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Semiactive suspension design taking into account the actuator delay

Manuela Ebau, Alessandro Giua, Carla Seatzu, Giampaolo Usai
Dip. di Ing. Elettrica ed Elettronica, Università di Cagliari, Italy
Phone: +39-070-675-5892 – Fax: +39-070-675-5900 – Email: {giua,seatzu,gusai}@diee.unica.it

Abstract

In this paper we present a design procedure for semiactive suspensions of road vehicles. In the first phase we design an asymptotic state observer that reveals to be particularly efficient in the presence of external disturbances. We also derive a target active control law that minimizes a quadratic performance index and takes the form of a feedback control law. Finally, in the second phase, we approximate the target law by controlling the damper coefficient f of the semiactive suspension. To improve the efficiency of the proposed system, we take into account the updating frequency of the coefficient f — imposed by present technology — and compute the expected value of f using a predictive procedure.

1 Introduction

The design of active suspensions for road vehicles aims to optimize the performance of the vehicle with regard to comfort, road holding, and rideability.

In a fully active suspension there are no passive elements, such as dampers and springs. The interaction between vehicle body and wheel is regulated by an actuator of variable length. The actuator is usually hydraulically controlled and applies between body and wheel a force that represents the control action generally determined with an optimization procedure.

Active suspensions [2, 8, 13] have better performance than passive suspensions. However, active suspension systems are rather complex, since they require several components such as actuators, servovalves, high-pressure tanks for the control fluid, either sensors for detecting the system state or appropriate system state observers, etc. Moreover, the associated power, that must be provided by the vehicle engine, may reach the order of several 10 KW [6] depending on the required performance. Thus, these suspension systems have a very high cost.

As a viable alternative to a purely active suspension system, the use of semiactive suspensions has been considered [1, 3, 6, 7, 9, 12]. Such a system consists of a spring whose stiffness is constant and of a damper whose characteristic coefficient f can be made to change within an interval $[f_{\min}, f_{\max}]$ controlling the opening of a valve. The time required to update f is usually less than 10^{-2}s [5].

A semiactive suspension is a valid engineering solution when it can reasonably approximate the performance of the active control. In fact, a semiactive suspension requires a low power controller that can be easily realized at a lower cost than that of a fully active one. In general, a semiactive suspension design consists of two phases [1]: (a) design a good active law, $u_t(\cdot)$ to be considered as a "target"; (b) design the semiactive suspension so that its control law $u_s(\cdot)$ approximates as close as possible the target law $u_t(\cdot)$.

In the following, we discuss in detail these two phases.

Target active law

Thompson [13] was the first to explore the use of optimal control techniques to design an active law so as to minimize a performance index of the form $J = \int_0^\infty (\boldsymbol{x}^T(t)\boldsymbol{Q}\boldsymbol{x}(t) + ru^2(t))dt$, where $\boldsymbol{x}(t)$ is the system state and $u(t)$ the control force provided by the actuator at the time instant t. This design technique is called LQR [10, 11] and has been used by many authors [1, 2, 12]. Its two main advantages are: a) the optimal solution can be easily computed solving an algebraic Riccati equation; b) it takes the form of a state feedback law with constant gains, i.e., $u(t) = -\boldsymbol{K}\boldsymbol{x}(t)$. Note, however, that in most cases the system state is not directly accessible or measuring it is too expensive. Thus, an asymptotic state observer needs to be used. This implies that the real control law takes the form $u(t) = -\boldsymbol{K}\hat{\boldsymbol{x}}(t)$ where $\hat{\boldsymbol{x}}(t)$ denotes the system state estimate at the generic time instant t.

In this paper we propose an original procedure for the design of an asymptotic state observer that well fits within the present application whose main requirement is that of reconstructing the system state when external disturbances are acting on it, while the initial state may always be assumed known. More precisely, the observer gain matrix is computed so as to minimize the H_2 norm of the transfer function matrix between the estimate error and the external disturbance. The results of various numerical simulations show the efficiency of the proposed approach that also provides a good estimate of the system state derivatives.

Semiactive approximation

On the base of the previous analysis, we propose to choose as target for the semiactive control law $u_s(\cdot)$ the law $u_t(\cdot)$. Every Δt time units the controller should select the damper coefficient f in the set $[f_{\min}, f_{\max}]$ so as to minimize the quadratic difference among the semiactive and the target active control force. The value of Δt cannot be chosen arbitrarily, but its lower bound is imposed by the physical limits on the updating frequency of the damper coefficient f.

In this paper we introduce a significant variation with respect to previous works [5] so as to improve the efficiency of the resulting suspension system. In particular, taking into account the time Δt required to update f, we compute the value of f at the generic time instant t so as to minimize the quadratic difference $(u_t(t + \Delta t) - u_s(t + \Delta t))^2$. In such a way, as proved via various numerical simulations, we are able to compensate the delay on the updating of f, thus producing a significant improvement on the system behaviour, that is evaluated in terms

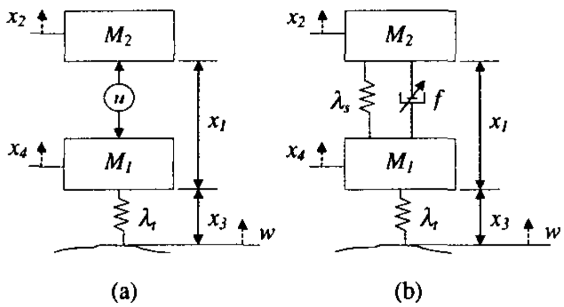

Figure 1: *Scheme of two degree-of-freedom suspension: (a) active suspension; (b) semiactive suspension.*

of the performance index J.

Note that this approach has been possible thanks to the efficiency of the proposed state observer that also provides a good estimate of the system state derivatives.

Different simulations have been carried out, considering the effect of input disturbances caused by the road profile and the effect of non–null initial conditions on the state. The results of these simulations show that the semiactive suspension performs reasonably well, and is a good approximation of the target active suspension, while it introduces significant improvements with respect to a completely passive suspension [4].

2 Dynamical model of the suspension system

Let us now consider the completely active suspension system with two degrees of freedom schematized in figure 1.a. We used the following notation: M_1 is the equivalent unsprung mass consisting of the wheel and its moving parts; M_2 is the sprung mass, i.e., the part of the whole body mass and the load mass pertaining to only one wheel; λ_t is the elastic constant of the tire, whose damping characteristics have been neglected. This is in line with almost all researchers who have investigated synthesis of active suspensions for motor vehicles as the tire damping is minimal; $x_1(t)$ is the deformation of the suspension with respect to (wrt) the static equilibrium configuration, taken as positive when elongating; $x_2(t)$ is the vertical absolute velocity of the sprung mass M_2; $x_3(t)$ is the movement of the unsprung mass wrt the static equilibrium configuration, taken as positive upwards. Under the assumption of flat road surface, this is also the deformation of the tire; $x_4(t)$ is the vertical absolute velocity of the unsprung mass M_1; $u(t)$ is the control force produced by the actuator; $w(t)$ is the function representing the disturbance. It coincides with the absolute vertical velocity of the point of contact of the tire with the road.

It is readily shown that the state variable mathematical model of the system under study is given by [2]

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t) + \boldsymbol{L}w(t) \qquad (1)$$

where $\boldsymbol{x}(t) = [x_1(t),\ x_2(t),\ x_3(t),\ x_4(t)]^T$ is the state, whereas the constant matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{L}$ have the following structure:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\lambda_t/M_1 & 0 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ 1/M_2 \\ 0 \\ -1/M_1 \end{bmatrix}, \ \boldsymbol{L} = \begin{bmatrix} 0 & 0 & -1 & 0 \end{bmatrix}.$$

The disturbance w is caused by the uneven road profile and is assumed to be a white noise signal, which is equivalent to saying that the longitudinal road profile x_0 can be represented by an integrated white noise [5, 8, 13]. Here, the road roughness characteristics are expressed by a signal whose PSD distribution function is [8]:

$$\Psi(\omega) = \frac{cV}{\omega^2 + \alpha^2 V^2} \qquad (2)$$

where $c = (\sigma^2/\pi)\alpha$. Here σ^2 denotes the road roughness variance and V the vehicle speed, whereas the coefficients c and α depend on the type of road surface. The product is the power spectrum of the white noise. The signal $x_0(t)$, whose PSD is given by (2), may be obtained as the output of a linear filter expressed by the differential equation [2]

$$\dot{x}_0(t) = -\alpha V x_0(t) + w(t). \qquad (3)$$

If we assume $\alpha^2V^2 << \omega^2$, we have $\Psi(\omega) = cV/\omega^2$ and $\dot{x}_0(t) = w(t)$, i.e., the road profile is integrated white noise.

3 Observer design via H_2 norm minimization

The control law we will design in the following section requires the knowledge of the system state $\boldsymbol{x}$. Since not every component of $\boldsymbol{x}$ is directly measured, we construct an appropriate state observer. To do this, we choose a suitable matrix $\boldsymbol{C}$ for the output equation

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t). \qquad (4)$$

If we assume

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (5)$$

which corresponds to measuring the suspension and the tire deformation, the observability of the pair $(\boldsymbol{A}, \boldsymbol{C})$ is ensured.

The asymptotic state observer we propose has the structure of a Luenberger observer, i.e., it takes the form

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{B}u(t) + \boldsymbol{K}_0(\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)) \qquad (6)$$

where $\hat{\boldsymbol{x}}(t)$ is the state estimate and $\hat{\boldsymbol{y}}(t) = \boldsymbol{C}\hat{\boldsymbol{x}}(t)$.

$\boldsymbol{K}_0$ is the gain matrix that has to be determined so as to impose the desired error dynamics:

$$\dot{\boldsymbol{e}}(t) = (\boldsymbol{A} - \boldsymbol{K}_0\boldsymbol{C})\boldsymbol{e}(t) + \boldsymbol{L}w(t). \qquad (7)$$

The gain matrix $\boldsymbol{K}_0$ may be chosen so as to impose a given set of eigenvalues to $(\boldsymbol{A} - \boldsymbol{K}_0\boldsymbol{C})$. Nevertheless, in the presence of external disturbances, as in the case at hand, this does not ensure a satisfactory behaviour. This motivates the non–standard procedure used in this paper for the design of the state observer, that is described in detail in the following.

Firstly, let us observe that we can always assume that the initial estimation error is null, i.e., $e(0) = \mathbf{0}$, being x_1 and x_3 measurable variables and x_2 and x_4 vertical velocities that are null at the very first time instant of evolution, when the car is motionless.

By virtue of this consideration, the Laplace–transform of the above equation (7) takes the form

$$E(s) = [sI - (A - K_0C)]^{-1}LW(s) \tag{8}$$

where I is the fourth order identity matrix, and $E(s)$ and $W(s)$ are the Laplace–transformation of $e(t)$ and $w(t)$, respectively.

Now, we determine the observer matrix K_0 by simply minimizing the H_2 norm[1] of the transfer function matrix

$$F(s) = [sI - (A - K_0C)]^{-1}L.$$

In such a way we can be sure that we are minimizing the effect of the disturbance on the error estimate.

Note that the resulting matrix K_0 does not guarantee a priori a satisfactory behaviour of the closed loop error dynamics in the presence of significant errors in the initial state estimate. Nevertheless, as already discussed above, this is not a problem in the particular case at hand where we can be sure that $e(0) = \mathbf{0}$ and the main requirement is that of reducing the effect of the disturbance on the state estimate, caused by the uneven road profile.

4 Semiactive suspension design

In this section we first discuss how the target active control law has been determined. Then we show how such a control law, that requires an actuator, may be approximated by a semiactive suspension, whose varying parameter is the characteristic coefficient of the damper f.

4.1 Target active control law

The design of the active suspension requires determining a suitable control law $u(\cdot)$ for system (1). To this end, we first determine the control law $u(\cdot)$ that minimizes a performance index of the form

$$J = \int_0^\infty (x^T(t)Qx(t) + ru^2(t))dt \tag{9}$$

where Q is positive semidefinite and $r > 0$. As well known from the literature [10], the solution of this problem can be easily computed by simply solving an algebraic Riccati equation, and takes the form of a feedback control law:

$$u(t) = -Kx(t). \tag{10}$$

[1]Let $G(s) : \mathbb{C} \to \mathbb{R}^{m,n}$ be a transfer function matrix. The H_2 norm of G is:

$$\| G \|_2 = \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} trace\{G^H(j\omega)G(j\omega)\}d\omega \right)^{1/2},$$

where ^{H} denotes complex conjugate transposition.

Obviously, when the system state is not directly measured, but is reconstructed via an asymptotic observer, the above control law is replaced by

$$u(t) = -K\hat{x}(t). \tag{11}$$

4.2 Semiactive approximation

In this section we show how the target control law u_t may be approximated by a semiactive suspension, whose varying parameter is the characteristic coefficient of the damper f.

In figure 1.b, we have represented a conventional semiactive suspension composed of a spring with elastic constant λ_s, and of a damper with adaptive characteristic coefficient f.

The effect of this suspension is equivalent [4] to that of a control force

$$u_s(t) = -\begin{bmatrix} \lambda_s & f & 0 & -f \end{bmatrix} x(t). \tag{12}$$

Note that, as f may vary, $u_s(t)$ is both a function of f and of $x(t)$.

In general, f can only take values in a real set $[f_{\min}, f_{\max}]$ and requires a certain time Δt to be updated, that is usually less than 10^{-2}s [5].

This implies that, if at the generic time instant t we determine a certain value of f, then such a value will only be imposed to f at the time instant $t + \Delta t$. In previous works the updated value of f is determined so as to minimize the quadratic difference $(u_t(t) - u_s(t))^2$ where $u_t(\cdot)$ is an appropriately selected control law [1, 5]. In such a way the delay on updating f never allows $u_s(\cdot)$ to be equal to $u_t(\cdot)$.

To overcome such a problem, in this paper we choose f so as to minimize the quadratic difference

$$\begin{aligned} &F[f, x(t + \Delta t), \hat{x}(t + \Delta t)] = \\ &(u_t(t + \Delta t) - u_s(t + \Delta t))^2 = \\ &(-K\hat{x}(t + \Delta t) + K_p x(t + \Delta t))^2 \end{aligned} \tag{13}$$

where $K_p = [\lambda_s \; f \; 0 \; -f]$.

Moreover, we make the realistic assumption that f varies linearly during the time interval Δt.

Let us first observe that the assumption of non–measurable state implies that instead of minimizing $F[f, x(t + \Delta t), \hat{x}(t + \Delta t)]$ as defined in equation (13), we have to minimize

$$\hat{F}[f, \hat{x}(t + \Delta t)] = (-K\hat{x}(t + \Delta t) + K_p\hat{x}(t + \Delta t))^2 . \tag{14}$$

Let us first assume that $\hat{x}_2(t + \Delta t) \neq \hat{x}_4(t + \Delta t)$; then the value $f^*(t)$ such that $\hat{F}[f^*(t), \hat{x}(t + \Delta t)] = 0$ is

$$f^*(t) = -\frac{K\hat{x}(t + \Delta t) - \lambda_s\hat{x}_1(t + \Delta t)}{\hat{x}_2(t + \Delta t) - \hat{x}_4(t + \Delta t)}. \tag{15}$$

It is easy to see that in this case

$$f(t) = \min \; \arg_{f \in [f_{\min}, f_{\max}]} \hat{F}[f, \hat{x}(t + \Delta t)] =$$

$$\begin{cases} f_{\max} & \text{if } \; f^*(t) > f_{\max} \\ f^*(k) & \text{if } \; f^*(t) \in [f_{\min}, f_{\max}] \\ f_{\min} & \text{if } \; f^*(t) < f_{\min} \end{cases} . \tag{16}$$

When $\hat{x}_2(t + \Delta t) = \hat{x}_4(t + \Delta t)$, regardless of the values of f the damper does not give any contribution to u_s. Thus, in this case we assume that if $t > \Delta t$ then $f(t) = f(t - \Delta t)$, i.e., the value of the damper coefficient is not updated, while if $t = 0$ we arbitrarily choose $f(0) = f_{\max}$.

Let us now discuss how the above updating procedure is made possible thanks to the presence of a good state observer. We first note that for sufficiently small values of Δt, like those of interest here, we may assume

$$\hat{\boldsymbol{x}}(t + \Delta t) \approxeq \hat{\boldsymbol{x}}(t) + \Delta t \cdot \dot{\hat{\boldsymbol{x}}}(t), \qquad (17)$$

and from equation (6)

$$\begin{aligned}\hat{\boldsymbol{x}}(t + \Delta t) &\approxeq (\boldsymbol{I} + \Delta t \cdot \boldsymbol{A}) \cdot \hat{\boldsymbol{x}}(t) + \\ &\Delta t \cdot \boldsymbol{B} u_s(t) + \Delta t \cdot \boldsymbol{K}_o \cdot (\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)) .\end{aligned} \qquad (18)$$

Thus, being $\boldsymbol{u}_s(t) \approxeq -\boldsymbol{K}_p \hat{\boldsymbol{x}}(t)$,

$$\begin{aligned}\hat{\boldsymbol{x}}(t + \Delta t) &\approxeq (\boldsymbol{I} + \Delta t \cdot \boldsymbol{A} - \Delta t \cdot \boldsymbol{B} \cdot \boldsymbol{K}_p) \cdot \hat{\boldsymbol{x}}(t) + \\ &\Delta t \cdot \boldsymbol{K}_o \cdot (\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)) ,\end{aligned} \qquad (19)$$

i.e., $\hat{x}(t + \Delta t)$ can be written as a function of known variables.

5 Application example

In this section we discuss the results of several simulations. First, however, we explain the choices we have made for the various parameters.

The proposed procedure has been applied to the quarter car suspension shown in figure 1, with values of the parameters taken from [13]: $M_1 = 28.58$Kg, $M_2 = 288.90$Kg, $\lambda_t = 155900$N/m. The value of $\lambda_s = 14345$N/m has been taken from [4].

To define completely the state equation (1) it is necessary to assume reference values for α and V. In the following we have taken $\alpha = 0.15\text{m}^{-1}$ and $V = 30$m/s. These values of α and V have been used in [8] to describe an asphalt road profile.

The matrices $\boldsymbol{Q}$ and r of the performance index J have been taken from [13] and are the same as those already used in [4, 5]:

$$\boldsymbol{Q} = diag\{1, 10, 1, 10\}, \; r = 0.8 \cdot 10^{-9}.$$

Thus, the resulting feedback control matrix is

$$\boldsymbol{K} = \begin{bmatrix} 35355 & 4827 & -21879 & -1386 \end{bmatrix}.$$

For the computation of the observer matrix we used the software tools available in Matlab: `fmins` is the minimization procedure and `normh2` computes the H_2 norm. We determined

$$\boldsymbol{K}_o = \begin{bmatrix} 29.72 & -560.73 & -11.52 & -1341.70 \\ -0.25 & 2.45 & 370.91 & -5449.00 \end{bmatrix}.$$

Finally, we have taken $\Delta t = 0.7 \cdot 10^{-2}$s, $f_{\min} = 800$Ns/m and $f_{\max} = 3000$Ns/m.

To show the performance of our semiactive suspension design, we have simulated two different situations.

5.1 Simulation 1

In the first simulation we considered null initial conditions, i.e., $\boldsymbol{x}(0) = \hat{\boldsymbol{x}}(0) = \boldsymbol{0}$ and we assumed that the only disturbance acting on the system is caused by an asphalt road profile, whose characteristic parameters are given above.

The results of this simulation are shown in figure 2.

Figure (a) shows the road profile x_0 (thin line) along with the unsprung mass displacement $x_3 + x_0$ (thick line). Figure (b) shows the road profile x_0 (thin line) along with the sprung mass displacement $x_1 + x_3 + x_0$ (thick line). It is possible to observe that the semiactive suspension filters the high frequencies smoothing the movement of the sprung mass.

Figure (c) compares the sprung mass displacement in the case of the semiactive suspension (thick line) and in the case of a completely passive suspension (thin line). In particular, we considered the passive suspension system proposed in [4] where all characteristic parameters are the same as those used in the semiactive design, apart from the damper coefficient that has been chosen constant and equal to $f = 1.918$Ns/m $\in [f_{\min}, f_{\max}]$. It is immediate to observe the significant improvements deriving from adapting f.

Figure (d) compares the sprung mass displacement in the case of the semiactive suspension (thick line) and in the case of the target active suspension (thin line). As it can be noted, the two curves are practically coincident, thus proving the satisfactory behaviour of the proposed design.

Figure (e) compares the target force (thin line) with the control force produced by the semiactive suspension (thick line). We can observe that the variation of f guarantees a satisfactory approximation.

Figure (f) shows the value of the damper coefficient f as a function of time.

Figures (g) – (l) show the efficiency of the proposed state observer that provides a good evaluation of both the state variables and their derivatives. As an example, in figure (g) we have reported the evolution of the first state variable x_1, while in figure (h) we have reported the evolution of its error estimate $e_1 = x_1 - \hat{x}_1$. Figure (i) shows the evolution of $\dot{x}_1$, while $\dot{e}_1 = \dot{x}_1 - \dot{\hat{x}}_1$ is reported in figure (l).

Finally, to prove the efficiency of the procedure used for the updating of f, we provide the values of the performance index J in the case of the proposed semiactive design and in the case of a semiactive suspension designed without taking into account such a delay and using the same target control law. In the first case $J = 0.95$, while in the second case $J = 0.98$, i.e., we have an improvement of the order of 3%.

5.2 Simulation 2

In the second simulation we considered an initial state different from zero and no disturbance. We assumed $\boldsymbol{x}(0) = [0.2\ 0\ 0.02\ 0]^T$.

The results of this simulation are shown in figure 3.

Figures (a) and (b) compare the unsprung and sprung mass displacement of the semiactive suspension (thick lines) with that of a completely passive one (thin lines). As we can note in both cases the semiactive system guarantees better performances that the passive one [4].

Figure (c) shows the comparison between the displacement of the sprung mass in the case of the semiactive suspension (thick line) and in the case of the target active suspension (thin line).

Figure (d) compares the target force (thin line) with the control force produced by the semiactive suspension (thick line) that represents a satisfactory approximation of the previous one.

Finally, as in the previous case, we compare the values of J corresponding to our procedure with the value of J computed

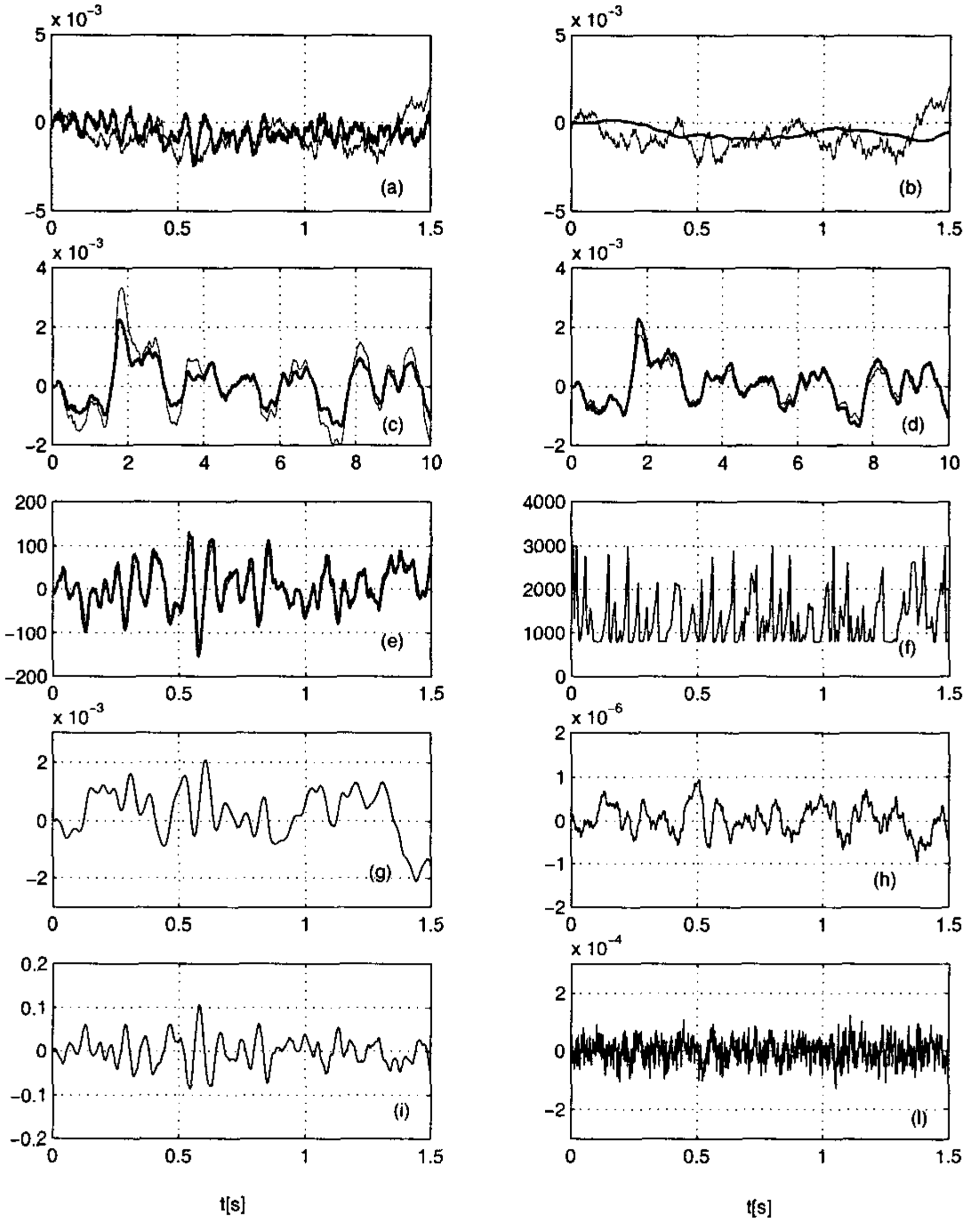

Figure 2: *The results of Simulation 1.*

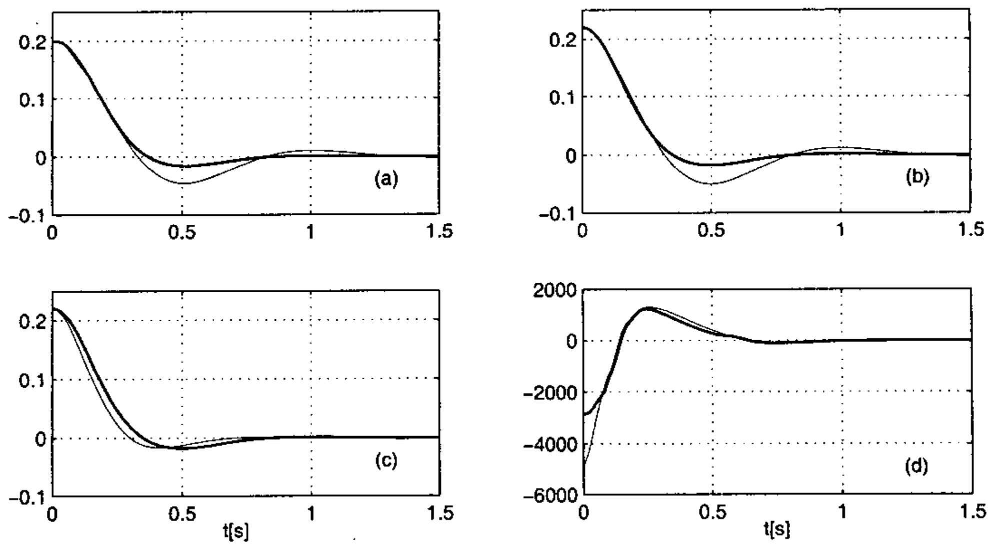

Figure 3: *The results of Simulation 2.*

in the case of a semiactive suspension designed without taking into account the delay on the updating of f. In the first case $J = 1.70$, while in the second case $J = 1.85$, i.e., our approach produces an improvement of the order of 10%.

6 Conclusions

This paper presents a two–phase design technique for semiactive suspensions.

The first phase of the project requires the design of an asymptotic state observer that has been computed by minimizing the H_2 norm of the transfer function matrix among the error state estimate and the external disturbance. Then, the target active control law has been obtained by solving an LQR problem.

In the second phase, this target is approximated by controlling the damper coefficient of the semiactive suspension. The novelty with respect to previous works is that we considered the delay Δt required for the updating of f, and we assumed that the new value of f is chosen so as to minimize the difference between the target and the semiactive control law at the time instant $t + \Delta t$, so as to be sure that when the computed value of f is really imposed, then the semiactive force is as close as possible to the target one.

The results of several numerical simulations shown that the use of a semiactive suspension leads to minimal loss with respect to the performance of the target active suspension, while the updating procedure produces a significant improvement in terms of the performance index J.

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