Simulation and Control of a Bottling Plant using First-Order Hybrid Petri Nets*

Roberta Armosini[1], Alessandro Giua[2], M. Teresa Pilloni[1], and Carla Seatzu[2]

[1] Dip. Ingegneria Meccanica, Università di Cagliari, Italy,
pilloni@dimeca.unica.it
[2] Dip. Ingegneria Elettrica ed Elettronica, Università di Cagliari, Italy,
{giua,seatzu}@diee.unica.it

Abstract. In this paper we show how First–Order Hybrid Petri nets, an hybrid positive model that combines fluid and discrete event dynamics, may be efficiently used to simulate the dynamic concurrent activities of manufacturing systems. In particular we deal with the performance analysis via simulation of a mineral water bottling plant according to the variations of the production controlling input parameters. The model allows a simulation of the productive line behavior through changes in the production capacity of the producing bottles and PET prototype machines, of the filling machines, of the volume and type of the bottles, of the silos dimensions, and so on.

1 Introduction

In this paper we show how *hybrid Petri nets* [5], a model for *positive systems* [3] that combines fluid and discrete event dynamics, may be efficiently used to simulate the concurrent activities of high-speed manufacturing systems.

The considered application. Problems related to production management and optimization become particularly critical in high-speed production plants, a particular example of which are mineral water bottling plants. Difficulties in production management arise, as a matter of fact, from two conflicting requirements: on one side we have the market, usually characterized by a very variable demand as far as formats and quantity outputs are concerned; on the other one we have the production system, whose best performances are obtained in stable conditions characterized by a constant output production.

Simulation techniques represent an important and valid support for coping with these problems, as they allow to estimate plant behavior and performances resulting from different market scenarios, in which variations in the

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number and size of PET units produced or of bottles filled may occur. Simulation is useful both in the design phase, providing important information for the subsequent decision choices, and in the management phase.

In the present work, a production line of an existing plant was simulated. The plant under study is the Sarda Acque Minerali (SAM) unit, a mineral water bottling plant located in southern Sardinia, at about 20 km from the city of Cagliari. The company production [6] achieves about 110 millions of bottles per year; several formats (0.25l, 0.5l, 1l, 1.5l, 2l) of bottles are produced, filled and finally sold, both with still mineral and sparkling water. Moreover four different mineral water brands are produced.

Petri nets as positive systems. Discrete Petri nets [7] are a discrete event model whose state space belongs to the set of non-negative integers. This is a major advantage with respect to other formalisms such as automata, where the state space is a symbolic unstructured set, and has been exploited to develop many analysis techniques that do not require to enumerate the state space (structural analysis [4]).

Recently, much work has been devoted to the extension of the classical discrete Petri net formalism to continuous Petri nets obtained by *fluidification* [8]. In fact, in many applications dealing with complex systems it happens that the model of the plant has a discrete event dynamics whose number of reachable states is typically very large. The analysis and optimization of these systems require large amount of computational efforts, and problems of realistic scale quickly become analytically and computationally untractable. To cope with this problem it is often possible to give a *fluid* (i.e., continuous) approximation of the "fast" discrete event dynamics [9].

Note that the discrete event dynamics that can be represented by a fluid model are usually related to the flow of materials, thus making fluid models essentially a type of *compartmental models* [3], a sub-class of *positive systems*.

In general different fluid approximations are necessary to describe the same system, depending on its discrete state. Thus, the resulting model can be better described as an *hybrid model*, where different dynamics are associated to each discrete state. This has recently lead to the definition of a new family of Petri net models that combine discrete and continuous subsystems into a so called hybrid Petri net [1, 5]. Note that the area of hybrid systems has received a lot of attention in the automatic control community, lately: we believe that in the next years much attention will also be devoted to hybrid positive systems, i.e., positive systems combining both discrete event and continuous dynamics, and hybrid Petri nets are a good example of these class of systems.

The hybrid Petri net model considered in this paper is called *First–Order* Hybrid Petri nets (FOHPN) because its continuous behavior is piece-wise constant. FOHPN were originally presented in [2].

2 First–Order Hybrid Petri Nets

In this paper we use the Petri net formalism firstly presented in [2].

Net structure. An (untimed) FOHPN is a structure $N = (P, T, Pre, Post, \mathcal{D}, \mathcal{C})$. The set of *places* $P = P_d \cup P_c$ is partitioned into a set of *discrete* places P_d (represented as circles) and a set of *continuous* places P_c (represented as double circles). The cardinality of P, P_d and P_c is denoted n, n_d and n_c. The set of *transitions* $T = T_d \cup T_c$ is partitioned into a set of discrete transitions T_d and a set of continuous transitions T_c (represented as double boxes). The cardinality of T, T_d and T_c is denoted q, q_d and q_c. The *pre-* and *post-incidence functions* that specify the arcs are (here $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$): $Pre, Post : P_c \times T \rightarrow \mathbb{R}_0^+$, $P_d \times T \rightarrow \mathbb{N}$. We require that $\forall t \in T_c$ and $\forall p \in P_d$, $Pre(p,t) = Post(p,t)$, so that the firing of continuous transitions does not change the marking of discrete places. The function $\mathcal{D} : T_d \rightarrow \mathbb{R}_0^+$ specifies the timing associated to timed discrete transitions. The function $\mathcal{C} : T_c \rightarrow \mathbb{R}_0^+ \times \mathbb{R}_\infty^+$ specifies the firing speeds associated to continuous transitions (here $\mathbb{R}_\infty^+ = \mathbb{R}^+ \cup \{\infty\}$). For any continuous transition $t_j \in T_c$ we let $\mathcal{C}(t_j) = (V'_j, V_j)$, with $V'_j \leq V_j$. Here V'_j represents the *minimum firing speed* (mfs) and V_j represents the *maximum firing speed* (MFS).

The *incidence matrix* of the net is defined as $C(p,t) = Post(p,t) - Pre(p,t)$. The restriction of $\boldsymbol{C}$ to P_X and T_Y is denoted $\boldsymbol{C}_{XY}$.

A *marking* is a function that assigns to each discrete place a non-negative number of tokens, represented by black dots and assigns to each continuous place a fluid volume. A continuous place can be seen as a tank that can fill up with fluid (marking). However, we also consider some connecting elements (such as a pipe) with a zero capacity where fluid can flow but not accumulate. Thus we partition the set of continuous places $P_c = P_0 \cup P_+$ into a set of places P_0 (represented as full dark circles) whose marking is always equal to zero (connecting elements), and a set of places P_+ (represented as double circles) whose marking may assume any nonnegative real number (tanks). Therefore $\boldsymbol{m} : P_+ \rightarrow \mathbb{R}_0^+$, $P_0 \rightarrow 0$, $P_d \rightarrow \mathbb{N}$. The marking of place p_i is denoted m_i, while the value of the marking at time τ is denoted $\boldsymbol{m}(\tau)$. The restriction of $\boldsymbol{m}$ to P_d and P_c are denoted with $\boldsymbol{m}^d$ and $\boldsymbol{m}^c$, respectively. An *FOHPN system* $\langle N, \boldsymbol{m}(\tau_0) \rangle$ is an FOHPN N with an initial marking $\boldsymbol{m}(\tau_0)$.

Net dynamics. The enabling of a discrete transition depends on the marking of all its input places, both discrete and continuous. More precisely, a discrete transition t is *enabled* at $\boldsymbol{m}$ if for all $p_i \in {}^\bullet t$, $m_i \geq Pre(p_i, t)$, where ${}^\bullet t$ denotes the preset of transition t.

If a discrete transition t_j fires at a certain time instant τ^-, then its firing at $\boldsymbol{m}(\tau^-)$ yields a new marking $\boldsymbol{m}(\tau)$ such that $\boldsymbol{m}^c(\tau) = \boldsymbol{m}^c(\tau^-) + \boldsymbol{C}_{cd}\boldsymbol{\sigma}$, and $\boldsymbol{m}^d(\tau) = \boldsymbol{m}^d(\tau^-) + \boldsymbol{C}_{dd}\boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ is the *firing count vector* associated to the firing of transition t_j.

To every continuous transition t_j is associated an instantaneous firing speed (IFS) $v_j(\tau)$. For all τ it should be $V'_j \leq v_j(\tau) \leq V_j$, and the IFS of each continuous transition is piecewise constant between events.

An empty continuous place p_i can be fed, i.e., supplied, by an input transition, which is enabled. Thus, as a flow can pass through an unmarked continuous place, this place can deliver a flow to its output transitions. Consequently, a continuous transition t_j is enabled at time τ if and only if all its input discrete places $p_k \in P_d$ have a marking $m_k(\tau)$ at least equal to $Pre(p_k, t_j)$, and all its input continuous places are either marked or fed. If all input continuous places of t_j have a not null marking, then t_j is called *strongly enabled*, else t_j is called *weakly enabled*. Finally, transition t_j is not enabled if one of its empty input places is not fed.

We can write the equation which governs the evolution in time of the marking of a place $p_i \in P_c$ as $\dot{m}_i(\tau) = \sum_{t_j \in T_c} C(p_i, t_j) v_j(\tau)$ where $\boldsymbol{v}(\tau) = [v_1(\tau), \ldots, v_{n_c}(\tau)]^T$ is the IFS vector at time τ.

The enabling state of a continuous transition t_j defines its admissible IFS v_j. If t_j is not enabled then $v_j = 0$. If t_j is strongly enabled, then it may fire with any firing speed $v_j \in [V'_j, V_j]$. If t_j is weakly enabled, then it may fire with any firing speed $v_j \in [V'_j, \overline{V}_j]$, where $\overline{V}_j \leq V_j$ since t_j cannot remove more fluid from any empty input continuous place $\overline{p}$ than the quantity entered in $\overline{p}$ by other transitions.

We say that a *macro–event* occurs when: (a) a discrete transition fires, thus changing the discrete marking and enabling/disabling a continuous transition; (b) a continuous place becomes empty, thus changing the enabling state of a continuous transition from strong to weak.

Let τ_k and τ_{k+1} be the occurrence times of two consecutive macro–events as defined above; we assume that within the interval of time $[\tau_k, \tau_{k+1})$, denoted as a *macro–period*, the IFS vector is constant and we denote it $\boldsymbol{v}(\tau_k)$. Then the continuous behavior of an FOHPN for $\tau \in [\tau_k, \tau_{k+1})$ is described by $\boldsymbol{m}^c(\tau) = \boldsymbol{m}^c(\tau_k) + \boldsymbol{C}_{cc}\boldsymbol{v}(\tau_k)(\tau - \tau_k)$, $\boldsymbol{m}^d(\tau) = \boldsymbol{m}^d(\tau_k)$.

3 Modeling plant subsystems with FOHPN

In this section we briefly describe some components of the considered plant and the corresponding net models.

Transportation lines and switches. Transportation lines consist of pipes of appropriate diameter, depending on the bottle sizes, where bottles are conveyed at high speed thanks to the force produced by the compressed air. Due to the high speed, the main feature of these elements is that there is no accumulation of bottles in their inside. Therefore, transportation lines may be seen as connecting elements and the corresponding places in the Petri net model are zero capacity places, i.e., places in P_0.

The connections among different lines may vary and this can be modeled by a MIMO (multi input - multi output) switch. In figure 1 a switch is represented in the case of two input and two output lines, where place p_c has been denoted as a dark circle because it is a zero capacity place. The discrete marking is such that one possible path at a time is enabled.

The delay times associated to discrete transitions determine the the paths that bottles follow at the different time intervals: thus they are design parameters to be optimized.

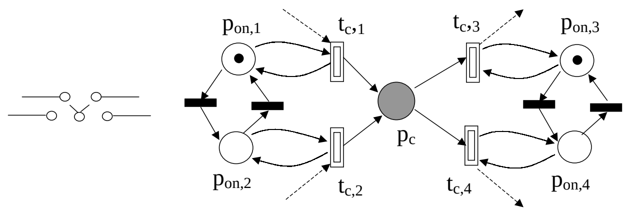

Fig. 1. *A MIMO switch with* 2 *input and* 2 *output lines.*

Machines. In this plant we have two different types of machines. The first type is involved in bottles production, while the second one is involved in bottles filling and corking.

Machines of the first type are equipped so as to produce bottles of different sizes. In the following, we consider the case of a machine that can be used to produce 1.5 lt bottles and 2 lt bottles. The Petri net model for such a machine is shown in figure 2.a. In particular, the firing of $t_{c,1}$ denotes the production of 1.5 lt bottles, whereas the firing of $t_{c,2}$ denotes the production of 2 lt bottles. Clearly, the productivity of the machine is not the same in the two cases, thus the weights of the input arcs to p_c are different.

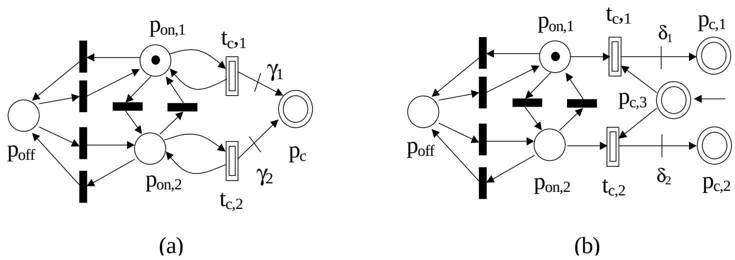

Fig. 2. *A machine that produces bottles (a). A machine that fills bottles (b).*

A dual scheme may be used to describe the functioning of those machines that are involved in the bottle filling and corking. An example in the case of bottles of two different sizes is reported in figure 2.b.

The delay times associated to discrete transitions determine the machine production cycle and are the design parameters to be optimized.

4 A real bottling plant

Plant description. The production cycle considered in this paper consists of several stages [6]. The first stage consists in the creation of the PET bottles and the last stage consists in self-filling and corking. More precisely, the first operational machine is M_1 that produces PET bottles starting from raw-material of PET granules (PET chips). Thanks to an appropriate equipment, this machine may be extremely versatile and may produce different bottle sizes, e.g., 1.5 lt and 2 lt. Then, the produced bottles are directed to appropriate lines of different diameter, depending on their size. The flow of bottles through the conveyor lines occurs at a high speed and is induced by a jet of compressed air. Bottles may follow different paths and may be assigned to different buffers. Path assignment may be seen as a decision problem whose solution aims to optimize the production process. In particular, in the case we are dealing with, there are 7 buffers (S_1, S_2, $\cdots$, S_7) and the partitioning is established so as to compensate as much as possible the delay due to the reduced productivity of the machines that fill bottles of mineral water with respect to those that produce them.

Finally, from buffers bottles are conveyed to the zone of self–filling through other appropriate flow lines. Even in this case, bottles may follow different paths so as to better exploit the filling machines. In particular, there are 3 filling machines that can be used to fill bottles of all sizes.

The FOHPN model. The FOHPN model of the above production process can be obtained by simply putting together all the elementary modules previously defined. The resulting Petri net model has not been reported here for brevity's sake but it can be seen by looking at [6].

5 A numerical optimization problem

In this section we present the results of several numerical simulations whose main goal is that of determining the operational configuration of the production process that enables us to optimize the efficiency of the bottling plant with respect to a given performance index. All simulations have been carried out using Simulink, a Toolbox of Matlab.

The design parameters are the following: the initial configuration of the plant, i.e., the initial marking of the net; the paths that bottles should follow at the different time intervals, i.e., the timing delays associated to discrete transitions in the Petri net model of switches; the time intervals at which machines should produce (fill) bottles of different formats, i.e., the timing delays associated to discrete transitions in the Petri net model of machines producing (filling) bottles.

Different numerical simulations have been carried out using the real data of the machines (namely, their productivity) and the buffers (namely, their maximum capacity). In the following we focus our attention to 1.5 and 2 lt bottles. A time period of 48 hours has been considered during simulation (the behavior of the plant is periodic with a period of 48 hours).

The main goal of the company is that of maximizing the net profit resulting from selling its end items. We first assume that all the produced bottles are sold. In such a case the net profit is

$$P = (SP_{1.5} - UC_{1.5}) \cdot N_{1.5} + (SP_2 - UC_2) \cdot N_2$$

where $SP_{1.5}$ (SP_2) is the *selling price* of 1.5 (2) lt bottles, while $UC_{1.5}$ (UC_2) is the *unitary cost* associated to 1.5 (2) lt bottles. The selling price is the price at which the end item is sold to the customer. In all numerical simulations we assumed $SP_{1.5} = 18\ c$ and $SP_2 = 22\ c$, where c denotes a cent of Euro. The unitary cost is the cost that the company pays for one unit of end item. It includes the cost that the company pays for the PET and the water, plus an additional term taking into account the production costs pertaining to one bottle. In particular, we assumed $UC_{1.5} = 5\ c$ and $UC_2 = 6\ c$.

The resulting net profit, computed under the assumption that all the produced bottles are sold, is that shown by the thin curve in figure 3. Thus we can conclude that the fifth simulation corresponds to the best configuration of the plant with respect to the considered performance index P. Note that it is possible to prove that the fifth simulation corresponds to the maximal productivity of no format. This means that the maximum profit is guaranteed by appropriately partitioning the production resources among bottles of different sizes.

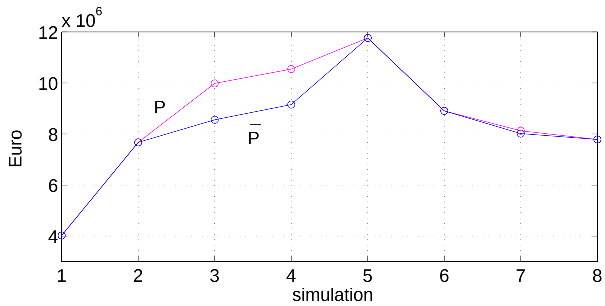

Fig. 3. *The net profit P under the assumption that all bottles are sold and the net profit $\overline{P}$ taking into account some constraints in the sale.*

Finally, we compute the net profit under the following two realistic assumptions. Firstly, we assume that there is an upper bound on the demand of bottles of each format: if the number of produced bottles is greater than such a limit, then there is a certain number of bottles that are not sold, thus producing no profit. Secondly, we assume that if the number of bottles is less than a given lower bound then the whole demand cannot be met. This produces a *shortage* which usually has many associated costs. Apart from the loss of profit, the effects of shortage include loss of goodwill, loss of future sales, and so on. In particular, in all numerical simulations we assumed that within the considered time period of simulation, the maximum number of bottles of each format that can be sold is $N_{\max} = 7 \cdot 10^5$, while the number of produced bottles under which there is shortage is $N_{\min} = 10^5$. Finally, we evaluated that shortage cost is equal to $SC = 2\ c$ for unit of end item for both formats. In such a case the net profit is equal to

$$\begin{aligned}\overline{P} = {} & SP_{1.5} \cdot \max\{N_{1.5}, N_{\max}\} - UC_{1.5} \cdot N_{1.5} - SC \cdot \max\{0, N_{\min} - N_{1.5}\} \\ & + SP_2 \cdot \max\{N_2, N_{\max}\} - UC_2 \cdot N_2 - SC \cdot \max\{0, N_{\min} - N_2\}.\end{aligned}$$

When the performance index to be maximized it $\overline{P}$ the resulting curve is the thick one in figure 3. Thus we can conclude that even in this case the best configuration of the plant is the fifth one.

6 Conclusions

An analysis of the operating conditions of a mineral water bottling plant was performed by means of a simulation model based on first order hybrid Petri nets and Simulink. The tests accomplished demonstrate the ability of the model to correctly describe the behavior of the single machines and of the global plant; it also allows to foresee the main plant performances for different operating plant conditions, so representing a valid instrument to cope with complex production optimization problems.

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