Modeling of a Bottling Plant using Hybrid Petri Nets

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Abstract— **In this paper we deal with the problem of modeling a bottling plant using First–Order Hybrid Petri nets. The model we use is an hybrid model that combines fluid and discrete event dynamics and enables us to simulate the dynamic concurrent activities of manufacturing systems. It also provides a modular representation of the considered plant, thus allowing one the bottom-up construction of models for large scale systems.**

I. Introduction

This paper shows how *First-Order Hybrid Petri nets* (FOHPN), a class of nets that combine fluid and discrete event dynamics, can be used to describe a flexible manufacturing system. This hybrid model was originally presented in [3] and adds to the formalism described by David and Alla [1], [2] linear algebraic techniques for analysis and control. In particular, in this paper we consider the problem of modeling and simulating the production process of a factory for the bottling of mineral water. The simulation model we develop may be used to evaluate the efficiency of the actual production process, and to highlight its main drawbacks so as to better determine in which directions the production cycle can be improved. Moreover, it can be used as an efficient tool for the solution of many management problems so as to optimize some of the system parameters via numerical simulation. The main advantage of this approach is that it enables us to easily take into account stochastic and unforeseen phenomena, such as failures and abrupt interruptions.

In this paper we consider an existing plant in Sardinia, Italy, belonging to *Sarda Acque Minerali* (Sardinian Mineral Waters). Its production process may be divided into four main steps: PET (polyethylene) bottle production, bottle filling, packaging, storage and transportation. Each stage of the production system may be further divided into an appropriate number of sub–models depending on the number and type of machines and/or buffers involved in it. Each elementary module is modeled trough a FOHPN. Finally, all sub-nets are put together so as to simulate the behavior of the whole process.

The advantage of the proposed approach originates from the following considerations. High–troughput manufacturing systems, like those of interest here, are discrete event dynamic systems whose number of reachable states is typically very large, thus the analysis and optimization of these systems requirès large amount of computational efforts, and problems of realistic scale quickly become analytically and computationally untractable. To cope with this problem, *fluid models* which are continuous–dynamics approximations of discrete event systems, may be successfully developed and applied to the inventory management domain. This has several advantages. Firstly, there is the possibility of considerable increase in computational efficiency, because the simulation of fluid models can often be done much more efficiently. Secondly, fluid approximations provide an aggregated formulation to deal with complex systems, thus reducing the dimension of the state space. Thirdly, the design parameters in fluid models are continuous hence there is the possibility of using gradient information to speed up optimization and perform sensitivity analysis.

It should be noted that in general different fluid approximations are necessary to describe the same management system, depending on its discrete state. Thus, the resulting model can be better described as an *hybrid model*, where different dynamics are associated to each discrete state.

FOHPN — and this is also generally true for Petri nets — have been used in many application domains such as manufacturing [4] and inventory management control [6]. Hybrid and Batch Petri net models have also been used for modeling packing and bottling plants by I. Demongodin in [5].

II. First–Order Hybrid Petri Nets

The Petri net formalism used in this paper can be seen as the "untimed" version of the model presented in [3], in the sense that no timing structure is associated to the firing of discrete transitions.

Net structure: An (untimed) FOHPN is a structure $N = (P, T, Pre, Post, \mathcal{C})$.

The set of *places* $P = P_d \cup P_c$ is partitioned into a set of *discrete* places P_d (represented as circles) and a set of *continuous* places P_c (represented as double circles). The cardinality of P, P_d and P_c is denoted n, n_d and n_c.

The set of *transitions* $T = T_d \cup T_c$ is partitioned into a set of discrete transitions T_d (represented as boxes) and a set of continuous transitions T_c (represented as double boxes). The cardinality of T, T_d and T_c is denoted q, q_d and q_c.

The *pre-* and *post-incidence functions* that specify the arcs are (here $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$): $Pre, Post : P_c \times T \rightarrow \mathbb{R}_0^+$, $P_d \times T \rightarrow \mathbb{N}$. We require (*well-formed nets*) that for all $t \in T_c$ and for all $p \in P_d$, $Pre(p,t) = Post(p,t)$. This ensures that the firing of continuous transitions does not change the marking of discrete places.

The function $\mathcal{C} : T_c \rightarrow \mathbb{R}_0^+ \times \mathbb{R}_\infty^+$ specifies the firing speeds associated to continuous transitions (here $\mathbb{R}_\infty^+ = \mathbb{R}^+ \cup \{\infty\}$). For any continuous transition $t_j \in T_c$ we let $\mathcal{C}(t_j) = (V_j', V_j)$, with $V_j' \leq V_j$. Here V_j' represents the *minimum firing speed* (mfs) and V_j represents the *maximum firing speed* (MFS). In the following, unless explicitly specified, the mfs of a continuous transition will be $V'_j = 0$.

The *incidence matrix* of the net is defined as $C(p,t) = Post(p,t) - Pre(p,t)$. The restriction of C to P_X and T_Y ($X, Y \in \{c, d\}$) is denoted C_{XY}.

A *marking* is a function that assigns to each discrete place a non-negative number of tokens, represented by black dots and assigns to each continuous place a fluid volume. A continuous place can be seen as a tank that can fill up with fluid (marking). However, we also consider some connecting elements (such as a pipe) with a zero capacity where fluid can flow but not accumulate. Thus we partition the set of continuous places $P_c = P_0 \cup P_+$ into a set of places P_0 (represented as full dark circles) whose marking is always equal to zero (connecting elements), and a set of places P_+ (represented as double circles) whose marking may assume any nonnegative real number (tanks). Therefore $m : P_+ \to \mathbb{R}_0^+$, $P_0 \to 0$, $P_d \to \mathbb{N}$. The marking of place p_i is denoted m_i, while the value of the marking at time τ is denoted $m(\tau)$. The restriction of m to P_d and P_c are denoted with m^d and m^c, respectively. An *FOHPN system* $\langle N, m(\tau_0)\rangle$ is an FOHPN N with an initial marking $m(\tau_0)$.

Note that in the original formalism used in [3], [4] no partition was introduced in the set of continuous places, thus $P_c \equiv P_+$, and places with a constant zero marking were modeled through zero–capacity buffers.

Example 1: Consider the net in figure 1.a. Places $p_{1,on}$, $p_{1,off}$, $p_{2,on}$, $p_{2,off}$, $p_{3,on}$ and $p_{3,off}$ are discrete places. Places p_1 and p_2 are continuous places, with $p_1 \in P_0$ and $p_2 \in P_+$. Transitions $t_{1,on}$, $t_{1,off}$, $t_{2,on}$, $t_{2,off}$, $t_{3,on}$ and $t_{3,off}$ are discrete transitions. Transitions t_1, t_2 and t_3 are continuous transitions whose mfs and MFS are specified between brackets.

The net in figure 1.a represents the manufacturing process sketched in figure 1.b. The three continuous transitions t_1, t_2 and t_3 represent three unreliable machines M_1, M_2 and M_3; parts produced by the first two machines are collected into a conveyor whose capacity may be assumed equal to zero, and are then sent to the third machine M_3 who processed them again before sending them to the buffer (modeled by place p_2).

In the net system in figure 1.a the discrete part of the net represents the failure model of the machines. When place $p_{1,on}$ is marked, transition t_1 is enabled, i.e., machine M_1 is operational; when place $p_{1,off}$ is marked, transition t_1 is not enabled, i.e., the machine is down. A similar interpretation applies to the other machines. The marking represented in the net shows that initially all machines are operational and the buffer is empty. ∎

Net dynamics: The enabling of a discrete transition depends on the marking of all its input places, both discrete and continuous. More precisely, a discrete transition t is *enabled* at m if for all $p_i \in {}^\bullet t$, $m_i \geq Pre(p_i, t)$, where ${}^\bullet t$ denotes the preset of transition t.

If a discrete transition t_j is enabled at a certain time instant τ^-, then it may fire and its firing at $m(\tau^-)$ yields a new marking $m(\tau)$. For each place p_i it holds $m_i(\tau) = m_i(\tau^-) + Post(p_i, t_j) - Pre(p_i, t_j) = m_i(\tau^-) + C(p_i, t_j)$, thus we can write $m^c(\tau) = m^c(\tau^-) + C_{cd}\sigma$, $m^d(\tau) = m^d(\tau^-) + C_{dd}\sigma$, where σ is the *firing count vector* associated to the firing of transition t_j, i.e., $\sigma \in \mathbb{N}^{n_d}$ and $\sigma_i = 1$ if $i = j$ else $\sigma_i = 0$.

To every continuous transition t_j is associated an instantaneous firing speed (IFS) $v_j(\tau)$. It represents the

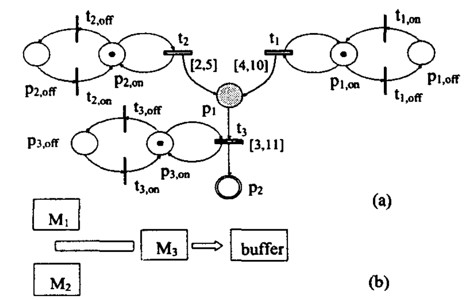

Fig. 1. A First–Order Hybrid Petri Net.

quantity of markings by time unit that fires the continuous transition at the generic time instant τ. For all τ it should be $V'_j \leq v_j(\tau) \leq V_j$, thus the IFS of each continuous transition is piecewise constant between events.

An empty continuous place p_i can be fed, i.e., supplied, by an input transition, which is enabled. Thus, as a flow can pass through an unmarked continuous place, this place can deliver a flow to its output transitions. Consequently, a continuous transition t_j is enabled at time τ if and only if all its input discrete places $p_k \in P_d$ have a marking $m_k(\tau)$ at least equal to $Pre(p_k, t_j)$, and all its input continuous places $p_i \in P_c$ satisfy the following condition: either $m_i(\tau) > 0$ or p_i is fed. If all input continuous places of t_j have a not null marking, then t_j is called *strongly enabled*, else t_j is called *weakly enabled*. Finally, transition t_j is not enabled if one of its empty input places is not fed.

We can write the equation which governs the evolution in time of the marking of a place $p_i \in P_c$ as

$$\dot{m}_i(\tau) = \sum_{t_j \in T_c} C(p_i, t_j) v_j(\tau) \tag{1}$$

where $v(\tau) = [v_1(\tau), \ldots, v_{n_c}(\tau)]^T$ is the IFS vector at time τ. Indeed Equation (1) holds assuming that at time τ no discrete transition is fired and that all speeds $v_j(\tau)$ are continuous in τ.

The enabling state of a continuous transition t_j defines its admissible IFS v_j.

- If t_j is not enabled then $v_j = 0$.
- If t_j is strongly enabled, then it may fire with any firing speed $v_j \in [V'_j, V_j]$.
- If t_j is weakly enabled, then it may fire with any firing speed $v_j \in [V'_j, \overline{V}_j]$, where $\overline{V}_j \leq V_j$ since t_j cannot remove more fluid from any empty input continuous place $\overline{p}$ than the quantity entered in $\overline{p}$ by other transitions.

The computation of the IFS of enabled transitions is not a trivial task. We will set up in the next subsection a linear–algebraic formalism to do this. Here we simply discuss the net evolution assuming that the IFS are given.

We say that a *macro–event* occurs when: (a) a discrete transition fires, thus changing the discrete marking and enabling/disabling a continuous transition; (b) a continuous place becomes empty, thus changing the enabling state of a continuous transition from strong to weak.

Let τ_k and τ_{k+1} be the occurrence times of two consecutive macro–events as defined above; we assume that within the interval of time $[\tau_k, \tau_{k+1})$, denoted as a *macro-period*, the IFS vector is constant and we denote it $\boldsymbol{v}(\tau_k)$. Then the continuous behavior of an FOHPN for $\tau \in [\tau_k, \tau_{k+1})$ is described by $\boldsymbol{m}^c(\tau) = \boldsymbol{m}^c(\tau_k) + \boldsymbol{C}_{cc}\boldsymbol{v}(\tau_k)(\tau - \tau_k)$, $\boldsymbol{m}^d(\tau) = \boldsymbol{m}^d(\tau_k)$.

Example 2: Let us consider again the net system in figure 1.a. Discrete transitions $t_{1,off}$, $t_{2,off}$ and $t_{3,off}$ are enabled, while transitions $t_{1,on}$, $t_{2,on}$ and $t_{3,on}$ are disabled. Continuous transitions t_1 and t_2 are strongly enabled, while transition t_3 is weakly enabled because it has an empty input continuous place p_1 that is fed by transitions t_1 and t_2. ■

We use linear inequalities to characterize the set of *all* admissible firing speed vectors $\mathcal{S}$. Each IFS vector $\boldsymbol{v} \in \mathcal{S}$ represents a particular mode of operation of the system described by the net, and among all possible modes of operation, the system operator may choose the best according to a given objective.

They form a convex set described by linear equations.

Definition 1 (admissible IFS vectors) Let $\langle N, \boldsymbol{m}\rangle$ be an FOHPN system with n_c continuous transitions and incidence matrix $\boldsymbol{C}$. Let $T_{\mathcal{E}}(\boldsymbol{m}) \subset T_c$ ($T_{\mathcal{N}}(\boldsymbol{m}) \subset T_c$) be the subset of continuous transitions enabled (not enabled) at $\boldsymbol{m}$, and $P_{\mathcal{E}}(\boldsymbol{m}) = \{p \in P_+ \mid m_p = 0\}$ be the subset of empty continuous places in P_+. Any *admissible IFS vector* $\boldsymbol{v} = [v_1, \cdots, v_{n_c}]^T$ at $\boldsymbol{m}$ is a feasible solution of the following linear set:

$$\left\{ \begin{array}{lll} (a) & V_j - v_j \geq 0 & \forall t_j \in T_{\mathcal{E}}(\boldsymbol{m}) \\ (b) & v_j - V_j' \geq 0 & \forall t_j \in T_{\mathcal{E}}(\boldsymbol{m}) \\ (c) & v_j = 0 & \forall t_j \in T_{\mathcal{N}}(\boldsymbol{m}) \\ (d) & \sum_{t_j \in T_{\mathcal{E}}} \boldsymbol{C}(p, t_j) \cdot v_j \geq 0 & \forall p \in P_{\mathcal{E}}(\boldsymbol{m}) \\ (e) & \sum_{t_j \in T_{\mathcal{E}}} \boldsymbol{C}(p, t_j) \cdot v_j = 0 & \forall p \in P_0 \end{array} \right. \tag{2}$$

Thus the total number of constraints that define this set is 2 card $\{T_{\mathcal{E}}(\boldsymbol{m})\}$ + card $\{T_{\mathcal{N}}(\boldsymbol{m})\}$ + card $\{P_{\mathcal{E}}(\boldsymbol{m})\}$ + card $\{P_0\}$. The set of all feasible solutions is denoted $\mathcal{S}(N, \boldsymbol{m})$. ■

Constraints of the form (2.a), (2.b), and (2.c) follow from the firing rules of continuous transitions. Constraints of the form (2.d) follow from (1), because if a continuous place is empty then its fluid content cannot decrease. Constraints of the form (2.e) follow from the fact that places in P_0 should always be empty by definition. Note that if $V_i' = 0$, then the constraint of the form (2.b) associated to t_i reduces to a non-negativity constraint on v_i.

Example 3: Let us consider the net N in figure 1.a. As already discussed above the set of admissible IFS depends on the actual marking of the net. In the particular case at hand, we first observe that a macro-event may only occur when a discrete transition fires, being p_1 an empty place by definition and p_2 a continuous place with no output arcs. This implies that the set of macro-periods may be uniquely characterized by the discrete marking of the net.

In figure 2 we have reported the set of admissible IFS for transitions t_1 and t_2. Note that, being $m_1(\tau) = 0$ for all time instants τ, it follows that $v_3(\tau) = v_1(\tau) + v_2(\tau)$ for all τ, thus the dark areas in figure 2 completely describe the set $\mathcal{S}(N, \boldsymbol{m})$ for all $\boldsymbol{m}$.

The sets of reachable discrete markings have been characterized by explicitly enumerating the set of marked discrete places and have been denoted as A, B, $\cdots$, H. As an example, $A = \{p_{1,on}, p_{2,on}, p_{3,on}\}$ is representative of the discrete marking $m(p_{1,on}) = m(p_{2,on}) = m(p_{3,on}) = 1$ and $m(p_{1,off}) = m(p_{2,off}) = m(p_{3,off}) = 0$.

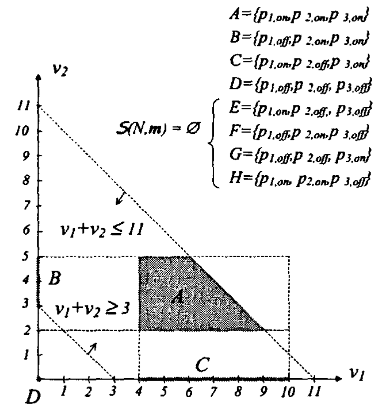

Fig. 2. The set of admissible IFS for transitions t_7 and t_8 in figure 1.

The plot in figure 2 has been obtained considering that whenever transitions t_1 and t_2 are enabled, it should be $4 \leq v_1 \leq 10$ and $2 \leq v_2 \leq 5$, respectively. Moreover, whenever transition t_3 is enabled it should be $3 \leq v_3 \leq 11$, thus implying two additional constraints in the set of IFS of transitions t_1 and t_2, i.e., $3 \leq v_1 + v_2 \leq 11$.

The set A denotes the macro-period in which all machines are operational. The larger dark region in figure 2 is representative of the set of admissible IFS for this discrete marking. Note that an operating mode with both transitions t_1 and t_2 firing at their MFS is not allowed (point $(10, 5)$ does not belong to this region).

The macro-period B corresponds to the situation in which t_2 and t_3 are enabled while t_1 is not enabled. We may observe that t_2 may never fire at its mfs. Similar considerations may be repeated for the macro-period C with the only difference that in this case the admissible IFS of t_3 imposes no additional constraint in the IFS of t_1 being $[V_1', V_1] = [4, 10] \subset [V_3', V_3] = [3, 11]$.

Macro-period D corresponds to the situation in which no machine is operational.

Finally, let us observe that no operational mode exists when the set of marked discrete places is any of the sets E, F, G and H. As an example, let us consider the set F. In this case the set of admissible IFS is

$$\mathcal{S}(N, \boldsymbol{m}) = \left\{ \begin{array}{l} v_1 = 0 \\ 2 \leq v_2 \leq 5 \\ v_3 = 0 \\ v_1 + v_2 - v_3 = 0 \end{array} \right. = \left\{ \begin{array}{l} v_1 = 0 \\ 2 \leq v_2 \leq 5 \\ v_3 = 0 \\ v_2 = 0 \end{array} \right. = \emptyset.$$

Similar conclusions may be drawn for the sets E, G and H. Physically this means that when a machine is operational then its IFS should be within its mfs and its MFS. If its mfs is strictly positive and we want its IFS be null, then the machine should be switched off. ■

Once the set of all admissible IFS vectors has been defined, we need a procedure to select one among them. One possible way of computing an optimal IFS vector consists in introducing an objective function that may

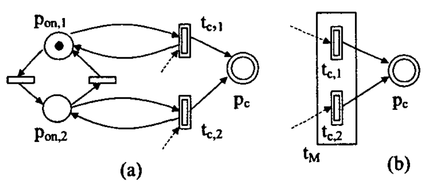

Fig. 3. The Petri net model of a macro transition: (a) detailed representation, (b) simplified representation.

be representative of a global performance index and solving the corresponding optimization problem with constraint set given by (2).

III. Modeling plant subsystems with FOHPN

In this section we present the Petri net model of the most important elementary modules of a bottling plant, namely transportation lines and switches, machines and buffers, that are then put together to make the whole Petri net model of a part of a real production plant.

We first introduce a novel elementary module of Petri nets, named *macro transition*, that will be useful in the following to get a more compact representation of the other elementary modules. A macro transition is represented with a large rectangle with some continuous transitions inside to denote that only one continuous transition at a time may be enabled. In such a way we can omit the representation of the discrete part of the net. An example of macro transition is reported in figure 3 in the case of two continuous transitions. When $p_{on,1}$ is marked, transition $t_{c,1}$ may fire. On the contrary, when the token is in $p_{on,2}$, only $t_{c,2}$ is enabled.

Transportation lines and switches: Transportation lines consist of pipes of appropriate diameter, depending on the bottle sizes, where bottles are conveyed at high speed thanks to the force produced by the compressed air. Due to the high speed, the main feature of these elements is that there is no accumulation of bottles in their inside. Therefore, transportation lines may be seen as connecting elements and the corresponding places in the Petri net model are zero capacity places, i.e., places in P_0.

In the general scheme the connections among different lines may vary: this corresponds to a switch that can be of different types: MIMO (multi input - multi output), MISO (multi input - single output) and SIMO (single input - multi output). In the MIMO case, we represent a switch with a macro transition at the input and a macro transition at the output, thus enabling one possible path at a time. In figure 4 a MIMO switch is represented in the case of two input and two output lines. Note that place p_c has been denoted as a dark circle because it is a zero capacity place.

Machines: In this plant we have two different types of machines. The first type is involved in bottles production, while the second one is involved in bottles filling and corking.

Machines of the first type are equipped so as to produce bottles of different sizes. In the following, we consider the case of a machine that can be used to produce 1.5 lt bottles and 2 lt bottles. A detailed and a reduced scheme of the Petri net model for such a machine is

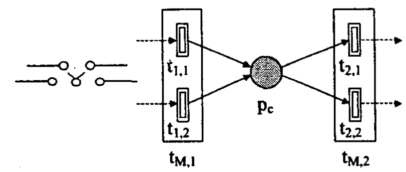

Fig. 4. A MIMO switch with 2 input and 2 output lines.

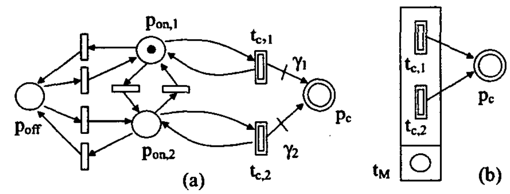

Fig. 5. The Petri net model of a machine that produces bottles: (a) detailed representation, (b) simplified representation.

shown in figure 5. In particular, the firing of $t_{c,1}$ denotes the production of 1.5 lt bottles, whereas the firing of $t_{c,2}$ denotes the production of 2 lt bottles. Clearly, the productivity of the machine is not the same in the two cases, thus the weights of the input arcs to p_c are different. Note that the machine may also be off, thus three discrete places have been introduced in the detailed Petri net model, as well as an empty circle has been included in the compact representation.

A dual scheme may be used to describe the functioning of those machines that are involved in the bottle filling and corking. An example in the case of bottles of two different sizes is reported in figure 6. A macro transition with an empty circle is used again to denote that the machine may also be off.

Buffers: A Petri net model of a buffer is reported in figure 7 in the case that bottles of two different sizes may be stored in it. For brevity's requirements the detailed model has been omitted, but it can be easily deduced from the previous ones. When effectively modeling a buffer we should take into account all sizes of bottles that can be stored in it. This can be easily done by simply introducing a continuous place for each possible

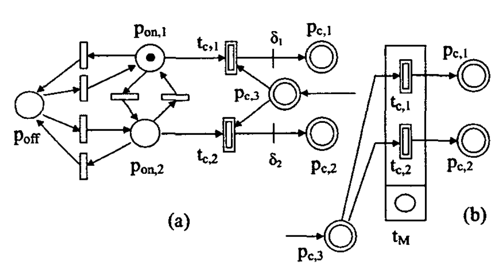

Fig. 6. The Petri net model of a machine that fills bottles: (a) detailed representation, (b) simplified representation.

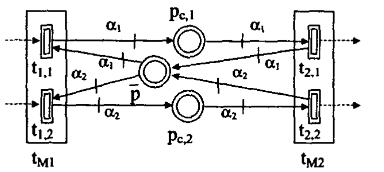

Fig. 7. The simplified Petri net model of a buffer.

format (see places $p_{c,1}$ and $p_{c,2}$). Then, an additional place ($\overline{p}$) should also be introduced to limit the total volume of bottles entering the buffer, according to its capacity. In this place the fluid content is complementary to the whole content of the buffer, i.e., it is empty when the buffer is full and is full when the buffer is empty. Clearly, the total number of bottles that can be introduced in the buffer depend on their size, and this is taken into account through the different values of α_1 and α_2. Moreover, we should also impose that bottles of different sizes are not put together. This implies that the following conditions should be verified:

- if $m(p_{c,1}) > 0$, then $t_{1,2}$ is not enabled;
- if $m(p_{c,2}) > 0$, then $t_{1,1}$ is not enabled.

These are safeness specifications that may be structurally enforced in the net (e.g., by inhibitory arcs) or may be imposed on-line by a supervisory controller.

Finally, let us observe that in all the examined cases, different ranges should be assigned to continuous transitions, depending on the physical system. Moreover, the actual firing speeds should be computed as the solution of on optimization problem that takes into account the main goal we want to achieve. This problem has not been dealt with in this paper, but it will be the object of our future research.

IV. The FOHPN model of a real bottling plant

In this section we first describe a part of the whole production process of a real bottling plant in Sardinia. Then, we show how it can be modeled through FOHPN by simply putting together the previous elementary modules. For more details on the whole production process we address to [7].

Plant description: Let us consider the flow diagram sketched in figure 8. It represents the production cycle whose first stage consists in the creation of the PET bottles and whose last stage consists in self-filling and corking. More precisely, the first operational machine is M_1 that produces PET bottles starting from raw-material of PET granules (PET chips). Thanks to an appropriate equipment, this machine may be extremely versatile and may produce different bottle sizes, e.g., 1.5 lt and 2 lt. The flow of bottles of the two types have been distinguished in figure 8 with two different colours, green and red, respectively. Then, the produced bottles are directed to appropriate lines of different diameter, depending on their size. The flow of bottles through the conveyor lines occurs at a high speed and is induced by a jet of compressed air. Bottles may follow different paths and may be assigned to different buffers. Path assignment may be seen as a decision problem whose solution aims to optimize the production process. In particular, in the case we are dealing with, there are 7 buffers (S_1, S_2, $\cdots$, S_7) and the partitioning is established so as to compensate as much as possible the delay due to the reduced productivity of the machines that fill bottles of mineral water with respect to those that produce them.

Finally, from buffers bottles are conveyed to the zone of self-filling through other appropriate flow lines. Even in this case, bottles may follow different paths so as to better exploit the filling machines. In particular, there are 3 filling machines that are denoted in figure 8 as M_2, M_3 and M_4, and that can be used to fill bottles of all sizes.

The FOHPN model: The FOHPN model of the above production process has been reported in figure 9, where all the elementary modules previously defined can be easily recognized. The same colour notation has been used in the two figures, so as to better distinguish the flow of bottles of different sizes, and their flow in the belt conveyor. We may also observe that all continuous places with a zero capacity have been denoted as full dark circles.

Note that two further colours with respect to those in figure 8, namely blue and yellow, have been introduced in figure 9 to denote that machine M_4 is also used for filling and corking bottles of two other sizes, namely 0.5 and 1 lt.

V. Conclusions

In this paper we have dealt with the problem of modeling a bottling plant. In particular, we have considered an existing plant in Sardinia.

To this aim we have used an hybrid Petri net model, named First-Order Hybrid Petri net. This model has been already used by the authors in other application fields, such as manufacturing and inventory management. Nevertheless, a slight variation in the continuous place definition has been introduced here, so as to better describe the behaviour of some elementary modules where no fluid content accumulation may occur.

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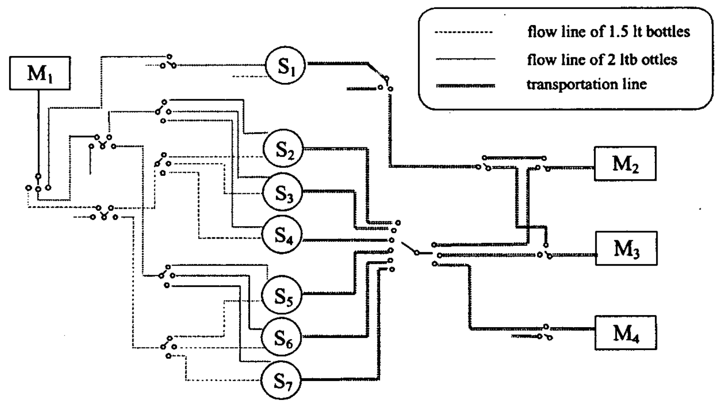

Fig. 8. A scheme representing a part of the bottling production process.

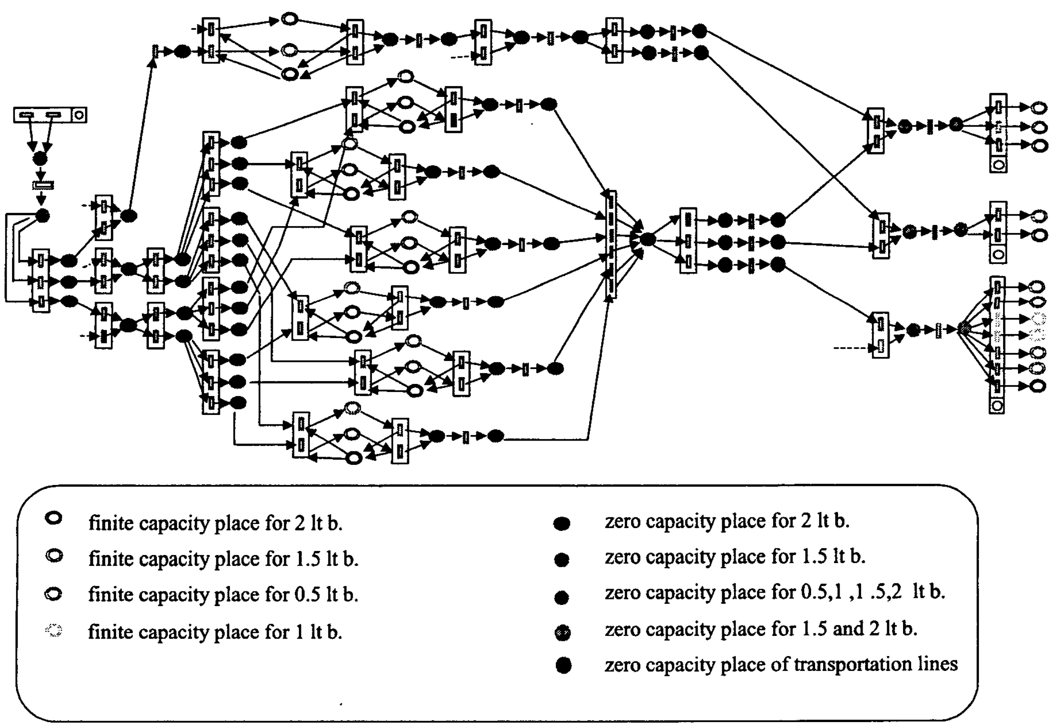

Fig. 9. The Petri net model of the production process in figure 8.