

# Design of a control law for a semiactive suspension system using a solenoid valve damper\*

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## Abstract

In this paper we present a design procedure for semiactive suspension systems using a solenoid valve damper as a shock absorber. We first derive a target active control law that takes the form of a feedback control law. Then, we approximate the target law by controlling the damper coefficient  $f$  of the semiactive suspension. The nonlinear characteristics force-velocity of the solenoid valve damper are used to approximate the target law. To improve the efficiency of the proposed system, we take into account the updating frequency of the coefficient  $f$  and compute the expected value of  $f$  using a predictive procedure.

*Keywords:* semi-active suspension, solenoid valve damper, linear quadratic regulator, vehicle dynamics.

## 1 Introduction

In this paper we deal with the problem of designing a control law for a semiactive suspension system where the shock absorber is a solenoid valve (SV) damper.

A semiactive suspension consists of a spring and a damper but, unlike a passive suspension, the value of the damper coefficient  $f$  can be controlled and updated. In some types of suspensions, but this case is not considered here, it may also be possible to control the elastic constant of the spring. The use of semiactive suspension systems is gaining more and more market share in the last years because it provides a valid tradeoff between purely active suspension systems and passive suspension systems. Many efficient and innovative contributions in this area have been recently proposed in the literature [1, 6, 7, 10].

In general, a semiactive suspension design consists of two phases: (a) design a good active law,  $u_t(\cdot)$  to be considered as a “target”; (b) choose at time  $t$  a suitable value of the damper coefficient  $f(t)$  so that the control force  $u_s(\cdot)$  generated by the suspension system approximates as close as possible the target law  $u_t(\cdot)$ .

In this paper, following [12], the **target active law** is designed so as to minimize a performance index of the form  $J = \int_0^\infty (\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + ru^2(t))dt$ , where  $\mathbf{x}(t)$  is the system state and  $u(t)$  the control force provided by the actuator at the time instant  $t$ . As well known [9] this implies that the control law takes the form of a state feedback law with constant gains, i.e.,  $u(t) = -\mathbf{K}\mathbf{x}(t)$ . However, being the system state not directly accessible (measuring it is too expensive) an asymptotic state observer must be used in the

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control loop. This implies that the real target control law takes the form  $u_t(t) = -\mathbf{K}\hat{\mathbf{x}}(t)$  where  $\hat{\mathbf{x}}(t)$  denotes the system state estimate at the generic time instant  $t$ .

In this paper we consider a procedure for the design of an asymptotic state observer firstly proposed in [3]. Such a procedure well fits within the present application whose main requirement is that of reconstructing the system state when external disturbances are acting on it, while the initial state may always be assumed known. The approach described in [3] has also been applied in [4] to a magnetorheological suspension.

To determine the **semiactive approximation**  $u_s(t)$ , every  $\Delta t$  time units the controller should select on the base of the current value of the suspension velocity, the new damper coefficient  $f$  using the nonlinear characteristics force-velocity of the damper. The new value of  $f$  is chosen so as to minimize the quadratic difference among the semiactive and the target active control force.

In this paper we presents two new original contributions with respect to our previous works.

### Non-linear behavior of the damper

Firstly, we take into account the detailed model of the solenoid valve damper when designing the semiactive suspension system. In such a damper the value of the coefficient  $f$  is updated by appropriately varying the opening section where the oil flows from one chamber to the other one within the damper body. The nonlinear behaviour of the damper that can be described through a family of nonlinear characteristics force-velocity, parametrized by constant values of the opening section, or equivalently by the position of the cylinder within the electro-valve. These characteristics are used to approximate the target active control law.

In all numerical simulations we considered a commercial damper whose physical characteristics are reported in [11]. Note however that in [11] the scaling of the axes is missing: we have reconstructed it making reasonable assumptions.

### Response time of the controller and actuators

The second (and more important) contribution consists in improving the performance of the resulting suspension system taking into account the delay time  $\Delta t$  that elapses between two updates of the damper coefficient.

First of all, it is important to observe that the on-board implementation of the controller is typically done using a microprocessor with a scan time  $\Delta t$  of the order of a few milliseconds. Every  $\Delta t$  time units the controller should choose — on the basis of the current value of the suspension velocity — the new damper coefficient  $f$  selecting a nonlinear characteristics force-velocity of the damper, so as to minimize the quadratic difference among the semiactive and the target control.

Furthermore, it is also necessary to take into account the physical limits on the updating frequency of the damper coefficient  $f$ . In fact, the actuators used to control the damper coefficient have a response time that cannot be neglected because it is usually larger than the scan time of the controller.

Depending on the response time of the actuators, two different approaches may be envisaged.

1. In the first approach we consider a scan time  $\Delta t$  comparable with the response time of the actuator. This means that if at time  $t$  the controller selects a value of  $f$ , i.e., a nonlinear characteristic, the damper will be able to switch to that characteristic only at time  $t + \Delta t$ .

The new value of  $f$  at the generic time instant  $t$  is selected so as to minimize the quadratic difference  $|u(t + \Delta t) - u_s(t + \Delta t)|$ . In such a way, as proved via various numerical simulations, we are able to compensate the delay on the updating of  $f$ , thus producing a significant improvement on the system behavior [8].

This approach is called *Unconstrained Update Control* (UUC) because at each step we may freely choose to update to any other nonlinear characteristic.

2. In the second approach we consider a scan time  $\Delta t$  of the controller significantly smaller than the response time of the damper actuator. This means that if at time  $t$  the damper is working along a particular nonlinear characteristic, at time  $t + \Delta t$  it cannot switch to an arbitrary one, but only to those "close enough" to the original one. In particular we consider the possibility of switching from a characteristic only to the one immediately over or under it.

This means that when minimizing the quadratic difference  $|u(t + \Delta t) - u_s(t + \Delta t)|$  an additional constraint is given from the fact that we may only move to an adjacent nonlinear characteristic. This approach is called *Incremental Update Control* (IUC).

The choice of the first or of the second approach depends on the particular damper technology, and on the scan time of microprocessor that computes the control law. As an example, in the case of the magnetorheological damper considered in [4] the updating frequency of  $f$  may take very high values, of the order of 250 Hz: for this suspension it only makes sense to consider the UUC taking a scan time of  $\Delta t = 4$  ms [?].

On the contrary, in the case of the SV damper considered in this paper the updating frequency of  $f$  is small when compared with the typical scan frequency of a microprocessor. Thus we apply both approaches. More precisely, we first consider a scan time  $\Delta t = 30$  ms that is large enough to allow the system to switch to any nonlinear characteristics [11] and use the UUC law. In a second simulation, we consider a significantly smaller value of  $\Delta t = 7$  ms such that the system may only switch to an adjacent nonlinear characteristic and use the IUC control law.

Different simulations have been carried out, considering the effect of input disturbances caused by the road profile and the effect of non-null initial conditions on the state. The results of these simulations show that the semiactive suspension performs reasonably well, and is a good approximation of the target active suspension, while it introduces significant improvements with respect to a completely passive suspension. Moreover, we can conclude that better performances are obtained imposing a small value of  $\Delta t$  and using the IUC control law.

## 2 Dynamical model of the suspension system

Let us now consider the completely active suspension system of a quarter car with two degrees of freedom schematized in Figure 1.a. We used the following notation:  $M_1$  is the equivalent unsprung mass consisting of the wheel and its moving parts;  $M_2$  is the sprung mass, i.e., the part of the whole body mass and the load mass pertaining to only one wheel;  $\lambda_t$  is the elastic constant of the tire, whose damping characteristics have been neglected. The state component  $x_1(t)$  is the deformation of the suspension with respect to (wrt) the static equilibrium configuration, taken as positive when elongating;  $x_2(t)$  is the vertical absolute velocity of the sprung mass  $M_2$ . The state component  $x_3(t)$  is the deformation of the tire wrt the static equilibrium configuration, taken as positive when elongating. The state component  $x_4(t)$  is the vertical absolute velocity of the unsprung mass  $M_1$ ;  $u(t)$  is the control force produced by the actuator. The signal  $w(t)$  represents the disturbance:

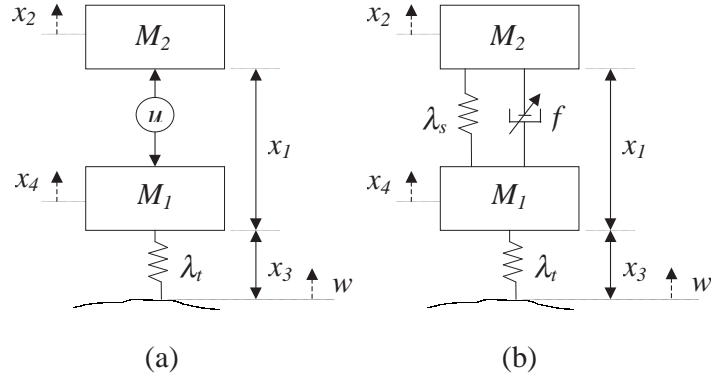


Figure 1: *Scheme of two degree-of-freedom suspension: (a) active suspension; (b) semiactive suspension.*

it coincides with the absolute vertical velocity of the point of contact of the tire with the road.

It is readily shown that the state variable mathematical model of the system under study is given by [2]

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{L}w(t) \quad (1)$$

where  $\mathbf{x}(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T$  is the state, and where the constant matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{L}$  have the following structure:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\lambda_t/M_1 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1/M_2 \\ 0 \\ -1/M_1 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}.$$

Now, let us consider Figure 1.b that represents a conventional semiactive suspension composed of a spring, whose characteristics force-deformation is nonlinear, and a damper with adaptive characteristic coefficient  $f = f(t)$ .

The effect of this suspension is equivalent to that of a control force

$$u_s(t) = - \begin{bmatrix} \lambda_s & f(t) & 0 & -f(t) \end{bmatrix} \mathbf{x}(t). \quad (2)$$

Note that, as  $f$  may vary,  $u_s(t)$  is both a function of  $f(t)$  and of  $\mathbf{x}(t)$ . It is immediate to verify that the state variable mathematical model of the semiactive suspension is still given by equation (1) where  $u(t)$  is replaced by  $u_s(t)$ .

### 3 The solenoid valve damper

In this paper we assume that the shock absorber of the semiactive suspension is a solenoid valve damper, frequently used in heavy vehicles. The main feature of this suspension system is the structure of the valves regulating the damping coefficient, that should be extremely rapid and precise, and at the same time should be capable of taking the stress due to the high values of the pressure within the damper.

The hydraulic circuit of the SV damper presented in [7] is sketched in Figure 2.a. A high speed electro-valve  $P_{v1}$  controls the pressure drop through the circuit, thereby appropriately updating the damping coefficient. A reservoir accommodates the oil displaced by the volume of the piston rod in the high pressure circuit and is pressurised with nitrogen

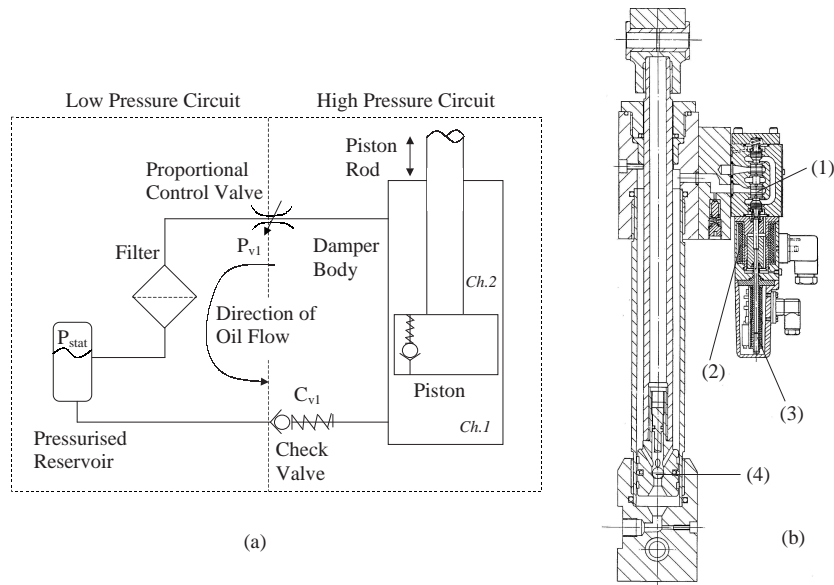


Figure 2: (a) The hydraulic circuit of the semiactive damper. (b) A cross-section of the semiactive damper (reservoir, filter and check valve are not shown) where (1) is the spool, (2) is the solenoid, (3) is the position transducer and (4) is the piston check valve.

gas (at a pressure  $P_{stat}$ ) to prevent cavitation occurring. Note that the flow of oil always occurs in the same direction, thus allowing to keep the structure of the system simple and compact. Moreover, the presence of the check valve and of the electro-valve at the opposite sides of the hydraulic circuit creates a physical separation among the low pressure circuit and the high pressure circuit.

A cross section view of the semiactive damper is shown in Figure 2.b where for simplicity the reservoir, the filter and the check valve are not shown. It is possible to distinguish three main parts that constitute the electro-valve: the hydraulic part, the electrical part and the position transducer. In the hydraulic part of the valve it occurs the variation of the cross section that allows to update the actual value of the damping coefficient. This updating can be realized thanks to a small cylinder that translates in the direction of its own axis. The electrical part is basically constituted by a solenoid that can modify the position of the cylinder by simply applying an axial force. Finally, the position transducer is particularly useful when the damping coefficient requires to be updated at a very high frequency. Note that the feedback control of the cylinder position is also necessary due to the disturbance on the cylinder produced by the flow of oil in the hydraulic part of the valve.

Summarizing, such a suspension system requires two different control devices [7]. The first one is used to determine the position of the cylinder on the base of the difference among the target force and the actual force produced by the damper. Finally, the goal of the second control device is that of modifying the intensity of the current though the solenoid so as to reduce the difference among the required and the actual position of the cylinder.

In this paper we refer to a real existing SV damper whose physical (static) characteristics force-velocity are reported in Figure 3 and have been taken from [11]. Using standard notation, in this figure positive deformation velocities correspond to positive forces. Each curve is parametrized by the position of the cylinder and consequently by the value of the

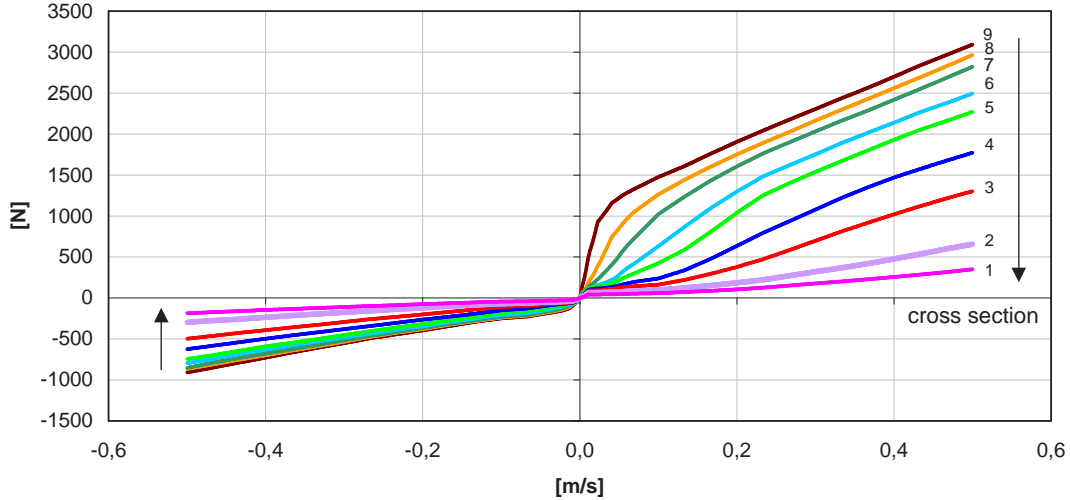


Figure 3: *The nonlinear characteristics of the SV damper.*

opening section where the oil flows from one chamber to the other one within the damper body.

## 4 Semiactive suspension design

In this section we first discuss how the target active control law has been determined. Then we show how such a control law, that requires an actuator, may be approximated by a semiactive suspension, whose varying parameter is the characteristic coefficient of the damper  $f$ .

### 4.1 Target active control law

The design of the active suspension requires determining a suitable control law  $u(\cdot)$  for system (1). To this end, we first determine the control law  $u(\cdot)$  that minimizes a performance index of the form

$$J = \int_0^{\infty} (\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + ru^2(t))dt \quad (3)$$

where  $\mathbf{Q}$  is positive semidefinite and  $r > 0$ . As well known from the literature [9], the solution of this problem can be easily computed by simply solving an algebraic Riccati equation, and takes the form of a feedback control law:

$$u(t) = -\mathbf{K}\mathbf{x}(t). \quad (4)$$

Obviously, when the system state is not directly measured, but is reconstructed via an asymptotic observer, the above control law is replaced by

$$u(t) = -\mathbf{K}\hat{\mathbf{x}}(t) \quad (5)$$

where  $\hat{\mathbf{x}}(t)$  is the state estimate.

In this paper the asymptotic state observer is designed using the procedure we proposed in [3]. We assume that the suspension and the tire deformation are measurable. This is equivalent to choosing

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

for the output equation  $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$ . This ensures the observability of the pair  $(\mathbf{A}, \mathbf{C})$ .

The considered asymptotic state observer has the structure of a Luenberger observer, i.e., it takes the form

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}u(t) + \mathbf{K}_0(\mathbf{y}(t) - \mathbf{C}\hat{\mathbf{x}}(t)). \quad (6)$$

The gain matrix  $\mathbf{K}_0$  is determined by simply minimizing the  $H_2$  norm of the transfer function matrix

$$\mathbf{F}(s) = [s\mathbf{I} - (\mathbf{A} - \mathbf{K}_0\mathbf{C})]^{-1}\mathbf{L}$$

between the estimate error  $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$  and the external disturbance  $w(t)$ . In such a way we can be sure that we are minimizing the effect of the disturbance on the error estimate.

## 4.2 Semiactive approximation

In this section we show how the active target control law  $u_t$  may be approximated using a semiactive suspension with a SV damper, taking into account the nonlinear characteristics force-velocity of the SV damper (see Figure 3). These characteristics are parametrized by the position of the cylinder within the electro-valve (the control action) that enables to modify the opening section where the oil flows from one chamber to the other one in the body damper. The aim of the controller is that of selecting the nonlinear characteristic that minimizes the difference among the resulting semiactive control force and the target active control force. The nonlinear characteristic force-deformation of the spring is also taken into account.

Note that a certain time  $\Delta t$ , depending on the physical system, is necessary to update the damper coefficient. In general this time interval also depends on the required variation of the force, and consequently on the required variation of the cylinder position. For small variations of the force the value of  $\Delta t$  is approximately equal to 7 ms, while for the largest admissible variations it may reach values of the order of 30 ms [7]. Thus, if we assume  $\Delta t = 30$  ms, we may be sure that within this time interval we can move from any characteristic to any other one, regardless of the particular characteristic at hand. On the contrary, if we assume  $\Delta t = 7$  ms, we may be sure that within this time interval we can only switch to adjacent characteristics.

In several previous works the delay time  $\Delta t$  has been neglected. This implies that, if at the generic time instant  $t$  we select a certain characteristic, then such a characteristic will only be reached at the time instant  $t + \Delta t$ , thus never allowing  $u_s(\cdot)$  to be equal to  $u_t(\cdot)$ .

To overcome such a problem, in this paper our goal at time  $t$  becomes that of minimizing the quadratic difference among the semiactive control force and the target active control force at the time instant  $t + \Delta t$ , namely

$$(u_t(t + \Delta t) - u_s(t + \Delta t))^2.$$

The target control force has been chosen equal to

$$u_t(t + \Delta t) = -\mathbf{K}\hat{\mathbf{x}}(t + \Delta t).$$

The semiactive control force may be written as:

$$\begin{aligned} u_s(t + \Delta t) &= -\lambda_s(x_1(t + \Delta t)) \cdot x_1(t + \Delta t) - f(t + \Delta t) \cdot (x_2(t + \Delta t) - x_4(t + \Delta t)) \\ &\simeq -\lambda_s(\hat{x}_1(t + \Delta t)) \cdot \hat{x}_1(t + \Delta t) - F_d(t + \Delta t) \end{aligned}$$

where the  $\hat{x}_i$  denotes the estimate of state  $x_i$  generated by the observer, thus

$$-\lambda_s(\hat{x}_1(t + \Delta t)) \cdot \hat{x}_1(t + \Delta t)$$

denotes the force due to the spring at time  $t + \Delta t$ , while  $F_d(t + \Delta t) = f(t + \Delta t) \cdot (\hat{x}_2(t + \Delta t) - \hat{x}_4(t + \Delta t))$  denotes the force due to the damper at the time instant  $t + \Delta t$ .

Thus, given the nonlinear characteristics of the damper, we restrict our attention to only those values of the force that can be generated when the suspension velocity deformation is equal to  $\hat{x}_2(t + \Delta t) - \hat{x}_4(t + \Delta t) \simeq \dot{x}_1(t + \Delta t)$ . We select the characteristic that generates the force that minimizes the quadratic difference:

$$(-\mathbf{K}\hat{\mathbf{x}}(t + \Delta t) + \lambda_s(\hat{x}_1(t + \Delta t)) \cdot \hat{x}_1(t + \Delta t) + F_d(t + \Delta t))^2$$

and we denote it  $F_d^*(t + \Delta t)$ .

At this point we can follow two different procedures, depending on the considered value of  $\Delta t$ .

**Procedure UUC (Unconstrained Updating Control)** We set the updating interval of the control to  $\Delta t = 30$  ms. Within this time we can move from any nonlinear characteristic to any other one. Thus, we impose the chosen characteristic selecting the corresponding position of the cylinder within the electro-valve. ■

**Procedure IUC (Incremental Updating Control)** We set the updating interval of the control to  $\Delta t = 7$  ms. Within this time we can only switch from one characteristic to an adjacent one. Thus, three different cases may occur.

- If  $u(t) < F_d^*(t + \Delta t)$ , where  $u(t)$  denotes the current value of the force at time  $t$ , then we switch to the adjacent superior characteristic, regardless of the required variation.
- If  $u(t) = F_d^*(t + \Delta t)$  we keep the actual characteristic unaltered.
- If  $u(t) > F_d^*(t + \Delta t)$  we switch to the adjacent inferior characteristic, regardless of the required variation.

■

## 5 Application example

In this section we discuss in detail the results of several simulations. First, however, we explain the choices we have made for the various parameters.

The proposed procedure has been applied to the quarter car suspension shown in Figure 1, with values of the parameters taken from [12]:  $M_1 = 28.58\text{Kg}$ ,  $M_2 = 288.90\text{Kg}$ ,  $\lambda_t = 155900\text{N/m}$ . In the simulation we used the nonlinear characteristic of the suspension spring given in Figure 4. Finally, the characteristics of the damper are those shown in Figure 3.

The matrices  $\mathbf{Q}$  and  $r$  of the performance index  $J$  have been taken from [12] and are the same as those already used in [5]:

$$\mathbf{Q} = \text{diag}\{1, 0, 10, 0\}, \quad r = 0.8 \cdot 10^{-9}.$$

Thus, the resulting feedback control matrix is

$$\mathbf{K} = \begin{bmatrix} 35355 & 4827 & -21879 & -1386 \end{bmatrix}.$$



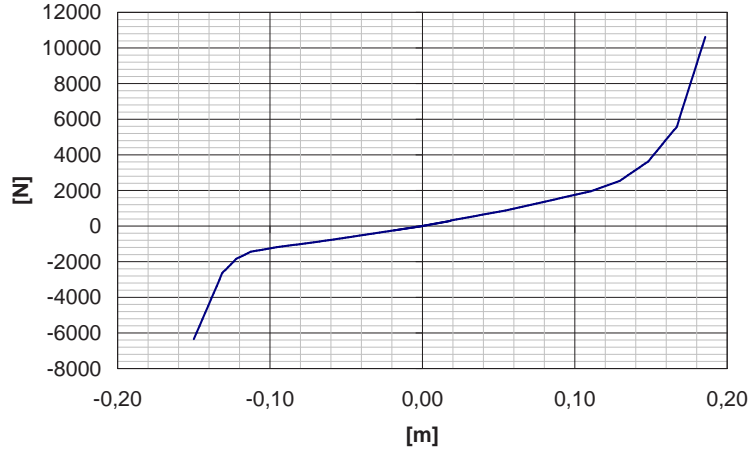


Figure 4: *The nonlinear characteristic of the suspension spring.*

For the computation of the observer matrix we used the software tools available in Matlab: `fmins` is the minimization procedure and `normh2` computes the  $H_2$  norm. We determined [3]

$$\mathbf{K}_o = \begin{bmatrix} 176.1 & 1334.4 & 1.9 & -145.7 \\ 51.3 & 426.1 & 1852.5 & -5501.4 \end{bmatrix}^T.$$

To show the performance of the proposed semiactive suspension design, we have simulated two different situations.

### 5.1 Simulation 1

In the first simulation we consider an initial state different from zero and no external disturbance. In particular, we assume  $\mathbf{x}(0) = \hat{\mathbf{x}}(0) = [0.1 \ 0 \ 0.01 \ 0]^T$ .

The same simulation is carried out with both the UUC procedure and the IUC procedure. In the first case we assume  $\Delta t = 30$  ms, while in the second case we assume  $\Delta t = 7$  ms.

The results of this simulation are shown in Figure 5. The upper two figures (a)-(b) compare the unsprung and the sprung mass displacement of the semiactive suspension (using both the UUC and the IUC procedure) with that of a completely passive suspension and a purely active one. Note that the spring of the passive suspension is the same as that used in the semiactive suspension, while the nonlinear characteristic of the damper is that one denoted with the number 5 in Figure 3.

We can observe that both in the case of the UUC and in the case of the IUC case, the semiactive system better approximates the active suspension than the passive one. We may also conclude that the IUC procedure provides a more satisfactory behaviour in terms of comfort with respect to the UUC procedure. In fact, in such a case the behaviour of the semiactive suspension system in terms of the sprung mass displacement, is practically the same as that obtained using the purely active system.

The lower left figure (c) compares the target force with the control force produced by the semiactive suspension, both in the case of the IUC and in the case of the UUC procedure. We can observe that in the case of the IUC procedure, the difference among the target active control force and the semiactive control force is quite negligible.

Finally, figure (d) shows the values of the index denoting the current nonlinear characteristic during the evolution of the semiactive suspension, both in the case of UUC and

in the case of IUC.

## 5.2 Simulation 2

In the second simulation we consider null initial conditions, i.e.,  $\mathbf{x}(0) = \hat{\mathbf{x}}(0) = \mathbf{0}$  and assume that an external disturbance is acting on the system, caused by a bump in the road profile. The geometrical characteristics of the bump are shown in Figure 6.a.

We make the hypothesis that the velocity of the vehicle keeps at a constant value  $V$  during all time period of interest.

Moreover, we assume that the point of contact of the tire with the road perfectly follows the road profile, or equivalently we assume that no loss of contact between wheel and road may occur. Finally, we assume that the damping of the tire is negligible and its dynamical behaviour may be modeled through a pure elastic constant.

Under these hypothesis the vertical position  $x_0$  of the point of contact of the tire with the road depends not only on the shape of the bump, but also on the velocity  $V$  of the vehicle. The value of  $x_0$  with respect to time  $t$  is shown in Figure 6.b where

$$(t_B - t_A) = (t_D - t_C) = H/V,$$

being the velocity of the vehicle equal to  $V$  during all the time period of interest.

As a consequence, the external disturbance  $w(t)$ , i.e., the vertical velocity of the point of contact of the tire with the road, varies with respect to time as shown in Figure 6.b.

The results of this simulation are shown in Figure 7, where we have taken  $H = 25$  mm,  $L = 50$  mm, and  $V = 10$  m/s. Note that for brevity's sake in this numerical simulation we limit our attention to the IUC procedure.

Figure 7.a shows the road profile  $x_0$  (thin line) along with the unsprung mass displacement  $x_3 + x_0$  (thick line). Figure 7.b shows the road profile  $x_0$  (thin line) along with the sprung mass displacement  $x_1 + x_3 + x_0$  (thick line). It is possible to observe that the semi-active suspension well behaves in front of the abrupt obstacle, smoothing the movement of the sprung mass.

Figure 7.c compares the sprung mass displacement in the case of the semiactive suspension (thick line) and in the case of a completely passive suspension (thin line), while Figure 7.d compares the sprung mass displacement in the case of the semiactive suspension (thick line) and in the case of the target active suspension (thin line). As it can be noted, the behaviour of the semiactive suspension is intermediate between that of the passive and active suspension.

Figure 7.e compares the target force (thin line) with the control force produced by the semiactive suspension (thick line). We can observe that the variation of  $f$  guarantees a satisfactory approximation.

Figure 7.f shows the values of the index denoting the current nonlinear characteristic during the evolution of the semiactive suspension.

Figures 7.g – j show the efficiency of the asymptotic state observer used during simulations. In Figure 7.g we have reported the evolution of the first state variable  $x_1$ , while in Figure 7.h we have reported the evolution of its error estimate  $e_1 = x_1 - \hat{x}_1$ . Figure 7.i shows the evolution of  $\dot{x}_1$ , while  $\dot{e}_1 = \dot{x}_1 - \dot{\hat{x}}_1$  is reported in Figure 7.j. We can observe that the observer provides a good evaluation of both the state variables and their derivatives.

## 5.3 A comparison among the suspension systems

To conclude we provide a resumptive comparison among the different kinds of suspension systems considered in this paper. Such a comparison is provided in Table 1 where we have

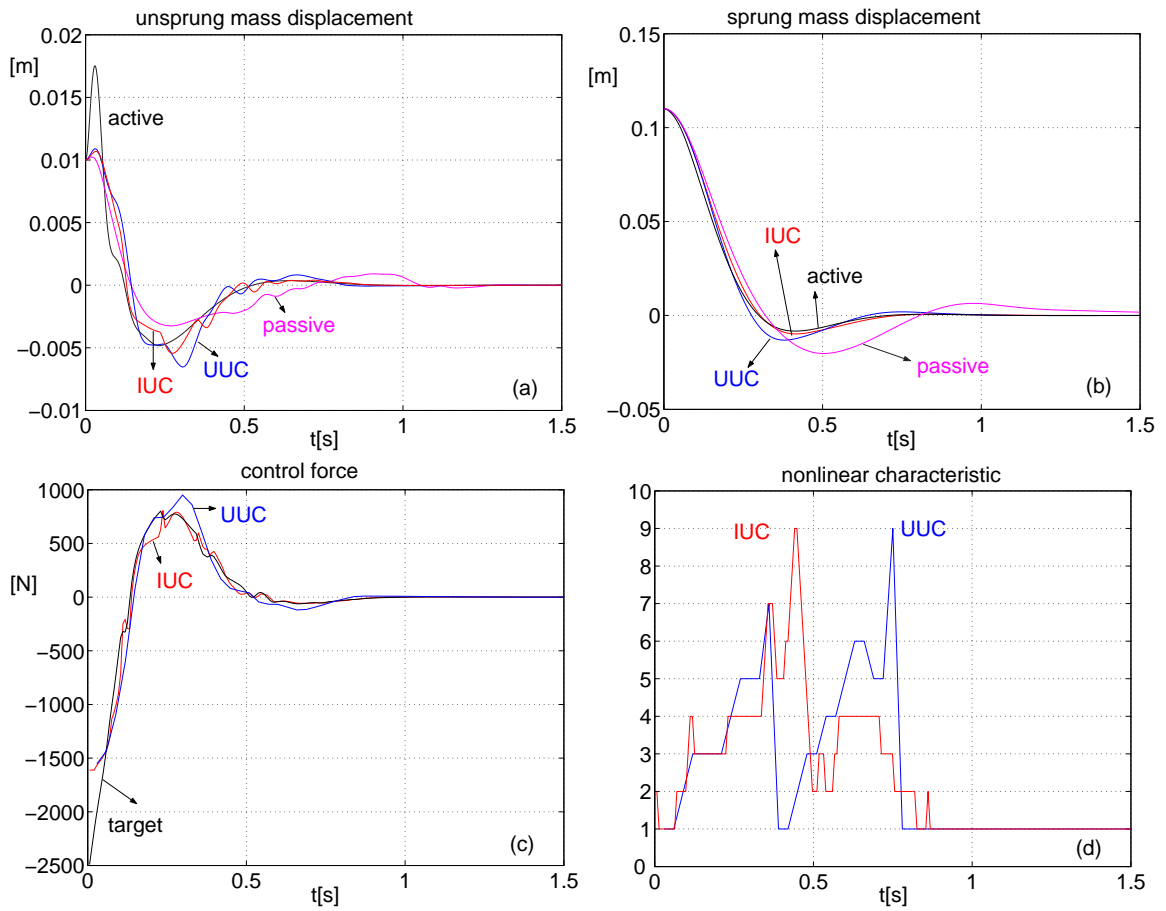


Figure 5: The results of simulation 1.

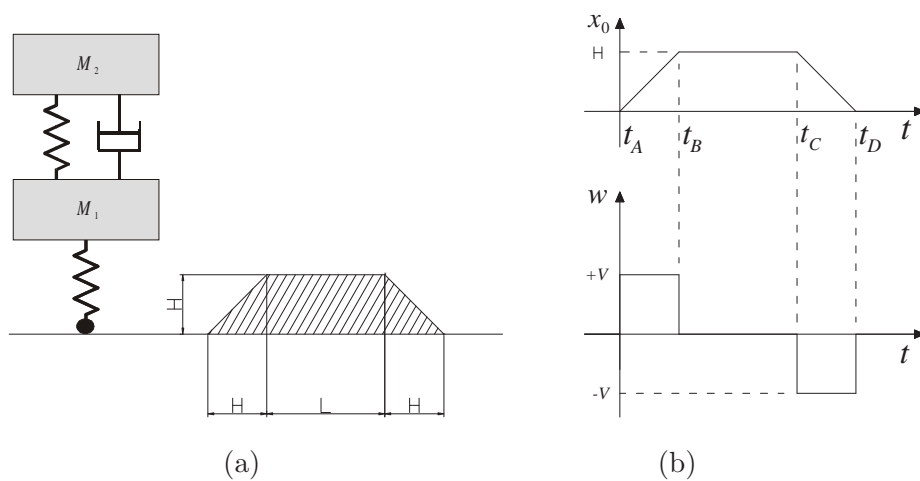


Figure 6: Geometrical characteristics of the bump (a) and the resulting disturbance  $w(t) = \dot{x}_0(t)$  (b).

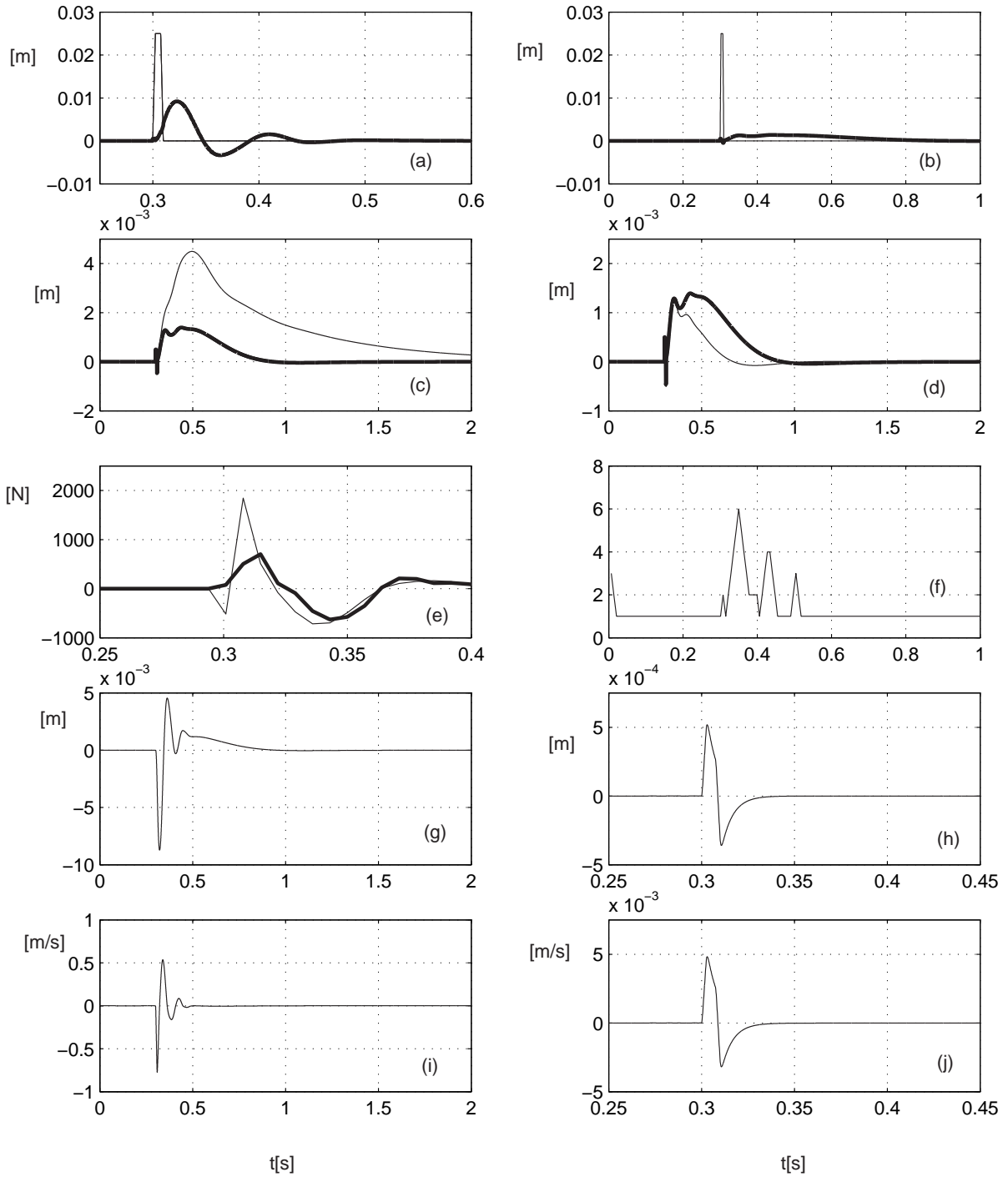


Figure 7: *The results of Simulation 2.*

	passive	active	UUC	IUC
Simulation 1	$1.736 \cdot 10^{-3}$	$1.606 \cdot 10^{-3}$	$1.675 \cdot 10^{-3}$	$1.652 \cdot 10^{-3}$
Simulation 2	$7.781 \cdot 10^{-5}$	$7.355 \cdot 10^{-5}$	$7.587 \cdot 10^{-5}$	$7.368 \cdot 10^{-5}$

Table 1: *A comparison among the different semiactive suspensions.*

reported the value of the performance index  $J$  corresponding to all the simulation test cases examined. Such an analysis enables us to conclude that in all cases examined the semiactive suspension system always provides a good approximation of the fully active suspension system, while producing significant improvements with respect to the purely passive suspension system. Finally, we may also conclude that the best results in terms of the performance index  $J$  are obtained when the semiactive suspension uses the IUC approach. This enables us to conclude that the updating frequency does not pose in practice a relevant limitation. The main drawback of the semiactive suspension systems examined lies in the fact that even the lowest obtainable values of the damper coefficient are often too high. Thus, to improve the damper performance it is necessary to have characteristic curves closer to the x axis than the one labeled "1" in Figure 3.

## 6 Conclusions

In this paper we have presented a two-phase design technique for semiactive suspensions where the shock absorber is a solenoid damper valve.

The first phase of the project consists in the design of a target active control law that has been obtained by solving an LQR problem. The assumption of non-measurable state required the introduction of an asymptotic state observer, that has been designed using a procedure proposed by the authors in a previous work.

In the second phase, this target law is approximated by controlling the damper coefficient of the semiactive suspension. In particular, we have taken into account the delay  $\Delta t$  required for the updating of  $f$ : we have assumed that the new value of  $f$  is chosen so as to minimize the difference between the target and the semiactive control law at the time instant  $t + \Delta t$ . In such a way we can be sure that when the computed value of  $f$  is really imposed, then the semiactive force is as close as possible to the target one. Two different procedures have been suggested depending on the particular value of  $\Delta t$ .

The nonlinear behaviour of both the damper and the spring is also taken into account to approximate the target active control law.

Several numerical simulations have been carried out considering a commercial SV damper.

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