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MONITOR DESIGN FOR COLORED PETRI NETS WITH UNCONTROLLABLE AND UNOBSERVABLE TRANSITIONS

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Abstract: In this paper we deal with the problem of designing a supervisory controller that enforces certain specifications on the marking of a Colored Petri net (CPN). In particular, we consider colored Generalized Mutual Exclusion Constraints (GMEC) that represent in a compact way several linear constraints that limit the weighted sum of colored tokens in subsets of places. In a previous work we have shown that, when all transitions are controllable and observable with respect to all colors, these constraints can be enforced by a colored monitor place that can be added to the net to obtain the closed-loop system. The novel contribution of this paper is that of showing how these results can be extended to the case of transitions that are uncontrollable and/or unobservable with respect to certain colors. In particular, we show that the parametrization and the tabular procedure proposed by Moody and Antsaklis for uncolored Petri nets can still be used to compute – when it does exist – a less permissive GMEC that can be forced by an admissible colored monitor place.

Keywords: Petri nets, colored Petri nets, generalized mutual exclusion constraints, monitor places.

1. INTRODUCTION

In (Fanti *et al.*, 2003) we showed how the control approach based on the construction of monitor places presented in (Giua *et al.*, 1992) for place/transition nets can be extended to the more general case of CPN (Jensen, 1992). A colored GMEC may represent in a compact way several constraints, and can be unfolded into a set of uncolored GMEC. In (Fanti *et al.*, 2003) it was shown that, when all transitions are controllable and observable, a GMEC can still be enforced by adding a monitor place p_c, and it was provided a systematic procedure to compute the incidence matrix defining such a monitor place, as well as its initial marking. When all transitions are controllable and observable with respect to (wrt) all colors, the monitor place minimally restricts the behavior of the closed-loop system, in the sense that it prevents only those transition firings that yield forbidden markings.

In the presence of transitions that are uncontrollable and/or unobservable wrt certain colors, it may well be the case that the monitor designed for a given GMEC is not admissible. In other words, it may occur that the monitor either disables a transition wrt an uncontrollable color, or observes the firing of a transition wrt an unobservable color. We show that in such a case it may be possible to construct a less permissive, but admissible monitor, extending to the case of colored nets the parametrization and the tabular procedure proposed by Moody and Antsaklis (1998) for uncolored Petri nets (PN).

It is important to observe that the computational complexity of the proposed approach is the same as that of computing the equivalent admissible GMEC and the corresponding monitor places for the unfolded net. There is still some advantages in using our approach with CPN rather than unfolding the net and use the standard approach in (Moody and Antsaklis, 1998).

— In many application cases it is much more convenient to deal with a CPN model rather than a PN model, so as to keep all the advantages deriving from the greater expressive power of CPN and the possibility of using other techniques developed to analyze and simulate CPN. In such a case an alternative to our procedure would be that of unfolding the net, converting the colored GMEC into a set of uncolored GMEC, then compute the admissible monitor places, and finally convert them into a single colored monitor place.

Our approach provides a systematic procedure to do this within the framework of CPN.

— Moreover, when computing an admissible GMEC using the proposed tabular procedure, we do not need to compute the unfolding of the whole CPN, but we only consider the incidence matrix of the unfolded subnet containing those transitions and those colors wrt whom transitions are either uncontrollable or unobservable.

— Finally, the proposed approach is a first step towards the formulation of a symbolic procedure for the computation of admissible monitor places for arbitrary high-level PN.

2. MULTISETS

Definition 2.1. Let D be a set. A *multiset* (resp., *non negative multiset*) $\boldsymbol{\alpha}$ over D is defined by a mapping $\alpha : D \to \mathbb{Z}$ ($\alpha : D \to \mathbb{N}$) and may be represented as $\boldsymbol{\alpha} = \sum_{d \in D} \alpha(d) \otimes d$ where the sum is limited to the elements such that $\alpha(d) \neq 0$.
Let $\mathcal{Z}(D)$ (resp., $\mathcal{N}(D)$) denote the set of all multisets (resp., non negative multisets) over D.
The multiset $\boldsymbol{\varepsilon}$ is the empty multiset such that for all $d \in D$, $\varepsilon(d) = 0$. ■

Now, given two sets D and D', let $\boldsymbol{F} : D \to \mathcal{Z}(D')$ be a function that associates to each element $d \in D$ a multiset on D':

$$\boldsymbol{F}(d) = \sum_{d' \in D'} F(d, d') \otimes d' \in \mathcal{Z}(D').$$

Definition 2.2. Given two sets D and D', a function $\boldsymbol{F} : D \to \mathcal{Z}(D')$, and a multiset $\boldsymbol{\alpha} \in \mathcal{Z}(D)$,

$$\boldsymbol{F}(\boldsymbol{\alpha}) \triangleq \boldsymbol{F} \circ \boldsymbol{\alpha} \triangleq \sum_{d \in D} \alpha(d) \boldsymbol{F}(d) = \sum_{d \in D} \sum_{d' \in D'} \alpha(d) F(d, d') \otimes d' \in \mathcal{Z}(D').$$

■

We finally observe that it is possible to give a matrix representation of multisets and of functions over multisets.
Given two sets D and D', let us arbitrary order their elements as follows: $D = \{d_1, \ldots, d_k\}$ and $D' = \{d'_1, \ldots, d'_{k'}\}$.
A multiset $\boldsymbol{\alpha} \in \mathcal{Z}(D)$ can be represented by a vector:

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha(d_1) \\ \alpha(d_2) \\ \vdots \\ \alpha(d_k) \end{bmatrix} \in \mathbb{Z}^k.$$

Thus, given a function $\boldsymbol{F} : D \to \mathcal{Z}(D')$ for all $d \in D$ we can write

$$\boldsymbol{F}(d) = \begin{bmatrix} F(d, d'_1) \\ F(d, d'_2) \\ \vdots \\ F(d, d'_{k'}) \end{bmatrix} \in \mathbb{Z}^{k'}.$$

while its extension $\boldsymbol{F} : \mathcal{Z}(D) \to \mathcal{Z}(D')$ can be represented by the matrix

$$\boldsymbol{F} = [\, \boldsymbol{F}(d_1) \ \boldsymbol{F}(d_2) \ \ldots \ \boldsymbol{F}(d_k) \,] \in \mathbb{Z}^{k' \times k}$$

and finally the multiset $\boldsymbol{F}(\boldsymbol{\alpha}) = \boldsymbol{F} \circ \boldsymbol{\alpha}$ can be computed with the usual matrix-vector product denoted by $\cdot$, i.e.,

$$\boldsymbol{F}(\boldsymbol{\alpha}) = \boldsymbol{F} \circ \boldsymbol{\alpha} = \boldsymbol{F} \cdot \boldsymbol{\alpha} = \begin{bmatrix} \sum_{i=1}^{k} \alpha(d_i) F(d_i, d'_1) \\ \sum_{i=1}^{k} \alpha(d_i) F(d_i, d'_2) \\ \vdots \\ \sum_{i=1}^{k} \alpha(d_i) F(d_i, d'_{k'}) \end{bmatrix} \in \mathbb{Z}^{k'}.$$

Finally, given a multiset $\boldsymbol{\alpha} \in \mathcal{Z}(D')$, where $D' \in \{d'_1, \ldots, d'_{k'}\}$, with the notation $\boldsymbol{\alpha}'$ we denote a multiset represented by a row vector, i.e.,

$$\boldsymbol{\alpha}' = \left[\, \alpha(d'_1) \ \ldots \ \alpha(d'_{k'}) \,\right] \in \mathbb{Z}^{1 \times k'}.$$

The multiset $\boldsymbol{\alpha}' \circ \boldsymbol{F}$ represented by a row vector, can be computed with the usual matrix-vector product, i.e.,

$$\boldsymbol{\alpha}' \circ \boldsymbol{F} = \boldsymbol{\alpha}' \cdot \boldsymbol{F} = \left[\sum_{i=1}^{k'} \alpha(d'_i) F(d'_i, d_1) \ \ldots \ \sum_{i=1}^{k'} \alpha(d'_i) F(d'_i, d_k) \right] \in \mathbb{Z}^{1 \times k'}.$$

3. COLORED PETRI NETS

A *Colored Petri Net* (CPN) is a bipartite directed graph represented by a quintuple $N = (P, T, Co, \boldsymbol{Pre}, \boldsymbol{Post})$ where P is the set of places, T is the set of transitions, $Co : P \cup T \to \mathcal{C}l$ is a color function that associates to each element in $P \cup T$ a non empty ordered set of colors in the set of possible colors $\mathcal{C}l$.
Therefore, for all $p_i \in P$, $Co(p_i) = \{a_{i,1}, \ldots, a_{i,u_i}\} \subseteq \mathcal{C}l$ is the ordered set of possible colors of tokens in p_i, and u_i is the number of possible colors of tokens in p_i. Analogously, for all $t_j \in T$, $Co(t_j) = \{b_{j,1}, \ldots, b_{j,v_j}\} \subseteq \mathcal{C}l$ is the ordered set of possible occurrence colors of t_j, and v_j is the number of possible occurrence colors in t_j.
We assume that $m = |P|$ and $n = |T|$.
Matrices $\boldsymbol{Pre}$ and $\boldsymbol{Post}$ are the pre-incidence and the post-incidence $m \times n$ dimensional matrices respectively. Each element $\boldsymbol{Pre}(p_i, t_j)$ (the same reasoning applies to $\boldsymbol{Post}$) is a mapping from the set of occurrence colors of t_j to a non negative multiset over the set of colors of p_i, namely, $\boldsymbol{Pre}(p_i, t_j) : Co(t_j) \to \mathcal{N}(Co(p_i))$, for $i = 1, \ldots, m$ and $j = 1, \ldots, n$. We denote $\boldsymbol{Pre}(p_i, t_j)$ as a matrix of $u_i \times v_j$ non negative integers, whose generic element $Pre(p_i, t_j)(h, k)$ is equal to the weight of the arc from place p_i wrt color $a_{i,h}$ to transition t_j wrt color $b_{j,k}$.
The incidence matrix $\boldsymbol{C}$ is an $m \times n$ matrix, whose generic element $\boldsymbol{C}(p_i, t_j) : Co(t_j) \to \mathcal{Z}(Co(p_i))$, for $i = 1, \ldots, m$ and $j = 1, \ldots n$. In particular $\boldsymbol{C}(p_i, t_j) = \boldsymbol{Post}(p_i, t_j) - \boldsymbol{Pre}(p_i, t_j)$.
For each place $p_i \in P$, we define the *marking* $\boldsymbol{m}_i$ of p_i as a *non negative multiset* over $Co(p_i)$. The mapping $m_i : Co(p_i) \to \mathbb{N}$ associates to each possible token color in p_i a non negative integer representing the number of tokens of that color that is contained in place p_i, and $\boldsymbol{m}_i = \sum_{d \in Co(p_i)} m_i(d) \otimes d$.
Here we denote $\boldsymbol{m}_i$ as a column vector of u_i non negative integers, whose h-th component $m_i(h)$ is equal to the number of tokens of color $a_{i,h}$ that are contained in p_i.
Finally, the marking $\boldsymbol{M}$ of a CPN is an m-dimensional column vector of multisets whose i-th entry is equal to $\boldsymbol{m}_i$.
A colored Petri net system $\langle N, \boldsymbol{M}_0 \rangle$ is a colored Petri net N with initial marking $\boldsymbol{M}_0$.
A transition $t_j \in T$ is *enabled* wrt color $b_{j,k}$ at a marking $\boldsymbol{M}$ if and only if for each place $p_i \in P$ and for all $h = 1, \ldots, u_i$, we have $m_i(h) \geq Pre(p_i, t_j)(h, k)$.
The firing of transition t_i wrt color b_{ij} follows the standard rules of CPN (Giua and Seatzu, 2004).

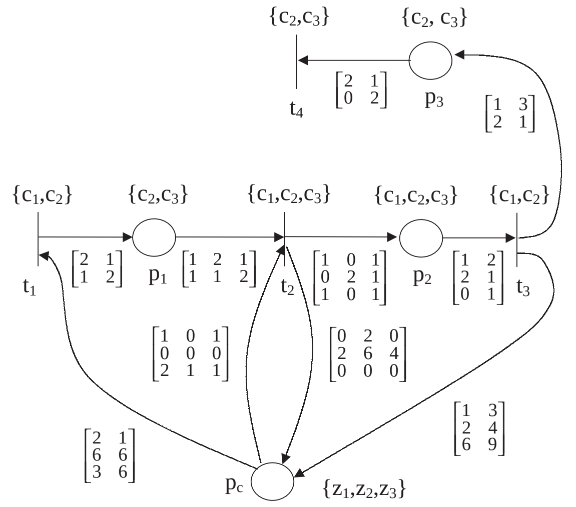

Fig. 1. The closed-loop colored Petri net of Example 1.

Example 3.1. Let us consider the CPN in Figure 1.a apart from place p_c and all connected arcs. The set of colors is $\mathcal{C}l = \{c_1, c_2, c_3\}$. Places p_1 and p_3 may only contain tokens of colors c_2 and c_3, while place p_2 may contain tokens of any color in $\mathcal{C}l$. Finally, transitions t_1 and t_3 may only fire wrt to colors c_1 and c_2, while transition t_2 may fire wrt any color in $\mathcal{C}l$, and transition t_4 may fire wrt colors c_2 and c_3. Given the structure of the net, the only non null matrices $\boldsymbol{Pre}$ and $\boldsymbol{Post}$ are those reported Figure 1.a using the matrix notation. ■

Finally, let us introduce the following definitions.

Definition 3.2. Given a CPN $N_p = (P, T, Co, \boldsymbol{Pre}_p, \boldsymbol{Post}_p)$, a transition $t_j \in T$ is *uncontrollable wrt color* $b_{j,k} \in Co(t_j)$ if its firing wrt $b_{j,k}$ cannot be inhibited by an external action. Thus, the set $Co(t_j)$ may be partitioned as $Co(t_j) = Co^{uc}(t_j) \cup Co^{c}(t_j)$ where $Co^{c}(t_j)$ $(Co^{uc}(t_j))$ is the set of colors wrt whom transition t_j is controllable (uncontrollable).

If $Co^{uc}(t_j) = \emptyset$ we say that t_j is *controllable*; if $Co^{c}(t_j) = \emptyset$ we say that t_j is *uncontrollable.* ■

Definition 3.3. Given a CPN $N_p = (P, T, Co, \boldsymbol{Pre}_p, \boldsymbol{Post}_p)$, a transition $t_j \in T$ is *unobservable wrt color* $b_{j,k} \in Co(t_j)$ if its firing wrt $b_{j,k}$ cannot be measured by an external action. Thus, the set $Co(t_j)$ may be partitioned as $Co(t_j) = Co^{uo}(t_j) \cup Co^{o}(t_j)$ where $Co^{o}(t_j)$ $(Co^{uo}(t_j))$ is the set of colors wrt whom transition t_j is observable (unobservable).

If $Co^{uo}(t_j) = \emptyset$ we say that t_j is *observable*; if $Co^{o}(t_j) = \emptyset$ we say that t_j is *unobservable.* ■

4. GMEC IN COLORED PETRI NETS

Now, we recall the notion of colored GMEC (Fanti *et al.*, 2003).

Definition 4.1. A GMEC is a couple $(\boldsymbol{W}, \boldsymbol{k})$ where $\boldsymbol{W} = [\boldsymbol{w}_1 \ \ldots \ \boldsymbol{w}_m]$, $\boldsymbol{k} \in \mathcal{Z}(D)$ and for all i, $\boldsymbol{w}_i : Co(p_i) \to \mathcal{Z}(D)$, while D is a set of colors different from $Co(p_i)$, $i = 1, \ldots, m$. Thus $\boldsymbol{W}$ can also be represented by a matrix with $|D|$ rows and $\sum_{i=1}^{m} |Co(p_i)|$ columns, and

$$\mathcal{M}(\boldsymbol{W}, \boldsymbol{k}) = \left\{ \boldsymbol{M} = \begin{bmatrix} \boldsymbol{m}_1 \\ \vdots \\ \boldsymbol{m}_m \end{bmatrix} \;\middle|\; \boldsymbol{m}_i \in \mathcal{N}(Co(p_i)), \right.$$
$$\left. \boldsymbol{W} \circ \boldsymbol{M} \triangleq \sum_{i=1}^{m} \boldsymbol{w}_i \circ \boldsymbol{m}_i \le \boldsymbol{k} \right\}.$$

is the set of legal markings. ■

Note that here we are extending the $\circ$ operator to the case of scalar product of vectors of multisets.

Example 4.2. Let us consider again the CPN in Figure 1.a apart from place p_c and all connected arcs. Assume $D = \{z_1, z_2, z_3\}$. Moreover, let $\boldsymbol{w}_1 = [\boldsymbol{w}_1(c_2) \ \boldsymbol{w}_1(c_3)]$, $\boldsymbol{w}_1(c_1) = 1 \otimes z_1 + 2 \otimes z_2$, $\boldsymbol{w}_1(c_3) = 2 \otimes z_2 + 3 \otimes z_3$, $\boldsymbol{w}_2 = [\boldsymbol{w}_2(c_1) \ \boldsymbol{w}_2(c_2) \ \boldsymbol{w}_2(c_3)]$, $\boldsymbol{w}_2(c_1) = 1 \otimes z_1 + 2 \otimes z_2 + 2 \otimes z_3$, $\boldsymbol{w}_2(c_2) = 2 \otimes z_3$, $\boldsymbol{w}_2(c_3) = 1 \otimes z_1 + 3 \otimes z_3$, $\boldsymbol{w}_3(c_2) = \boldsymbol{w}_3(c_3) = \boldsymbol{\varepsilon}$ and $\boldsymbol{k} = 3 \otimes z_1 + 5 \otimes z_2 + 6 \otimes z_3$. Therefore,

$$\boldsymbol{W} \circ \boldsymbol{M} \triangleq \sum_{i=1}^{m} \boldsymbol{w}_i \circ \boldsymbol{m}_i = \begin{bmatrix} 1 & 0 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} m_1(c_2) \\ m_1(c_3) \end{bmatrix} +$$
$$+ \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 0 \\ 2 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} m_2(c_1) \\ m_2(c_2) \\ m_2(c_3) \end{bmatrix} \le \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix} \implies$$
$$\mathcal{M}(\boldsymbol{W}, \boldsymbol{k}) = \{ \ \boldsymbol{M} = \begin{bmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \\ \boldsymbol{m}_3 \end{bmatrix} \ | \ \boldsymbol{m}_i \in \mathcal{N}(Co(p_i)),$$
$$m_1(c_2) + m_2(c_1) + m_2(c_3) \le 3,$$
$$2m_1(c_2) + 2m_1(c_3) + 2m_2(c_1) \le 5$$
$$3m_1(c_3) + 2m_2(c_1) + 2m_2(c_2) + 3m_2(c_3) \le 6\}.$$

■

5. MONITORS FOR COLORED PETRI NETS

Definition 5.1. Given a CPN system $\langle N_p, \boldsymbol{M}_{p,0} \rangle$, with $N_p = (P, T, Co, \boldsymbol{Pre}_p, \boldsymbol{Post}_p)$, and a GMEC $(\boldsymbol{W}, \boldsymbol{k})$ with $\boldsymbol{k} \in \mathcal{Z}(D)$, the *monitor* that enforces this constraint is a new place p_c with $Co(p_c) = D$, to be added to N_p. The resulting system is denoted $\langle N, \boldsymbol{M}_0 \rangle$, with $N = (P \cup \{p_c\}, T, Co, \boldsymbol{Pre}, \boldsymbol{Post})$. Then N will have incidence matrix

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_p \\ \boldsymbol{C}_c \end{bmatrix}, \quad \boldsymbol{C}_c = -\boldsymbol{W} \circ \boldsymbol{C}_p. \qquad (1)$$

We are assuming that there are no selfloops containing p_c in N, hence $\boldsymbol{Pre}$ and $\boldsymbol{Post}$ may be uniquely determined by $\boldsymbol{C}$. The initial marking of $\langle N, \boldsymbol{M}_0 \rangle$ is

$$\boldsymbol{M}_0 = \begin{bmatrix} \boldsymbol{M}_{p,0} \\ \boldsymbol{m}_{c,0} \end{bmatrix}, \quad \boldsymbol{m}_{c,0} = \boldsymbol{k} - \boldsymbol{W} \circ \boldsymbol{M}_{p,0}. \qquad (2)$$

We assume that the initial marking $\boldsymbol{M}_{p,0}$ of the system satisfies the constraint $(\boldsymbol{W}, \boldsymbol{k})$. ■

In the case of controllable and observable transitions we proved the following result (Fanti *et al.*, 2003).

Theorem 5.2. Let $\langle N_p, \boldsymbol{M}_{p,0} \rangle$ be a CPN system, and $(\boldsymbol{W}, \boldsymbol{k})$ a colored GMEC. Let $\langle N, \boldsymbol{M}_0 \rangle$ be the system with the addition of the monitor place p_c.

(1) The monitor place p_c enforces the GMEC $(\boldsymbol{W}, \boldsymbol{k})$ when included in the closed-loop system $\langle N, \boldsymbol{M}_0 \rangle$.

(2) The monitor place p_c minimally restricts the behavior of the closed-loop system $\langle N, \boldsymbol{M}_0 \rangle$, in the sense that it prevents only transition firings that yield forbidden markings.

6. UNCONTROLLABLE AND UNOBSERVABLE GMEC

In this section we first introduce the definition of controllable and observable GMEC. Then we provide necessary and sufficient conditions for its controllability and observability.

Definition 6.1. Given a CPN system $\langle N_p, \boldsymbol{M}_{p,0}\rangle$, with $N_p = (P, T, Co, \boldsymbol{Pre}_p, \boldsymbol{Post}_p)$, and a GMEC $(\boldsymbol{W}, \boldsymbol{k})$ with $\boldsymbol{k} \in \mathcal{Z}(D)$, let p_c with $Co(p_c) = D$ be the corresponding monitor place with (pre-)incidence matrix $(\boldsymbol{Pre}_c)$ $\boldsymbol{C}_c$.

— The monitor place p_c is said *structurally controllable* (or simply, *controllable*) if for all transitions $t_j \in T$ and for all colors $b_{j,k} \in Co^{uc}(t_j)$, $\boldsymbol{Pre}_c(\cdot, b_{j,k}) = \boldsymbol{\varepsilon}$, i.e., the monitor place is controllable if the weights of the input arcs to transitions that are uncontrollable wrt certain colors, are null wrt those colors.

— The monitor place p_c is said *structurally observable* (or simply, *observable*) if for all transitions $t_j \in T$ and for all colors $b_{j,k} \in Co^{uo}(t_j)$, $\boldsymbol{C}_c(\cdot, b_{j,k}) = \boldsymbol{\varepsilon}$, i.e., the monitor place is observable if the weights of the input and the output arcs to transitions that are uncontrollable wrt certain colors, are null wrt those colors. ■

A monitor place that is controllable and observable is said *admissible.*

Analogously, we say that a GMEC is *controllable* and *observable* (*admissible*) if the corresponding monitor place is controllable and observable (*admissible*).

Proposition 6.2. Given a CPN system $\langle N_p, \boldsymbol{M}_{p,0}\rangle$ with incidence matrix $\boldsymbol{C}_p$, let $(\boldsymbol{W}, \boldsymbol{k})$ with $\boldsymbol{k} \in \mathcal{Z}(D)$ be a GMEC that we want to force, and p_c be the corresponding monitor place with incidence matrix $\boldsymbol{C}_c$. Let $\boldsymbol{C}_p^{uc}$ be the matrix that we obtain selecting from $\boldsymbol{C}_p$ the only columns relative to those transitions and those colours such that the considered transitions are uncontrollable wrt the considered colors.

(i) The GMEC and the monitor place are controllable *if and only if*

$$\boldsymbol{C}_c^{uc} = -\boldsymbol{W} \circ \boldsymbol{C}_p^{uc} \geq \underbrace{[\,\boldsymbol{\varepsilon} \;\; \ldots \;\; \boldsymbol{\varepsilon}\,]}_{\sum_{j:\; t_j \in T} |Co^{uc}(t_j)|}.$$

(ii) The GMEC and the monitor place are observable *if and only if*

$$\boldsymbol{C}_c^{uo} = -\boldsymbol{W} \circ \boldsymbol{C}_p^{uo} = \underbrace{[\,\boldsymbol{\varepsilon} \;\; \ldots \;\; \boldsymbol{\varepsilon}\,]}_{\sum_{j:\; t_j \in T} |Co^{uo}(t_j)|}.$$

Proof. See (Giua and Seatzu, 2004). □

Example 6.3. Let us consider again the CPN system in Figure 1.a apart from place p_c and all connected arcs. Assume that transition t_2 is uncontrollable wrt colors c_2 and c_3. In such a case the GMEC $(\boldsymbol{W}, \boldsymbol{k})$ defined in the previous Example 4.2 is not controllable being

$$\boldsymbol{C}_c^{uc} = -\boldsymbol{W} \circ \boldsymbol{C}_p^{uc} = \begin{array}{c} \\ \end{array}\!\!\begin{array}{c} \begin{array}{cc} c_2 & c_3 \end{array} \\ \begin{bmatrix} 2 & -1 \\ 6 & 4 \\ -1 & -1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \not\geq [\,\boldsymbol{\varepsilon}\; \boldsymbol{\varepsilon}\,]$$

Now, assume that transition t_2 is unobservable wrt color c_1, and transition t_3 is unobservable wrt color c_2. Being

$$\boldsymbol{C}_c^{uo} = -\boldsymbol{W} \circ \boldsymbol{C}_p^{uo} = \begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ -2 & 9 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \neq [\,\boldsymbol{\varepsilon}\; \boldsymbol{\varepsilon}\,]$$

the GMEC $(\boldsymbol{W}, \boldsymbol{k})$ is not observable ■

7. MONITORS FOR UNCONTROLLABLE AND UNOBSERVABLE GMEC

As well known from the uncolored PN theory, given an uncontrollable or unobservable GMEC, the maximal permissive supervisor may not be a monitor place (Giua *et al.*, 1992). An important contribution in this area is due to Moody and Antsaklis (1998) who proposed a very simple and efficient procedure to compute a family of more restrictive *controllable and observable* GMEC that force the original GMEC. Clearly, these GMEC are not in general maximally permissive. Here we show how the procedure by Moody and Antsaklis can be easily extended to the case of CPN.

Proposition 7.1. Given a GMEC $(\boldsymbol{W}, \boldsymbol{k})$ with

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{w}_1 & \ldots & \boldsymbol{w}_m \end{bmatrix}, \quad \boldsymbol{w}_i : Co(p_i) \to \mathcal{Z}(D), \quad i = 1, \ldots, m,$$
$$\boldsymbol{k} \in \mathcal{Z}(D), \quad D = \{z_1, \;\ldots,\; z_q\},$$

let us consider the modified GMEC $(\tilde{\boldsymbol{W}}, \tilde{\boldsymbol{k}})$ where

$$\tilde{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{s}_1' \circ \boldsymbol{W} \\ \vdots \\ \boldsymbol{s}_q' \circ \boldsymbol{W} \end{bmatrix} + \boldsymbol{R},$$
$$\boldsymbol{s}_i \in \mathcal{N}(D), \quad s_i(z_j) \in \mathbb{N}^+ \text{ if } i = j, \quad s_i(z_j) = 0 \text{ if } i \neq j,$$
$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{r}_1 & \ldots & \boldsymbol{r}_m \end{bmatrix}, \quad r_i : Co(p_i) \to \mathcal{N}(D), \quad i = 1, \;\ldots,\; m,$$
$$\tilde{\boldsymbol{k}} = \begin{bmatrix} \boldsymbol{s}_1' \circ (\boldsymbol{k} + \boldsymbol{1}) \\ \vdots \\ \boldsymbol{s}_q' \circ (\boldsymbol{k} + \boldsymbol{1}) \end{bmatrix} - \boldsymbol{1}$$

where $\boldsymbol{1} \in \mathcal{N}(D)$ is the multiset of ones over D. It holds $\mathcal{M}(\tilde{\boldsymbol{W}}, \tilde{\boldsymbol{k}}) \subseteq \mathcal{M}(\boldsymbol{W}, \boldsymbol{k})$.

Proof. If $\boldsymbol{M} \in \mathcal{M}(\tilde{\boldsymbol{W}}, \tilde{\boldsymbol{k}})$, then $\tilde{\boldsymbol{W}} \circ \boldsymbol{M} \leq \tilde{\boldsymbol{k}}$, or equivalently, by definition,

$$\begin{bmatrix} \boldsymbol{s}_1' \circ \boldsymbol{W} \\ \vdots \\ \boldsymbol{s}_q' \circ \boldsymbol{W} \end{bmatrix} \circ \boldsymbol{M} + \boldsymbol{R} \circ \boldsymbol{M} \leq \begin{bmatrix} \boldsymbol{s}_1' \circ (\boldsymbol{k} + \boldsymbol{1}) \\ \vdots \\ \boldsymbol{s}_q' \circ (\boldsymbol{k} + \boldsymbol{1}) \end{bmatrix} - \boldsymbol{1}. \tag{3}$$

Therefore,

$$\begin{bmatrix} \boldsymbol{s}'_1 \circ \boldsymbol{W} \\ \vdots \\ \boldsymbol{s}'_q \circ \boldsymbol{W} \end{bmatrix} \circ \boldsymbol{M} \leq \begin{bmatrix} \boldsymbol{s}'_1 \circ (\boldsymbol{k}+\boldsymbol{1}) \\ \vdots \\ \boldsymbol{s}'_q \circ (\boldsymbol{k}+\boldsymbol{1}) \end{bmatrix} - \boldsymbol{1} - \boldsymbol{R} \circ \boldsymbol{M}$$

$$\leq \begin{bmatrix} \boldsymbol{s}'_1 \circ (\boldsymbol{k}+\boldsymbol{1}) \\ \vdots \\ \boldsymbol{s}'_q \circ (\boldsymbol{k}+\boldsymbol{1}) \end{bmatrix} - \boldsymbol{1} \tag{4}$$

being $\boldsymbol{R} \circ \boldsymbol{M} \geq \boldsymbol{\varepsilon}$. Now, let us rewrite the weighting matrix $\boldsymbol{W}$ as

$$\boldsymbol{W} = \begin{bmatrix} \bar{\boldsymbol{w}}'_1 \\ \vdots \\ \bar{\boldsymbol{w}}'_q \end{bmatrix}.$$

Because each multiset $\boldsymbol{s}_i$ has only one non-zero (positive) element in correspondence to the i-th position, the inequality (4) can be rewritten as

$$s_i(z_i)\ \bar{\boldsymbol{w}}'_i \circ \boldsymbol{M} \leq s_i(z_i)\ (k(z_i)+1) - 1, \quad i = 1,\ \ldots,\ q.$$

Now, because $s_i(z_i) \in \mathbb{N}^+$, it holds

$$\bar{\boldsymbol{w}}'_i \circ \boldsymbol{M} \leq k(z_i) + 1 - \frac{1}{s_i(z_i)} < k(z_i) + 1, \quad i = 1,\ \ldots,\ q.$$

Being $\bar{\boldsymbol{w}}'_i \circ \boldsymbol{M} \in \mathbb{Z}$ for all $i = 1,\ \ldots,\ q$, it holds that $\bar{\boldsymbol{w}}'_i \circ \boldsymbol{M} \leq k(z_i)$, for $i = 1,\ \ldots,\ q$, or equivalently $\boldsymbol{W} \circ \boldsymbol{M} \leq \boldsymbol{k}$, i.e., $\boldsymbol{M} \in \mathcal{M}(\boldsymbol{W}, \boldsymbol{k})$, thus proving the statement. □

A modified GMEC can be easily determined using the Algorithm 7.2 that is an extension of the algorithm presented by Moody and Antsaklis in (1998).

Note that with no ambiguity in the notation we often refer to the matrix representation of multisets and of matrices of multisets. Moreover, we denote as $\boldsymbol{0}_{k \times q}$ the zero matrix of dimension $k \times q$ and with $\boldsymbol{0}_k$ the zero column vector of dimension k.

If the algorithm stops with $k = q+1$ and $flag = 0$, then consider the transformed GMEC $(\tilde{\boldsymbol{W}}, \tilde{\boldsymbol{k}})$ defined as follows

$$\tilde{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{s}'_1 \circ \boldsymbol{W} \\ \vdots \\ \boldsymbol{s}'_q \circ \boldsymbol{W} \end{bmatrix} + \boldsymbol{R}, \quad \tilde{\boldsymbol{k}} = \begin{bmatrix} \boldsymbol{s}'_1 \circ (\boldsymbol{k}+\boldsymbol{1}) \\ \vdots \\ \boldsymbol{s}'_q \circ (\boldsymbol{k}+\boldsymbol{1}) \end{bmatrix} - \boldsymbol{1}$$

that is controllable and observable being

$$-\tilde{\boldsymbol{W}} \circ \boldsymbol{C}_p^{uc} \geq \underbrace{[\,\varepsilon \ \ \ldots \ \ \varepsilon\,]}_{n_{uc}}$$

$$-\tilde{\boldsymbol{W}} \circ \boldsymbol{C}_p^{uo} = \underbrace{[\,\varepsilon \ \ \ldots \ \ \varepsilon\,]}_{n_{uo}}$$

Moreover, by Proposition 7.1, the controllable and observable monitor $\tilde{p}_c$ corresponding to the GMEC $(\tilde{\boldsymbol{W}}, \tilde{\boldsymbol{k}})$, if physically implementable, is sub-optimal (or even optimal) for the GMEC $(\boldsymbol{W}, \boldsymbol{k})$.

Example 7.3. Let us consider the CPN system in Figure 1.a. apart from place p_c and all connected arcs. Assume that transition t_2 is uncontrollable wrt colors c_2 and c_3 and unobservable wrt color c_1, while transition t_3 is unobservable wrt color c_2. Let us apply Algorithm 7.2 to compute an admissible GMEC $(\tilde{\boldsymbol{W}}, \tilde{\boldsymbol{k}})$. In such a case $n_{uc} = n_{uo} = 2$, and $M = 7$. Moreover, at step 1 of Algorithm 7.2 we define

$$\boldsymbol{V} = \begin{bmatrix} -2 & 1 \\ -6 & -4 \\ 1 & 1 \end{bmatrix}, \quad \boldsymbol{U} = \begin{bmatrix} 1 & -3 \\ -2 & -4 \\ 2 & -9 \end{bmatrix},$$

$$\boldsymbol{R} = [\,\varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\,], \quad \boldsymbol{s}_i = \boldsymbol{\varepsilon},$$

$i = 1, \ldots, 3$. Thus, for $k = 1$ we consider the table

$$\boldsymbol{A} = \left|\begin{array}{cc|cc|ccccccc|c|} -2 & -1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hline -2 & 1 & 1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right.$$

To make null the element $v(2) = 1$ we update the last row of $\boldsymbol{A}$ as: $\boldsymbol{A}(8,\cdot) = \boldsymbol{A}(8,\cdot) + \boldsymbol{A}(1,\cdot)$. To make null the element $u(2) = -3$ we update the last row of $\boldsymbol{A}$ as: $\boldsymbol{A}(8,\cdot) = \boldsymbol{A}(8,\cdot) + \boldsymbol{A}(6,\cdot)$.

Now, $\boldsymbol{v} \leq [0\ \ 0]'$ and $\boldsymbol{u} = [0\ \ 0]'$, thus we update $s_1(1) = 1$, $\boldsymbol{R}(1,\cdot) = [1\ 0\ 0\ 0\ 0\ 1\ 0]$, and repeat the procedure for $k = 2$.

An admissible GMEC is $(\tilde{\boldsymbol{W}}, \tilde{\boldsymbol{k}})$ where

$$\tilde{\boldsymbol{w}}_1 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \quad \tilde{\boldsymbol{w}}_2 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

$$\tilde{\boldsymbol{w}}_3 = \begin{bmatrix} 1 & 0 \\ 0 & 6 \\ 3 & 0 \end{bmatrix} \quad \tilde{\boldsymbol{k}} = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$$

Thus, the resulting monitor place is defined by

$$\tilde{\boldsymbol{C}}_c = \begin{array}{c} \begin{array}{cc|ccc|cc|cc} c_1 & c_2 & c_1 & c_2 & c_3 & c_1 & c_2 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cc|ccc|cc|cc} -4 & -2 & 0 & 4 & 0 & 0 & 0 & 2 & 0 \\ -6 & -6 & 0 & 6 & 2 & -10 & 0 & 6 & 12 \\ -6 & -9 & 0 & 2 & 2 & 3 & 0 & 6 & 0 \end{array}\right] \end{array} \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array}$$

Let $\boldsymbol{M}_{p,0} = [\boldsymbol{\varepsilon}\ \ 1 \otimes c_1 + 1 \otimes c_3\ \ \boldsymbol{\varepsilon}]'$ that satisfies the modified GMEC $(\tilde{\boldsymbol{W}}, \tilde{\boldsymbol{k}})$. The initial marking of $\tilde{p}_c$ should be taken equal to $\tilde{\boldsymbol{m}}_{c,0} = \tilde{\boldsymbol{k}} - \tilde{\boldsymbol{W}} \circ \boldsymbol{M}_{p,0} = \tilde{\boldsymbol{k}} - \sum_{i=1}^{3} \tilde{\boldsymbol{w}}_i \circ \boldsymbol{m}_{p,0,i} = 1 \otimes z_1 + 1 \otimes z_2 + 1 \otimes z_3$. ■

8. CONCLUSIONS

In this paper we first recalled some results in (Fanti *et al.*, 2003) where we shown that the classic PN control approach based on GMEC and monitor places can be extended to the case of CPN. Then, we focused our attention to the case in which not all transitions are controllable and observable thus a colored monitor designed for a given colored GMEC may not be admissible (it either disables a transition wrt an uncontrollable color, or observes the firing of a transition wrt an unobservable color). We shown how it may be possible to construct a less permissive, but admissible monitor, extending to the case of CPN the parametrization and the tabular procedure proposed by Moody and Antsaklis (1998) for uncolored PN.

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Algorithm 7.2 (Design of a suboptimal monitor place for CPN).
Let us consider a CPN system $\langle N_p, \boldsymbol{M}_{p,0}\rangle$ with incidence matrix $\boldsymbol{C}_p$. Let $\boldsymbol{C}_p^{uc}$ ($\boldsymbol{C}_p^{uo}$) be the matrices obtained selecting from $\boldsymbol{C}_p$ the only columns relative to those transitions and those colors such that the considered transitions are uncontrollable (unobservable) wrt those colors.
Let $(\boldsymbol{W}, \boldsymbol{k})$ with $\boldsymbol{k} \in \mathcal{Z}(D)$, $D = \{z_1, \ldots, z_q\}$, be the GMEC that we want to force.
Finally, let $n_{uc} = \sum_{j:t_j \in T} |Co^{uc}(t_j)|$, $n_{uo} = \sum_{j:t_j \in T} |Co^{uo}(t_j)|$, and $M = \sum_{i=1}^{m} |Co(p_i)|$.

1. Let $\boldsymbol{V} := \boldsymbol{W} \circ \boldsymbol{C}_p^{uc}$, $\boldsymbol{U} := \boldsymbol{W} \circ \boldsymbol{C}_p^{uo}$,

$$\boldsymbol{R} := \underbrace{[\,\boldsymbol{\varepsilon} \quad \ldots \quad \boldsymbol{\varepsilon}\,]}_{M} \quad \boldsymbol{s}_i := \boldsymbol{\varepsilon}, \quad i = 1, \ldots, q, \quad \boldsymbol{\varepsilon} \in \mathcal{Z}(D), \quad k := 1, \quad flag := 0.$$

2. While $k \le q$ **and** $flag = 0$, **do**
if $\boldsymbol{V}(k, \cdot) \le \boldsymbol{0}'_{n_{uc}}$ and $\boldsymbol{U}(k, \cdot) = \boldsymbol{0}'_{n_{uo}}$, then **let** $k := k + 1$, **else**
begin

2.1. let $\boldsymbol{A} := \left|\begin{array}{c|c|c|c} \boldsymbol{C}_p^{uc} & \boldsymbol{C}_p^{uo} & \boldsymbol{I}_M & \boldsymbol{0}_M \\ \hline \boldsymbol{v} & \boldsymbol{u} & \bar{\boldsymbol{r}}_1 & r_2 \end{array}\right|$ where $\boldsymbol{v} := \boldsymbol{V}(k, \cdot)$, $\boldsymbol{u} := \boldsymbol{U}(k, \cdot)$, $\bar{\boldsymbol{r}}_1 := \boldsymbol{0}'_M$, $r_2 := 1$.

2.2. Let $\mathcal{J}_{uc} := \{j \mid \boldsymbol{v}(j) > 0\}$ be the set of indices of columns of $\boldsymbol{A}$ corresponding to positive elements of $\boldsymbol{v}$.
Let $\mathcal{J}_{uo}^{+} := \{j + n_{uc} \mid \boldsymbol{u}(j) > 0\}$ be the set of indices of columns of $\boldsymbol{A}$ corresponding to positive elements of $\boldsymbol{u}$.
Let $\mathcal{J}_{uo}^{-} := \{j + n_{uc} \mid \boldsymbol{u}(j) < 0\}$ be the set of indices of columns of $\boldsymbol{A}$ corresponding to negative elements of $\boldsymbol{u}$.
Let $\mathcal{J} := \mathcal{J}_{uc} \cup \mathcal{J}_{uo}^{+} \cup \mathcal{J}_{uo}^{-}$. **If** $\mathcal{J} = \emptyset$, then **goto** 2.4.

2.3 If $\mathcal{J} \neq \emptyset$, then **choose** a value $\bar{\jmath} \in \mathcal{J}$.
If $\bar{\jmath} \in \mathcal{J}_{uc}$ then try to reduce $v(\bar{\jmath})$ to zero using the following procedure.
(a) Let $\mathcal{I} := \{i \mid C_p^{uc}(i, \bar{\jmath}) < 0\}$ be the set of row indices of the elements of $C_p^{uc}(\cdot, \bar{\jmath})$ that are negative.
If $\mathcal{I} = \emptyset$ it is not possible to get null the positive element $v(\bar{\jmath})$,
then **let** $flag := 1$ and **goto** 2.4.
(b) Choose an index $\bar{\imath} \in \mathcal{I}$ and compute $d := \text{l.c.m.}\{-C_p^{uc}(\bar{\imath}, \bar{\jmath}), v(\bar{\jmath})\}$
the least common multiple among the two elements $-C_p^{uc}(\bar{\imath}, \bar{\jmath})$ and $v(\bar{\jmath})$.
(c) Update the $(M + 1)$–th row of $\boldsymbol{A}$ as follows:

$$\boldsymbol{A}(M + 1, \cdot) := \frac{d}{v(\bar{\jmath})} \boldsymbol{A}(M + 1, \cdot) + \frac{d}{C_p^{uc}(\bar{\imath}, \bar{\jmath})} \boldsymbol{A}(\bar{\imath}, \cdot).$$

By construction $v(\bar{\jmath})$ is null.
If $\bar{\jmath} \in \mathcal{J}_{uo}^{+}$ then try to reduce $u(\bar{\jmath} - n_{uc})$ to zero using the following procedure.
(a) Let $\mathcal{I}^{-} := \{i \mid C_p^{uo}(i, \bar{\jmath} - n_{uc}) < 0\}$ be the set of row indices of the elements of $C_p^{uo}(\cdot, \bar{\jmath} - n_{uc})$ that are negative.
If $\mathcal{I}^{-} = \emptyset$ it is not possible to get null the positive element of $\boldsymbol{u}(\bar{\jmath} - n_{uc})$,
then let $flag := 1$ and goto 2.4.
(b) Choose an index $\bar{\imath} \in \mathcal{I}^{-}$ and compute $d^{+} := \text{l.c.m.}\{-C_p^{uo}(\bar{\imath}, \bar{\jmath} - n_{uc}), u(\bar{\jmath} - n_{uc})\}$.
(c) Update the $(M + 1)$–th row of $\boldsymbol{A}$ as follows:

$$\boldsymbol{A}(M + 1, \cdot) := \frac{d^{+}}{u(\bar{\jmath} - n_{uc})} \boldsymbol{A}(M + 1, \cdot) + \frac{d^{+}}{C_p^{uo}(\bar{\imath}, \bar{\jmath} - n_{uc})} \boldsymbol{A}(\bar{\imath}, \cdot).$$

By construction $u(\bar{\jmath} - n_{uc})$ is null.
If $\bar{\jmath} \in \mathcal{J}_{uo}^{-}$ then try to reduce $u(\bar{\jmath} - n_{uc})$ to zero using the following procedure.
(a) Let $\mathcal{I}^{+} := \{i \mid C_p^{uo}(i, \bar{\jmath} - n_{uc}) > 0\}$ be the set of row indices of the elements of $C_p^{uo}(\cdot, \bar{\jmath} - n_{uc})$ that are positive.
If $\mathcal{I}^{+} = \emptyset$ it is not possible to get null the negative element $u(\bar{\jmath} - n_{uc})$,
then let $flag := 1$ and **goto** 2.4.
(b) Choose an index $\bar{\imath} \in \mathcal{I}^{+}$ and compute $d^{-} := \text{l.c.m.}\{C_p^{uo}(\bar{\imath}, \bar{\jmath} - n_{uc}), -u(\bar{\jmath} - n_{uc})\}$.
(c) Update the $(M + 1)$–th row of $\boldsymbol{A}$ as follows:

$$\boldsymbol{A}(M + 1, \cdot) := \frac{d^{-}}{u(\bar{\jmath} - n_{uc})} \boldsymbol{A}(M + 1, \cdot) + \frac{d^{-}}{C_p^{uo}(\bar{\imath}, \bar{\jmath} - n_{uc})} \boldsymbol{A}(\bar{\imath}, \cdot).$$

By construction $u(\bar{\jmath} - n_{uc})$ is null.

2.4. If $flag = 1$ then the algorithm stops without providing a solution,
else
the last row of the table is in the form $|\,\boldsymbol{v}\,|\,\boldsymbol{u}\,|\,\bar{\boldsymbol{r}}_1\,|\,r_2\,|$
where $\boldsymbol{v} \le \boldsymbol{0}_{n_{uc}}$, $\boldsymbol{u} = \boldsymbol{0}_{n_{uo}}$, $\boldsymbol{r}_1 \in \mathbb{N}^m$ and $r_2 \in \mathbb{N}$.
Let $s_k(k) := r_2$, $\boldsymbol{R}(k, \cdot) := \bar{\boldsymbol{r}}_1$,
If $k < q$, then **let** $k := k + 1$ and **goto** 2.2.
endif
end ■

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