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Synthesis of State-Feedback Optimal Controllers for Continuous-Time Switched Linear Systems

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Abstract

The paper deals with the optimal control of switched piecewise linear autonomous systems, where the objective is to minimize a performance index over an infinite time horizon. We assume that the switching sequence has a finite length: the unknown switching times and the switching sequence are the optimization parameters. We also assume that a cost may be associated to each switch.

The optimal control for this class of systems takes the form of a state feedback, i.e., it is possible to identify a set of regions of the state space such that an optimal switch should occur if and only if the present state belongs to one of them. We show how the tables containing these regions can be computed off-line through a numerical procedure.

1 Introduction

Switched systems are a particular class of hybrid systems that switch between many operating modes, where each mode is governed by its own characteristic dynamical law [6]. Typically, mode transitions are triggered by variables crossing specific thresholds (state events), by the elapse of certain time periods (time events), or by external inputs (input events). The problem of determining optimal control laws for this class of hybrid systems has been widely investigated in the last years and many results can be found in the control and computer science literature. For continuous-time hybrid systems, most of the literature is focused on the study of necessary conditions for a trajectory to be optimal [11, 14], and on the computation of optimal/suboptimal solutions by means of dynamic programming or the maximum principle [4, 5, 10, 12, 13, 15]. For determining the optimal feedback control law some of these techniques require the discretization of the state space in order to solve the corresponding Hamilton-Jacobi-Bellman equations. In [9] the authors use a hierarchical decomposition approach to break down the overall problem into smaller ones. In so doing, discretization is not involved and the main computational complexity arises from a higher-level nonlinear programming problem.

In the continuous-time case, and in particular for switched linear systems composed by stable autonomous dynamics, by assuming that the switching sequence is pre-assigned (and, therefore, that the only decision variables to be optimized are the switching instants), in [7, 8] we proved an important result, namely that the control law is a state-feedback and there exists a numerically viable procedure to compute the switching tables $\mathcal{C}_{k,N}$ showing the points of the state space where the k-th switch of a sequence of length N should occur. A similar state-feedback result was also proved in [1] for optimal control problems based on discrete-time hybrid models and linear cost functions, which leads to piecewise affine optimal control laws.

In [2, 3] we generalized the optimization problem of [7, 8] by taking both the switching instants and the switching sequence as decision variables. The procedure we used to solve the generalized problem exploits a synergy of discrete-time and continuous-time techniques, by alternating between a "master" procedure that finds an optimal switching sequence and a "slave" procedure that finds the optimal switching instants still using the approach described in [7, 8].

In this paper we present a different solution to the generalized optimization problem that is still based on the construction of switching tables. Using a simple procedure inspired by dynamic programming, we show how it is possibile to avoid the exponential growth of the computational cost as the length of the switching sequence is increased.

To motivate the interest for pursuing two different approaches to solve essentially the same problem, we compare the procedure presented in this paper with the master-slave algorithm in [2, 3]. More details about the computational complexity of both procedures will be given in Section 4.

The procedure presented in this paper is based on the generation of a set of "switching tables" and has the following properties:

- is guaranteed to find the optimal solution;
- has a computational cost of the order $O(Ns^2)$, where N is the length of the switching sequence and s is the number of possible operating modes (or dynamics);
- provides a "global" closed-loop solution, i.e., the tables may be used to determine the optimal state feedback law for all initial states.

On the other hand, the master-slave algorithm presented in [2, 3]:

- is not guaranteed to converge to a global optimum;

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- has a computational cost of the order $O(\alpha N)$, where N is the length of the switching sequence and α is usually very small (≤ 5);
- provides a "local" closed-loop solution, i.e., a by-product of the slave procedure consists of tables that may be used to determine the optimal state feedback law which, however, is only valid for small perturbation around a given initial state.

This brief analysis shows that the two procedures have different pros and cons. For this reason, we believe they may both be useful depending on the application.

As a final remark, we also mention that the approach presented in this paper allows one to also take into account a fixed cost associated to each switch as in [7].

2 Optimal Control Problem

In this paper we consider the following class of hybrid systems

$$\dot{x}(t) = A_{i(t)}x(t), \quad i(t) \in \mathcal{S} \tag{1}$$

that we denote as *switched linear systems*, where $x(t) \in \mathbb{R}^n$, $i(t) \in \mathcal{S}$ is the control variable, and $\mathcal{S} \triangleq \{1, 2, \ldots, s\}$ is a finite set of integers, each one associated with a linear dynamics.

Assume that a positive semi-definite weight matrix Q_i, $i \in \mathcal{S}$, is associated to each dynamics and a cost $H_{i,j}$ is associated to a switch from mode i to mode j. For such a class of hybrid systems we want to solve the following optimal control problem

$$\begin{array}{lll} V_N^* \triangleq \min\limits_{I,\mathcal{T}} & \{F(I,\mathcal{T}) \\ & \triangleq \int_0^\infty x'(t)Q_{i(t)}x(t)dt + \sum_{k=1}^N h_k(\tau_k)\Big\} \\ \text{s.t.} & \dot{x}(t) = A_{i(t)}x(t) \\ & x(0) = x_0 \\ & i(t) = i_k \text{ for } \tau_{k-1} \leq t < \tau_k \\ & i_k \in \mathcal{S},\ k = 1, \ldots, N+1 \\ & \tau_0 = 0,\ \tau_{N+1} = +\infty \\ & \tau_k \in \mathbb{R}_{\geq 0}\ \forall k = 1, \ldots, N \\ & h_k(\tau_k) = H_{i_k, i_{k+1}} \text{ if } \tau_k < +\infty, \\ & h_k(\tau_k) = 0 \text{ if } \tau_k = +\infty. \end{array} \tag{2}$$

In this problem the initial state x_0 of the system is given, while the control variables are $\mathcal{T} \triangleq \{\tau_1, \ldots, \tau_N\}$, a finite sequence of switching times, $I \triangleq \{i_1, \ldots, i_{N+1}\}$, a finite sequence of switching indices, and N is the maximum allowed number of switches (fixed a priori). The cost consists of two components: a quadratic cost that depends on the time evolution (the integral) and a cost that depends on the switches (the sum). Note that $\tau_k < +\infty$ means that the $k-$th switch occurs after a finite amount of time, while $\tau_k = +\infty$ means that the $k-$th does not occur: in the latter case $h_k(\tau_k) = 0$ thus its cost is not considered.

We denote by $i^*(t)$, $t \in [0, +\infty)$, $i^*(t) = i_k^*$ for $\tau_{k-1}^* \leq t < \tau_k^*$ the switching trajectory solving (2), and I^*, $\mathcal{T}^*$ the corresponding optimal sequences.

In order to make the problem solvable with finite cost V_N^*, we assume the following:

Assumption 1 *There exists at least one index $i \in \mathcal{S}$ such that A_i is strictly Hurwitz.*

The optimal control problem may be easily generalized. One may assume that whenever at time τ_k a switch from i_k to i_{k+1} occurs, the state should jump from $x(\tau_k^-)$ to $x(\tau_k^+) = M_k x(\tau_k^-)$ as in [8]. Affine dynamics of the form $\dot{x}(t) = A_{i(t)}x(t) + f_{i(t)}$ have also been studied in [3]. However, to avoid heavy notation in this paper we only restrict to the basic framework (2).

Let us define $\delta_k = \tau_k - \tau_{k-1}$. The optimal control problem (2) may also be rewritten as:

$$\begin{array}{ll} \min\limits_{I,\mathcal{T}} & \left\{ \sum\limits_{k=1}^{N+1} x'_{k-1}\bar{Q}_{i_k}(\delta_k)x_{k-1} + \sum\limits_{k=1}^{N} h_k(\tau_k) \right\} \\ \text{s.t.} & x_k = \bar{A}_{i_k}(\delta_k)x_{k-1},\ k = 1, \ldots, N+1 \\ & x_0 = x(0) \end{array} \tag{3}$$

where

$$\bar{A}_i(\delta_k) \triangleq e^{A_i \delta_k}, \tag{4}$$

and $\bar{Q}_i(\delta_k)$ can be obtained by simple integration and linear algebra. In [8] we showed that when A_i is asymptotically stable it is possible to write $\bar{Q}_i(\delta_k) = Z_i - \bar{A}'_i(\delta_k)Z_i\bar{A}_i(\delta_k)$, where Z_i is the unique solution of the Lyapunov equation $A'_iZ_i + Z_iA_i = -Q_i$.

Since we are not making any assumptions here on A_i we may resort to direct integration as in [3]. Indeed, if for simplicity we assume that A_i is diagonalizable, $A_i = T_i^{-1}\Lambda_i T_i$, $\Lambda_i = \text{Diag}\{\lambda_i^1, \ldots, \lambda_i^n\}$, we obtain

$$\int_{\tau_{k-1}}^{\tau_k} x'(t)Q_{i_k}x(t)dt = x'_{k-1} \underbrace{T'_{i_k} \left(\int_{\tau_{k-1}}^{\tau_k} e^{\Lambda_{i_k} t}(T_{i_k}^{-1})' Q_{i_k} T_{i_k}^{-1} e^{\Lambda_{i_k} t} dt \right) T_{i_k}}_{\bar{Q}_{i_k}(\delta_k)} x_{k-1}$$

The case of A_i not diagonalizable is similar.

3 State-Feedback Control Law

In this section we show that the optimal control law for the optimization problem described in the previous section takes the form of a state-feedback, i.e., it is only necessary to look at the current system state x in order to to determine if a switch from A_{i_k} to $A_{i_{k+1}}$ should occur.

In particular, for a given mode $i \in \mathcal{S}$ and for a given switch $k \in 1, \ldots, N$ it is possible to construct a table $\mathcal{C}_{k,N}^i$ that collects a partition of the state space $\mathbb{R}^n$ into s regions

$\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_s$. Whenever $i_k = i$ we use table $\mathcal{C}^i_{k,N}$ to determine if a switch should occur: as soon as the state reaches a point in the region $\mathcal{R}_j$ (with $j \neq i$) we will switch to mode $i_{k+1} = j$, while no switch will occur while the system's state belong to $\mathcal{R}_i$.

This is an important result because it is well known that a state-feedback control law has many advantages over an open-loop control law, including the fact that the computation of the control law can be done off line as opposed to being performed on line.

To prove this result, we show constructively how the tables $\mathcal{C}^i_{k,N}$ can be computed using a dynamic programming argument. We first show how the tables $\mathcal{C}^i_{N,N}$ ($i \in \mathcal{S}$) for the last switch can be determined. Then, we show by induction how the tables $\mathcal{C}^i_{k,N}$ can be computed once the tables $\mathcal{C}^i_{k+1,N}$ are known.

3.1 Computation of the Tables for the Last Switch

Let us assume that $i_N = i$, i.e., after $N-1$ switches the current system dynamics is that corresponding to matrix A_i, and the current state vector is y with $||y|| = 1$. We show how to compute the table $\mathcal{C}^i_{N,N}$.

The optimal remaining cost starting from y will consist of two terms: a term due to the time-driven evolution, plus (if the $N-$th switch occurs and $i_{N+1} = j$) the switching cost $H_{i,j}$.

- Let us first consider the case in which no switch occurs. The remaining cost starting from y is only due to the time-driven evolution and is

$$T^*_i(y, i) = y'\bar{Q}_i(+\infty)y. \tag{5}$$

- If the system evolves with dynamics A_i for a time ϱ and then a switch to A_j ($j \neq i$) occurs, the remaining cost starting from y only due to the time-driven evolution (disregarding the switching cost) is

$$T_i(y, \varrho, j) = y'\bar{Q}_i(\varrho)y + y'\bar{A}'_i(\varrho)\bar{Q}_j(+\infty)\bar{A}_i(\varrho)y. \tag{6}$$

Let us denote $\varrho_i = +\infty$, while for $j \neq i$ we denote

$$\varrho_j = \arg\min_{\varrho} T_i(y, \varrho, j), \tag{7}$$

the value of ϱ that minimizes (6) while the corresponding minimum is

$$T^*_i(y, j) = T_i(y, \varrho_j, j). \tag{8}$$

Let us now consider any other vector x such that $x = \lambda y$, with $\lambda \in \mathbb{R}$. We can compute for this new vector the equivalent of (5) and (6), i.e.,

$$T^*_i(x, i) = x'\bar{Q}_i(+\infty)x = \lambda^2 T^*_i(y, i) \tag{9}$$

and for $j \neq i$

$$T_i(x, \varrho, j) = \lambda^2 T_i(y, \varrho, j), \tag{10}$$

Equation (10) is minimized by the same value $\varrho = \varrho_j$ that minimizes (6) and its minimal value is

$$T^*_i(x, j) = \lambda^2 T^*_i(y, j). \tag{11}$$

We discuss separately two cases.

1. If all switching costs are null, the optimal remaining cost starting from x and allowing at most one switch is

$$F^*_{i,N}(x) = \min_{j \in \mathcal{S}} \{T^*_i(x, j)\} = \lambda^2 \min_{j \in \mathcal{S}} \{T^*_i(y, j)\}, \tag{12}$$

while the value of j that minimizes the previous equation is denoted $j^*(y)$. Thus the optimal switch from mode i to mode j should occur after a delay

$$\delta^*_{i,N}(x) = \delta^*_{i,N}(y) = \varrho_{j^*(y)} \tag{13}$$

that for $x = \lambda y$ is a function of y but not of λ.

We can say that a vector $x = \lambda y$ belongs to $\mathcal{R}_j$ ($j \neq i$) if and only if $j = j^*(y)$ and $\delta^*_{i,N}(y) = 0$, because in this case the optimal remaining cost can be obtained switching as soon as we reach x with no delay. Finally, $\mathcal{R}_i = \mathbb{R}^n \setminus \cup_{j \neq i} \mathcal{R}_j$. Since the value of $\delta^*_{i,N}(\lambda y)$ in this case does not depend on λ, it immediately follows that these regions are homogeneous[1], i.e., if $x \in \mathcal{R}_j$ then $\lambda x \in \mathcal{R}_j$, for all real numbers λ. This property may be exploited in the construction of the table since it is only necessary to compute $F^*_{i,N}(y)$ and $\delta^*_{i,N}(y)$ for all vectors y that belong to the unitary semi-sphere.

2. Assume that not all $H_{i,j}$ (this is the cost of switching from mode i to mode j) are null and let us define $H_{i,i} = 0$. Taking also into account the switching cost, the optimal remaining cost starting from x and allowing at most one switch is

$$F^*_{i,N}(x) = \min_{j \in \mathcal{S}} \{T^*_i(x, j) + H_{i,j}\}, \tag{14}$$

while the value of j that minimize the previous equation is denoted $j^*(x)$. Thus the optimal switch should occur after a delay

$$\delta^*_{i,N}(x) = \varrho_{j^*(x)}. \tag{15}$$

We can say that a vector $x = \lambda y$ belongs to $\mathcal{R}_j$ ($j \neq i$) if and only if $j = j^*(x)$ and $\delta^*_{i,1}(x) = 0$. Finally, $\mathcal{R}_i = \mathbb{R}^n \setminus \cup_{j \neq i} \mathcal{R}_j$. In this case it is not sufficient to compute $F^*_{i,N}(y)$ and $\delta^*_{i,N}(y)$ for all vectors y that belong to the unitary semi-sphere but we also have to take into account the norm λ of a vector $x = \lambda y$ (at least for small values of λ: for λ large enough the effect of the switching cost becomes negligible).

[1] A term also used to define the special form of these regions is *conic*.

Note that in order to compute the switching regions $\mathcal{R}_j$ and to determine the optimal remaining cost $F^*_{i,N}(x)$, we only need to compute the values $\varrho_i(j)$ with $s-1$ one-parameter optimization (see equations (6) and (7)) for all y on the unitary semi-sphere. The corresponding values of $T^*_i(y,i)$ and $T^*_i(y,j)$ can be obtained applying equations (5) and (8), while to determine if a vector $x = \lambda y$ belongs to $\mathcal{R}_j$ and to compute the corresponding optimal remaining cost we only need to apply equations (14) and (15).

3.2 Computation of the Tables for the Intermediate Switches

We now generalize the previous approach to determine the tables $\mathcal{C}^i_{k,N}$, for $k = 1, \ldots, N-1$.

Assume that:

- we have already computed the tables $\mathcal{C}^j_{k+1,N}$ for all $j \in \mathcal{S}$;
- for each vector x and each mode $j \in \mathcal{S}$ we know the optimal cost $F^*_{j,k+1}(x)$ for the remaining time-driven evolution that starts from x with dynamics A_j and allows $N-k$ more switches.

Then with the same argument of the previous subsection we can write that

$$F^*_{i,k}(x) = \min_{j\in\mathcal{S}} \{T^*_i(x,j) + H_{i,j}\}, \qquad (16)$$

where $T^*_i(x,i) = x'\bar{Q}_i(+\infty)x$ and for $i \neq j$

$$T^*_i(x,j) = \min_{\varrho} \left\{ x'\bar{Q}_i(\varrho)x + F^*_{i,k+1}(\bar{A}_i(\varrho)x) \right\},$$

and compute the new table $\mathcal{C}^i_{k,N}$, as we did before.

3.3 Computation of the Table for the Initial Mode

Once all tables are constructed off-line, we can use them on-line to decide if a switch should occur.

To decide the optimal initial mode i_1 we may use the knowledge of the cost $F^*_{i,1}(x)$ that is evaluated during the construction of the table $\mathcal{C}^i_{1,N}$. Thus we define a new table $\mathcal{C}_{0,N}$ that shows a partition of the state space $\mathbb{R}^n$ into s regions $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_s$.

Each region in this table is defined as follows:

$$\mathcal{R}_i = \{x \mid (\forall j \in \mathcal{S}) F^*_{i,1}(x) \leq F^*_{j,1}(x)\}$$

i.e., if the initial state belongs to region $\mathcal{R}_i$ we must choose $i_1 = i$ to minimize the total cost.

4 Computational complexity

We discuss here the computational complexity involved in the construction of the tables following the approach sketched in the previous section.

Let c be the computational complexity to compute one table using the algorithm given in [8, 7]. This algorithm assumes that the switching sequence is pre-assigned. Hence there exist just one table $\mathcal{C}_{k,N}$ associated to the k-th switch and this table contains only two regions: $\mathcal{R}_{i_{k-1}}$, i.e., the set of state vectors where we continue using mode i_{k-1}, and $\mathcal{R}_{i_k}$, i.e., the set of state vectors where we switch to mode i_k. These regions can be determined by solving a one-parameter optimization problem for each vector x on the unitary semi-sphere.

Thus the complexity of solving the optimal control problem for a pre-assigned sequence of length N is $f_1(N) = Nc$, because for each switch a new table must be determined.

This result may be used to solve by brute force an optimal control problem of the form (2), where the sequence is not pre-assigned: we have to consider all possible words of length N composed by symbols taken from an alphabet of cardinality s: the complexity becomes $f'_1(N,s) = Ns^Nc$.

Using the algorithm given in the previous section, for each switch it is necessary to compute s tables. Furthermore the cost of computing each table is not c but $(s-1)c$: in fact each table contains s regions that can be determined solving $s-1$ one-parameter optimization problems for each vector x on the unitary semi-sphere. Thus the complexity of solving the optimal control problem (2) for a sequence of length N is $f_2(N,s) = Ns(s-1)c$. In this case the complexity is a quadratic function of the number of possible dynamics.

Let us finally discuss the computational complexity of the master-slave algorithm presented in [3] where a not pre-assigned sequence of length N was still considered. In that case, if we ignore the complexity of solving the master problem (that is always solved much faster than the slave problem) we can write that its complexity is $f_3(N) = \alpha f_1(N) = \alpha Nc$ where α is a small number that denotes the number of steps required to converge (usually this number is rather small, e.g., $\alpha \leq 5$).

¿From all these considerations, one may conclude that from a computationally point of view the master-slave algorithm offers the best performance. Note however, that the master-slave algorithm solves the control problem only for a *given* initial state $x(0)$, while the construction of the tables as in the algorithm given in the previous section, once done, allows one to solve the problem for any value of the initial state.

5 Numerical simulations

Let us consider a second order linear system with $s = 3$. i.e., the systems dynamics may only be chosen within the set $\{A_1, A_2, A_3\}$. In particular, we assume $A_1 = \begin{bmatrix} 1 & -10 \\ 100 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & -100 \\ 10 & 1 \end{bmatrix}$, and $A_3 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$. Note that A_1 and A_2 are unstable matrices while A_3 is strictly Hurwitz, thus Assumption 1 is verified. We associate the same weighting matrix to each dynamics. In particular, we take $Q_1 = Q_2 = Q_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. We also assume that only $N = 3$ switches are possible. Finally, we associate a null cost to each switch, i.e., we take $H_{i,j} = 0$ for all $i, j = 1, \cdots, s$.

We first execute the off-line part of the procedure, consisting in the construction of the $N \times s = 9$ tables $\mathcal{C}^i_{k,N}$, for $k, i = 1, 2, 3$. Results are reported in Figure 1 where the following color notation has been used: Red color (medium gray) is used to denote region $\mathcal{R}_1$, i.e., the set of states where the

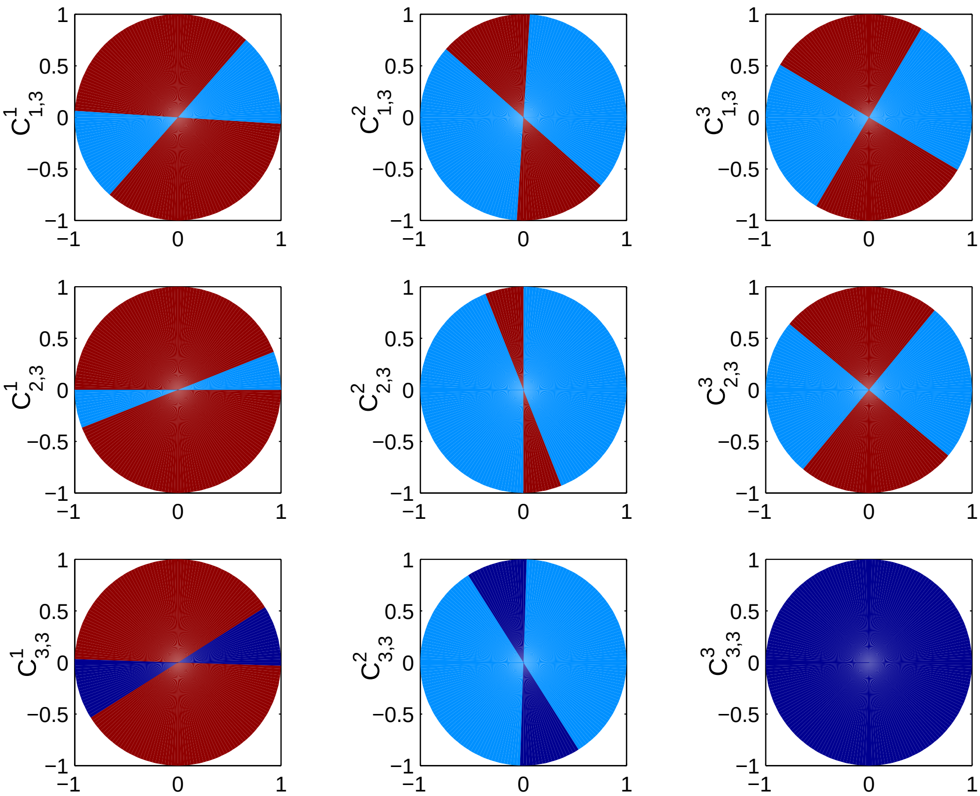

Figure 1: *The set of tables for the numerical example where $N = 3$ and $\mathcal{S} = \{1, 2, 3\}$.*

system either switches to A_1 if the current variable of the control variable is $i(t) \neq 1$, or no switch occur if $i(t) = 1$; light blue (light gray) denotes region $\mathcal{R}_2$, and dark blue (dark gray) is used to denote $\mathcal{R}_3$.

As an example, by looking at $\mathcal{C}^1_{2,3}$ we know that, if the first dynamics is A_1, then the system may either switch to A_2 or still evolve with the same dynamics A_1: on the contrary a switch to dynamics A_3 may never occur.

In Figure 2 we have reported table $\mathcal{C}_{0,3}$ that shows the partition of the state space introduced in subsection 3.3. The same color notation has been used. In particular, this table enables us to conclude that the global optimum may only be reached when the initial system dynamics is either A_1 or A_2. On the contrary, whenever the initial system dynamics is A_3, we may only reach a suboptimal value of the performance index.

Now, let us present the results of some numerical simulation. Let us assume that the initial state is $x_0 = \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]$. We compute the optimal index sequence for all admissible initial system dynamics, i.e., we assume $i_1 = 1, 2, 3$, respectively. The results of numerical simulations are reported in Figure 3 where switches are highlighted trough a small black square. More detailed results may be read in Table 1 where we have reported the optimal index sequence, the optimal timing sequence and the corresponding

i_1	i_2	i_3	i_4	τ_1	τ_2	τ_3	V_3
1	2	1	3	0.000	0.009	0.060	0.669
2	1	2	3	0.009	0.062	0.116	0.126
3	2	1	3	0.000	0.009	0.060	0.669

Table 1: *Detailed results of the numerical example when the initial state is $x_0 = [1\ 1]^T$.*

cost value for the different initial dynamics. We may observe that the best solution may only be reached when the initial system dynamic is the second one. In the other cases only a suboptimal value of the cost may be obtained. Note that these results are in accordance with those of figure 2 being $x_0 \in \mathcal{R}_1$.

The correctness of the solution has been validated through an exhaustive inspection of all admissible index sequences. More precisely, for each admissible index sequence we have computed the optimizing timing sequence and the corresponding cost value. In such a way we have verified that $V_3^* = 0.126$ is indeed the global optimum. Numerical results have also been compared with those obtained using the master-slave procedure [2, 3], that in the actual case requires an auxiliary heuristic to get the optimal solution.

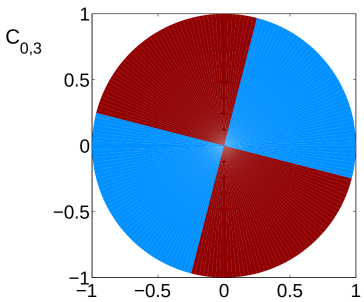

Figure 2: *Table* $\mathcal{C}_{0,3}$.

6 Conclusions

We have considered a special class of switched systems where the switching sequence is finite and each subsystem is autonomous.

We showed that the optimal control for this class takes the form of a state feedback, i.e., it is possible to identify a set of regions of the state space such that an optimal switch should occur if and only if the present state belongs to this region. This set of regions can be computed via an off-line numerical procedure.

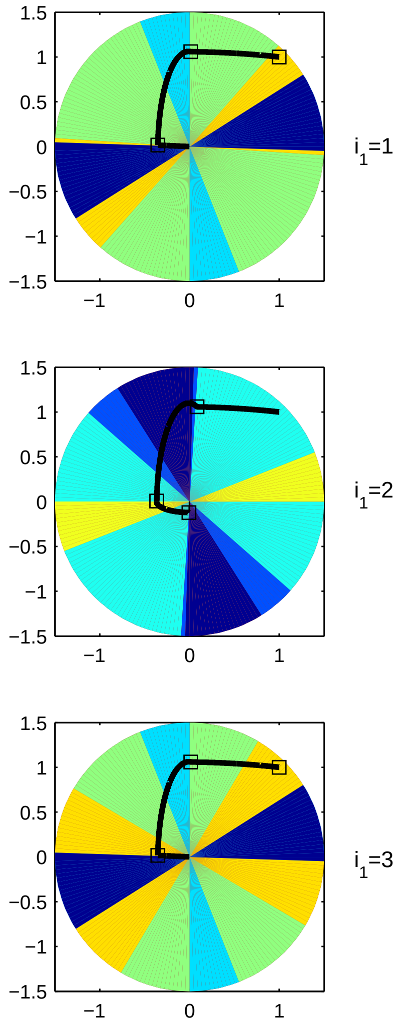

Figure 3: *The system evolution for* $x_0 = [1\ 1]^T$ *and* i_1 *varying in* $\mathcal{S}$.

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