario, let us formulate the problem as follows.

Problem Statement (I): "Given the system described in (9) with the assumptions given in (I-B), we want to determine a set of rules which can be applied locally by each single agent in order to guarantee the overall stability of the formation".

A possible way to solve this problem in a decentralized fashion is to let each agent implement the following algorithm.

***Algorithm** 1:*

Let us consider a generic agent i, the following steps must be performed:

1) Neighboring discovery in order to identify $\mathcal{N}_i$,
2) Construnction of a control law $\mathbf{K}$ so that:
    a) The dynamics $A_i + B_i K_{x_i}$ is asymptotically stable,
    b) $\min_{\lambda_i \in \sigma(A_i + B_i K_{x_i})} \{|\, \lambda_i \,|\} \geq \sum_{j \in \mathcal{N}_i} \|B_i K_{ij}\|$
3) Notify to the neighborhood $\mathcal{N}_i$ the status "ready to go".

In the following a theorem by which the stability of the formation can be proved when each single agent implements algorithm 1 is given.

***Theorem** 1:* A sufficient condition for the stabilization of the formation is that each agent i locally applies a control law $\mathbf{K}$ of the form given in (8) such that the dynamics $A_i + B_i K_{x_i}$ is asymptotically stable and:

$$\min_{\lambda_i \in \sigma(A_i + B_i K_{x_i})} \{|\, \lambda_i \,|\} \geq \sum_{j \in \mathcal{N}_i} \|B_i K_{ij}\|. \tag{10}$$

Proof: In order to prove the theorem, the generalization of the Gerschgorin disk theorem for block matrices given in Section II-B is exploited. In particular, since for any given block-matrix A all the eigenvalues lie within the set S, a sufficient condition for the stabilization of the formation given in (9) is that each Gerschgorin set S_i must be located on the left-half of the Gauss plane ($S_i \in \mathbb{C}^-$). Indeed, by noticing that the quantity defined on the right hand of the (5) can be computed (as suggested in by exploiting the Euclidean vector norm $\|x\|_2 = (\sum_i |x_i|^2)^{\frac{1}{2}}$ as [**?**]:

$$\left(\|(A_{ii} - zI_i)^{-1}\|\right)^{-1} = \min_{\lambda_j \in \sigma(A_{ii})} \{|\lambda_j - z|\},$$

it follows that $S_i \in \mathbb{C}^-$ is achieved by guaranteeing :

$$\min_{\lambda_i \in \sigma(A_i + B_i K_{x_i})} \{|\, \lambda_i \,|\} \geq \sum_{j \in \mathcal{N}_i} \|B_i K_{ij}\|,$$

with *Re*$\{\lambda\} < 0$ thus proving the theorem. ■

***Remark** 1:* Theorem 1 provides a policy by which each agent i can locally tune the effect of the interaction with the neighboring agents with respect to its own dynamics A_i so that at least from its point of view the formation is stable. Indeed, each agent i with its action can only influence the i-th Gerschgorin Set S_i, and therefore it has only a local view of the possible location of the eigenvalues related to the formation dynamics. Note that, this approach if fully decentralized as only local information, i.e., interconnection matrices P_{ij}s, is required to satisfy the condition (10). Moreover, it turns out to be a robust approach as the lost of an interconnection does not bring instability, i.e., the quantity on the right side of the inequality (10) does not increase.

***Remark** 2:* Theorem 1 can be equivalently stated by taking into account the concept of diagonal dominance of a block-matrix as in [**?**]. In particular, the formation is stable if each agent

i locally applies a control law $\mathbf{K}$ of the form given in (8) such that the dynamics $A_i + B_i K_{x_i}$ is asymptotically stable and the overall matrix A is block diagonally dominant, that is:

$$\left(\|A_{ii}^{-1}\|\right)^{-1} \geq \sum_{k=0, k\neq i}^{N} \|A_{ik}\|.$$

This allows to point out the analogy with the case of a scalar matrix, i.e., agents with a scalar dynamics, for which by exploiting the Gerschgorin circle theorem, the stability is guaranteed by the diagonal dominance and negative definiteness.

V. Multi-Agent System with Switching Topology

In Section IV a sufficient condition for the stabilization of the formation under the assumption of having interconnection matrices P_{ij}s that do not vary over time has been given. In the following, an enhanced scenario in which the interconnection among two agents might be temporarily available or not over time is investigated.

In particular, let us assume that the topology is switching but any time an interconnection from agent i to agent j is available it is always described by the same interconnection matrix P_{ij}. Indeed, this seems a reasonable assumptions, as these matrices usually describe sensing capability of the agents which normally do not vary over time, apart from malfunctioning or re-calibration issues which are not considered in this scenario. This leads to the following definition for the interconnection matrices for this enhanced scenario:

$$P_{ij}(t) = \begin{cases} P_{ij} & \textit{for some } t \\ \mathbf{0} & \textit{otherwise} \end{cases}, \tag{11}$$

where $\mathbf{0}$ is obviously a matrix of appropriate dimensions. This leads to a new slightly different formulation for this enhanced scenario as follows.

Problem Statement (II): “Given the system described in (9) with the assumptions given in (I-B) under switching topology where an interconnection matrix is defined as given in (11), we want to determine a set of rules which can be applied locally by each single agent in order to guarantee the overall stability of the formation under switching topology”.

***Theorem** 2:* A sufficient condition for the stabilization of the formation under switching topology is that each agent i locally applies a switching control law $\mathbf{K}$ of the form given in (8) such that for a given topology $\mathcal{G}(t)$ the dynamics $A_{\mathcal{G}(t),i} = A_i + B_i K_{x_i}^{\mathcal{G}(t)}$ is negative definite and:

$$\min_{\lambda_i \in \sigma\left(A^+_{\mathcal{G}(t),i}\right)} \{|\lambda_i|\} > \frac{1}{2} \sum_{j \in \mathcal{N}_i} \left(\|B_i K_{ij}^{\mathcal{G}(t)}\| + \|B_j K_{ji}^{\mathcal{G}(t)}\| \right), \tag{12}$$

where $A^+_{\mathcal{G}(t),i}$ is the symmetric part of the dynamics $A_{\mathcal{G}(t),i}$.

Proof: In order to prove the theorem, it is sufficient to show that for any topology $\mathcal{G}(t)$ by applying a control law of the form given in (12), the matrix $A_{\mathcal{G}(t)}$ describing the formation dynamics under topology $\mathcal{G}(t)$ is negative definite. Therefore, by exploiting a well-known result coming from the switching control theory we can use the identity matrix to build a common quadratic Lyapunov function (CQLF) to prove the stability of the formation under switching topology [**?**].

In order to prove the negative definiteness, we use a similar argument as in Theorem (1). In particular, for any non-hermitian matrix A with real coefficients we have that A is negative definite if and only if its symmetric part $A^+ = \left(\frac{A+A^T}{2}\right)$ is negative definite. Moreover, according to the generalization of the Gerschgorin circle theorem for block matrices given before, this can be obtained by forcing any Gerschgorin set S_i to be located on the left-half of the Gauss plane ($S_i \in \mathbb{C}^-$). Indeed, this can be achieved by guaranteeing that:

$$\min_{\lambda_i \in \sigma\left(A^+_{\mathcal{G}(t),i}\right)} \{|\lambda_i|\} > \frac{1}{2} \sum_{j \in \mathcal{N}_i} \left(\|B_i K_{ij}^{\mathcal{G}(t)}\| + \|B_j K_{ji}^{\mathcal{G}(t)}\| \right), \tag{13}$$

with $Re\{\lambda\} < 0$ thus proving the theorem. ■

***Remark** 3:* Theorem 2 represents a natural extension to the switching scenario of the result given by Theorem 1. However, the requirement of negative definiteness of the formation dynamics $A_{\mathcal{G}(t)}$ for any given topology $\mathcal{G}(t)$ demands for a bigger effort compared to the static scenario, that is the negative definiteness of each single agent dynamic $A_{\mathcal{G}(t),i}$ and a stronger interaction among the agents to fulfill the constraint (12). Note that, from a computational perspective a distributed collaborative technique must be developed to fulfill the constraint (12). Indeed, such a negotiation process might be significantly limited by the scale of the system.

A simple condition to guarantee the overall stability of the switching formation in a distributed fashion is provided by the following corollary.

Corollary *1:* The condition (12) is fulfilled if the following holds:

$$k_{ij} \;\leq\; \bar{k} < \frac{\min_{\lambda_i \in \sigma\left(A^{+}_{\mathcal{G}(t),i}\right)} \{\mid \lambda_i \mid\}}{n\ \max_{i,j} \|B_i\, P_{ij}\|},\ \forall i, j \in V \tag{14}$$

Note that, corollary 1 provides a *conservative* but decentralized solution to satisfy the sufficient condition given in Theorem 2. It requires each agent to reach consensus on a lower bound of their smallest eigenvalue along with a consensus on an upper bound on their interconnection matrices norms. Then if all the agents know the total number of interconnected agents n, the condition on Theorem 2 is satisfied.

VI. Simulation Results

In the following, simulations concerning the formation acquisition for a multi-agent system are presented. Each agent has a dynamics characterized by the following differential equation.

$$\dot{x}_i \;=\; A_i\, x_i + B_i\, u_i,$$

and the following inter-agent sensing model:

$$z_{ij} \;=\; P_{ij}(x_i - x_j),$$

where z_{ij} is the distance among agents i and j respectively. For sake of simplicity the identity matrix will be chosen as interconnection matrix P_{ij}. The local controller for each agent is defined as follows:

$$\begin{aligned} u_i &= \sum_{j \in \mathcal{N}_i} K_{ij} z_{ij}, \\ &= \sum_{j \in \mathcal{N}_i} K_{ij} P_{ij}(x_i - x_j + \delta_{ij}). \end{aligned}$$

Two different scenarios are considered; in the first one the goal of the multi-agent system is to acquire a formation with the shape of a hexagon, while in the second one the goal is to acquire

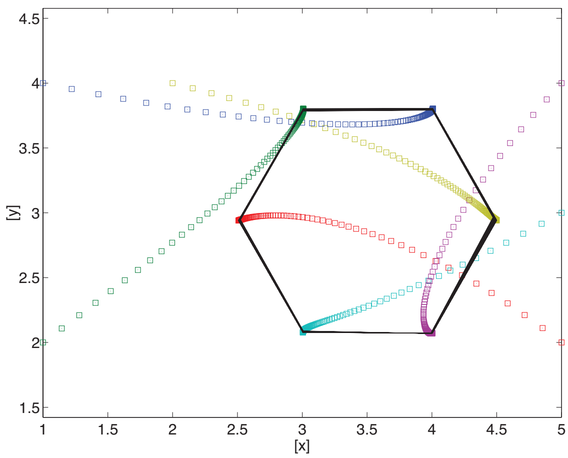

Fig. 1. Formation acquisition: agents' trajectories.

a formation with the shape of a regular lattice. Note that, a control law built according to the condition (10) given in Theorem 1 simply drives the state of the agents to a common value. According to [?], [?], in order to reach a formation with a desired shape, a proper offset must be added to the inter-agent distance z_{ij} for each pair of agents i, j, namely the term δ_{ij}. A simple way to achieve it is to define a proper vector of offsets $\delta = \left[\delta_{10}, \cdots, \delta_{i0}, \cdots, \delta_{N0}\right]$ that will act as an input for the dynamical system.

Regarding the first scenario, Fig. 1 shows the trajectories for the six agents team: agents start from random positions and move till the hexagon shaped formation is reached, while Fig. 2 shows the state evolution of the six agents dynamics.

In the same way, regarding the second scenario, Fig. 3 shows the trajectories for the nine agents team: agents start from random positions and move till the regular lattice shaped formation is reached, while Fig. 2 shows the state evolution of the nine agents dynamics.

Note that, in order to reach such a formation behavior, in both cases an inner control loop to provide an integral action has been designed for each agent. This can be explained by the fact, that the problem of formation acquisition can be thought as a regulation problem for which the presence of an integrator is required in order to drive the dynamics of the error with respect to a constant reference to zero.

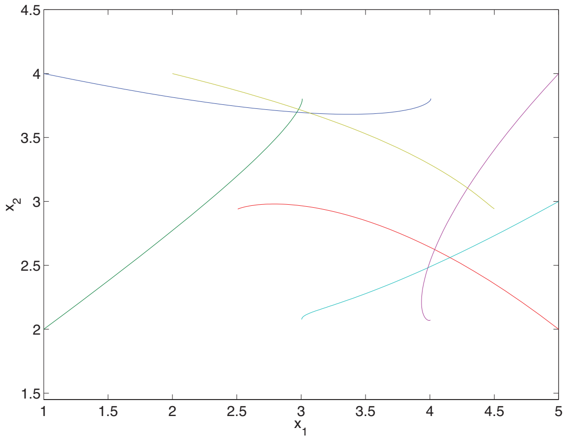

Fig. 2. Formation acquisition: agents' state evolution.

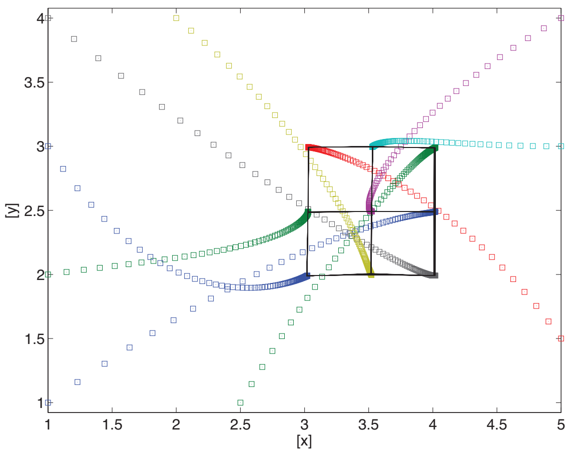

Fig. 3. Formation acquisition: agents' trajectories.

VII. CONCLUSION

In this paper the formation stabilization problem for a system of heterogeneous agents has been addressed. In the proposed scenario, agents are characterized by different dynamics and assumed to be able to collaborate by exchanging information according to their range of communication. A sufficient algebraic condition for the stabilization of the formation in the case of static topology has been provided. Furthermore, conditions under which the formation remains stable under switching topology have been investigated. The key idea is to use a well-known result on the generalization of the Gerschgorin circle theorem for block matrices to define a set of rules which can be applied locally by each agent to build a control law in a complete distributed fashion so

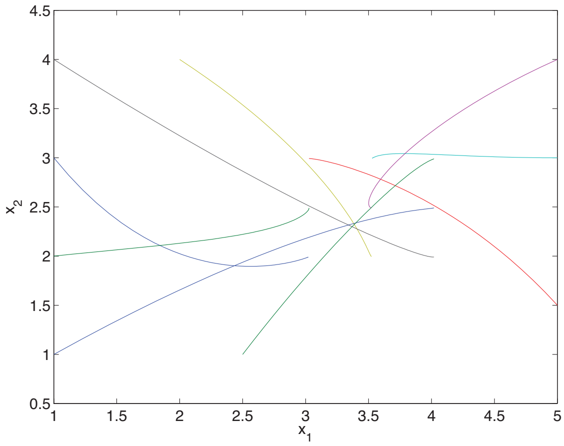

Fig. 4. Formation acquisition: agents' state evolution.

that the global stability of the formation is guaranteed. Simulation results have been proposed to corroborate the theoretical results. Future work, will be mainly focused on the investigation of additional, hopefully simpler condition for the stability of the formation under switching topology.