OPTIMAL TOKEN ALLOCATION IN TIMED CYCLIC EVENT–GRAPHS*

Alessandro Giua, Aldo Piccaluga, Carla Seatzu
Department of Electrical and Electronic Engineering, University of Cagliari, Italy
giua@diee.unica.it

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Abstract *In this paper we deal with the problem of allocating a given number of tokens in a cyclic timed event graph (CTEG) so as to maximize the firing rate of the net. We propose two different procedures, both involving the solution of a mixed integer linear programming problem. The first one needs the knowledge of the elementary cycles, thus it is convenient only for those classes of CTEGs whose number of elementary cycles is limited by the number of places, like kanban systems. On the contrary, the second one enables us to overcome this difficulty, thus providing an efficient tool for the solution of allocation problems in complex manufacturing systems like job–shop systems.*

Introduction

Cyclic timed event–graphs (CTEG) are a special class of timed ordinary Petri nets. They are often used for modeling and analyzing manufacturing systems assuming a cyclic manufacturing of the parts, since it has been shown that choice–free job–shop, kanban systems, and assembly systems, can be modeled using event graphs. In the case of deterministic CTEGs it is possible to evaluate the steady state performance of the net in terms of its *cycle time*.

In this paper we deal with the problem of allocating a given number of tokens in a CTEG so as to maximize the *firing rate* (i.e., the inverse of the cycle time) of the net. Note that both the initial marking *and* the firing rate are decision variables in this approach. This problem has a practical relevance: as an example, in the manufacturing domain

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it corresponds to determining the optimal allocation of a finite set of resources so as to maximize the throughput.

Many different optimization problems in the setting of CTEGs have been studied in the literature. We distinguish among problem statements requiring the knowledge of all elementary cycles and problem statements where this knowledge is not necessary. The solutions proposed by Hillion and Proth (Hillion and Proth, 1989), and Di Febbraro *et al.* (Di Febbraro et al., 1997) belong to the first class; on the contrary, the solutions proposed by Campos *et al.* (Campos et al., 1992), Nakamura and Silva (Nakamura and Silva, 1999), Magott (Margott, 1984), Morioka and Yamada (Morioka and Yamada, 1991), and Laftit *et al.* (Laftit et al., 1992), belong to the second class.

In (Giua et al., 2000), we considered a simplified version of the allocation problem we deal with in this paper. In fact, we defined a special class of allocations and we proved that for this class the firing rate is a *generalized smooth* performance index. Then, following Panayiotou and Cassandras (Panayiotou and Cassandras, 1999), we proved that whenever a performance index is generalized smooth, an incremental optimization procedure — that adds one token at a time — can be used to compute the optimal allocation.

In this paper we study the same problem in a more general setting, posing no restriction on the class of allocations considered. We derive two different approaches to solve the optimal allocation problem in this general case.

The first procedure involves the solution of a mixed ILPP and is based on the knowledge of all elementary cycles. Thus, it is convenient for those classes of CTEGs where the number of elementary cycles does not increase exponentially with the size of the net, such as kanban–systems where the number of elementary cycles is limited by the number of places.

The second procedure requires solving a mixed ILPP. It does not need the knowledge of the elementary cycles and the constraint set only involves the computation of the incidence matrix, thus resulting to be efficient for all classes of CTEGs.

1. BACKGROUND

In this section we recall the formalism used in the paper. For more details on Petri nets and CTEGs we refer to (Proth et al., 1993); (Hillion and Proth, 1989); (Laftit et al., 1992); (Murata, 1989); (Panayiotou and Cassandras, 1999).

A *Place/Transition net* (P/T net) is a structure $N = (P, T, \boldsymbol{Pre}, \boldsymbol{Post})$, where P is a set of n places; T is a set of m transitions; $\boldsymbol{Pre} : P \times T \to \mathbb{N}$ and $\boldsymbol{Post} : P \times T \to \mathbb{N}$; $\boldsymbol{C} = \boldsymbol{Post} - \boldsymbol{Pre}$ is the incidence matrix.

A *marking* is a vector $\boldsymbol{M} : P \to \mathbb{N}$ (M_i is the marking of place p_i).

A *P/T system* or *net system* $\langle N, M_0 \rangle$ is a net N with an initial marking $\boldsymbol{M}_0$.

A transition t is enabled at $\boldsymbol{M}$ if $\boldsymbol{M} \geq \boldsymbol{Pre}(\cdot\,, t)$ and may fire yielding the marking $\boldsymbol{M}' = \boldsymbol{M} + \boldsymbol{C}(\cdot\,, t)$.

A P/T net is called *ordinary* when all of its arc weights are 1's.

An *event graph* is an ordinary Petri net such that each place p has exactly one input transition and exactly one output transition.

We define an *elementary circuit* in a strongly connected event graph as a directed path that goes from one node back to the same node, while any other node is not repeated. A strongly connected event graph is also called *cyclic* because each node belongs to a cycle. In a cyclic event graph the total number of tokens in any elementary circuit is invariant by transition firing (Commoner et al., 1971).

A deterministic Timed P/T net is a pair (N, τ), where N is a standard P/T net, and $\tau : T \to \mathbb{R}^+$, called release delay, assigns a positive fixed firing duration to each transition. We consider an infinite–server semantics, i.e., we assume that each enabled transition can fire as many times as its enabling degree.

For deterministic timed cyclic event graphs we can compute, for any elementary circuit γ, the following ratio called the *cycle time* of the circuit: $c_\gamma = \mu_\gamma / x_\gamma$, where μ_γ denotes the sum of the firing times related to the transitions belonging to γ, and x_γ the number of tokens circulating in γ. We assume $\mu_\gamma > 0 \ \forall \ \gamma$.

Let Γ represent the set of elementary circuits of a cyclic event graph and $\hat{c} = \max_{\gamma \in \Gamma} c_\gamma$. Any $\gamma \in \Gamma$ such that $c_\gamma = \hat{c}$ is a *critical circuit.* These circuits are the ones that actually bind the speed of the system. Under an operational mode where transitions fire as soon as they are enabled, the *firing rate* of each transition in steady state is given by $\varrho = 1/\hat{c}$. As a consequence, if we want to increase the speed (i.e., the firing rate of the system), we have to add one (or several) token(s) to the critical circuits. Adding tokens in other circuits would be worthless.

2. PROBLEM STATEMENT

Let us consider a timed cyclic event graph with n places, m transitions and ℓ elementary circuits. We associate to each elementary circuit γ an n dimensional vector $\boldsymbol{a}_\gamma$ of zeros and ones. In particular, $a_\gamma(i) = 1$ if

$p_i \in \gamma$, $a_\gamma(i) = 0$ otherwise. Thus $\boldsymbol{a}_\gamma^T \boldsymbol{M}$ is the number of tokens in γ and $\mu_\gamma / (\boldsymbol{a}_\gamma^T \boldsymbol{M})$ is the cycle time of circuit γ.

We assume that tokens may only be allocated within a given subset of places $P_a \subseteq P$, while the number of tokens in all places in $P_r = P \setminus P_a$ is given. We denote as q the cardinality of P_a, and $r = n - q$ the cardinality of P_r. For simplicity of presentation, we assume that place labeling is such that $P_a = \{p_i \,|\, i = 1, \cdots, q\}$ and $P_n = \{p_i \,|\, i = q+1, \cdots, n\}$, thus a marking can be written as $\boldsymbol{M} = [\boldsymbol{M}_a^T \boldsymbol{M}_r^T]^T$, where $\boldsymbol{M}_a \in \mathbb{N}^q$ and $\boldsymbol{M}_r \in \mathbb{N}^r$. Finally, let Γ_a be the set of elementary circuits that contain at least a place in P_a, i.e., $\Gamma_a = \{\gamma \in \Gamma \mid \gamma \cap P_a \neq \emptyset\}$.

In this paper we shall deal with the problem of allocating a given number of tokens in P_a so as to maximize the firing rate of the net. We also assume that the allocation must satisfy a given set of s' linear inequalities each one of the form $\boldsymbol{g}^T \boldsymbol{M}_a \leq k$. Any admissible allocation thus must satisfy:

$$\begin{cases} (a) & \boldsymbol{M}_r = \boldsymbol{M}_{r,0} \\ (b) & \boldsymbol{G}\boldsymbol{M}_a \leq \boldsymbol{k} \end{cases} \tag{1}$$

where $\boldsymbol{M}_{r,0} \in \mathbb{N}^r$, $\boldsymbol{G} \in \mathbb{Z}^{s' \times q}$, and $\boldsymbol{k} \in \mathbb{Z}^{s'}$ are given. Constraints (a) express the fact that the marking of all places in P_r is assigned. Each equation in (b) may either express an upper/lower bound on the number of tokens in a place $p \in P_a$, or an upper/lower bound on the number of tokens in a circuit $\gamma \in \Gamma_a$ or in a generic subset of places in P_a. Generalizing, our optimization problem can be formally written as a nonlinear integer programming problem of the form:

$$\begin{cases} \max J = \min\limits_{\gamma \in \Gamma} \dfrac{\boldsymbol{a}_\gamma \boldsymbol{M}}{\mu_\gamma} \\ s.t. \quad \boldsymbol{A}\boldsymbol{M} \leq \boldsymbol{b} \end{cases} \tag{2}$$

where $\boldsymbol{M} \in \mathbb{N}^n$ is the unknown variable, and $\boldsymbol{A} \in \mathbb{Z}^{s \times n}$, and $\boldsymbol{b} \in \mathbb{Z}^s$ are given.

3. MAIN RESULTS

A special case of problem (2) has already been studied by the authors in (Giua et al., 2000). In particular, we considered a special class of allocation problems where one has to allocate a given number K of tokens (i.e., we had just one constraint of the form 1.b) and the set of places P_a was given so as to satisfy the following assumption.

[A1] If γ and γ' are two elementary circuits sharing a place in P_a, then they must have the same set of places in P_a, i.e.,$(\exists p \in P_a)\ p \in \gamma \cap \gamma' \implies \gamma \cap P_a = \gamma' \cap P_a$.

In (Giua et al., 2000) we proved that if assumption [A1] is satisfied then the optimal allocation can be efficiently computed with an incremental procedure that adds one token at a time. In this paper we want to consider more general allocation problems that may not satisfy assumption [A1]. More precisely, we propose two different solutions to the allocation problem (2) whose validity is not related to the chosen set of places P_a.

3.1. FIRST PROCEDURE

The first procedure we propose involves the solution of a mixed ILPP and is based on the knowledge of the elementary cycles. As it is well known, such an assumption is often unrealistic, thus making it not always useful in real applications.

The mixed ILPP formulation originates from the following folk theorem 1 that needs not to be proven.

Theorem 1. *Consider the two programming problems:*

$$\begin{cases} \max J_I = \min\limits_{i=1,\cdots,p} \{\boldsymbol{c}_i^T \boldsymbol{x}\} \\ s.t. \quad \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b} \end{cases} \tag{3}$$

with integer variables $\boldsymbol{x} \in \mathbb{N}^N$ *and*

$$\begin{cases} \max J_{II} = \beta \\ s.t. \quad \boldsymbol{c}_i^T \boldsymbol{y} - \beta \geq 0, \qquad i = 1, \cdots, p, \\ \qquad\ \ \boldsymbol{A}\boldsymbol{y} \leq \boldsymbol{b} \end{cases} \tag{4}$$

with integer variables $\boldsymbol{y} \in \mathbb{N}^N$ *and real variable* $\beta \in \mathbb{R}^+$.

Here $\boldsymbol{c}_i \in \mathbb{R}_0^N$, $i = 1, \cdots, p$, $\boldsymbol{A} \in \mathbb{R}^{s \times N}$, *and* $\boldsymbol{b} \in \mathbb{R}^s$ *are given.*

Then $\boldsymbol{x}^*$ *is an optimal solution of (3) with performance index* J_I^* *iff* $(\boldsymbol{x}^*, J_I^*)$ *is an optimal solution of (4).*

Proposition 2. The optimal solution $(\boldsymbol{M}^*, \beta^*)$ of the mixed ILPP:

$$\begin{cases} \max \beta \\ s.t. \quad \boldsymbol{a}_\gamma^T \boldsymbol{M}/\mu_\gamma - \beta \geq 0, \qquad \gamma \in \Gamma, \\ \qquad\ \ \boldsymbol{A}\boldsymbol{M} \leq \boldsymbol{b} \end{cases} \tag{5}$$

with variables $\boldsymbol{M} \in \mathbb{N}^n$, $\beta \in \mathbb{R}^+$, provides the optimal solution $\boldsymbol{M}^*$ and the corresponding optimal performance index value $J^* = \beta^*$ of the nonlinear integer programming problem (2). ■

Using this new formulation, we have to solve a simpler mixed ILPP with $n + 1$ variables and $\ell + s$ constraints.

Obviously, the main drawback of the above procedure lies in the requirement of computing all elementary cycles.

3.2. SECOND PROCEDURE

In this subsection we propose another solution to our allocation problem (2) that still involves the solution of a mixed ILPP, but presents two main advantages with respect to the previous one: (a) it does not require the computation of the elementary circuits, and (b) the number of constraints in the LPP is equal to $n+s$.

It is inspired by a result firstly proposed by Magott in (Margott, 1984), where he dealt with the problem of determining the cycle time of an event–graph, given the initial marking $\boldsymbol{M}_0$:

$$\left\{\begin{array}{ll} \max \dfrac{\boldsymbol{a}_\gamma^T \boldsymbol{Pre}\,\boldsymbol{\theta}}{\boldsymbol{a}_\gamma^T \boldsymbol{M}_0} & \\ & \\ s.t. & \boldsymbol{a}_\gamma^T \boldsymbol{C} = \boldsymbol{0} \\ & \boldsymbol{a}_\gamma \geq \boldsymbol{0} \end{array}\right. \tag{6}$$

where $\boldsymbol{Pre}^T \boldsymbol{a}_\gamma$ is the characteristic vector of the set of transitions that belong to cycle γ, and $\boldsymbol{\theta} \in \mathbb{N}^m$ is the vector containing all firing times of timed transitions (recall that $m = |T|$).

The two constraints in problem (6) force $\boldsymbol{a}_\gamma$ to be a P–invariant, i.e., $\boldsymbol{a}_\gamma$ represents (but for a scalar factor) the characteristic vector of the places along a cycle.

Magott (Margott, 1984) also observed that, the same optimal solution of (6) can also be obtained by means of the following LPP:

$$\left\{\begin{array}{ll} \max \boldsymbol{a}_\gamma^T \boldsymbol{Pre}\,\boldsymbol{\theta} & \\ s.t. & \boldsymbol{a}_\gamma^T \boldsymbol{C} = \boldsymbol{0} \\ & \boldsymbol{a}_\gamma^T \boldsymbol{M}_0 = 1 \\ & \boldsymbol{a}_\gamma \geq \boldsymbol{0}. \end{array}\right. \tag{7}$$

whose dual problem is:

$$\left\{\begin{array}{ll} \min v & \\ s.t. & \boldsymbol{C}\boldsymbol{z} + v\boldsymbol{M}_0 \geq \boldsymbol{Pre}\,\boldsymbol{\theta} \end{array}\right. \tag{8}$$

where the optimal value of the variable $v \in \mathbb{R}^+$ is the cycle time and the unconstrained vector $\boldsymbol{z} \in \mathbb{R}^m$ has no physical meaning.

Now, let us consider problem (8). This problem can be easily converted into the problem of determining the optimal firing rate of the net, given the initial marking. For this purpose we only need to replace v with its inverse $\beta = 1/v$, thus obtaining:

$$\left\{\begin{array}{ll} \max \beta & \\ s.t. & \boldsymbol{C}(\beta \boldsymbol{z}) + \boldsymbol{M}_0 \geq \boldsymbol{Pre}\,\boldsymbol{\theta}\ \beta \end{array}\right. \tag{9}$$

where $\beta \in \mathbb{R}^+$, and $\beta \boldsymbol{z} \in \mathbb{R}^m$, i.e.,

$$\begin{cases} \max \beta \\ s.t. \quad \boldsymbol{C}\boldsymbol{y} - \boldsymbol{Pre}\,\boldsymbol{\theta}\,\beta \geq -\boldsymbol{M}_0 \end{cases} \tag{10}$$

where $\boldsymbol{y} \in \mathbb{R}^m$ and $\beta \in \mathbb{R}^+$ are the new variables.

Finally, if we assume, as in problem (2), that $\boldsymbol{M}_0$ is not known but must satisfy a set of given inequalities, we have the following result.

Proposition 3. The optimal solution $(\boldsymbol{M}^*, \beta^*, \boldsymbol{y}^*)$ of the mixed ILPP:

$$\begin{cases} \max \beta \\ s.t. \quad \boldsymbol{C}\boldsymbol{y} - \boldsymbol{Pre}\,\boldsymbol{\theta}\,\beta + \boldsymbol{M} \geq \boldsymbol{0} \\ \qquad\;\; \boldsymbol{A}\boldsymbol{M} \leq \boldsymbol{b} \end{cases} \tag{11}$$

with variables $\boldsymbol{M} \in \mathbb{N}^n$, $\beta \in \mathbb{R}^+$, and $\boldsymbol{y} \in \mathbb{R}^m$, provides the optimal solution $\boldsymbol{M}^*$ and the corresponding optimal performance index value $J^* = \beta^*$ of the nonlinear integer programming problem (2). ■

In this way our optimization problem has been reduced to the solution of a mixed ILPP with $n+m+1$ variables and $n+s$ constraints. Obviously, in the most general cases, i.e., when the number of elementary cycles is larger than the number of places, the approach herein proposed is computationally more convenient with respect to the other one discussed in the previous subsection.

3.3. A JOB–SHOP EXAMPLE

In this subsection we deal with an example taken from the literature (Proth et al. 1997). We consider a job–shop composed of four machines $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$ and $\mathcal{M}_4$, which can manufacture three products denoted by $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$. The production mix is 25%, 25%, 50% for $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$, respectively. The production processes of the products and the corresponding cycles (the cycle for $\mathcal{R}_3$ is repeated) are:

$$\begin{array}{lll} \mathcal{R}_1: & (\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4) & \{p_1, t_1, p_9, t_2, p_{10}, t_3, p_{11}, t_4\} \\ \mathcal{R}_2: & (\mathcal{M}_1, \mathcal{M}_4, \mathcal{M}_3) & \{p_2, t_5, p_{12}, t_6, p_{13}, t_7\} \\ \mathcal{R}_3: & (\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_4) & \{p_3, t_8, p_{14}, t_9, p_{15}, t_{10}\} \\ & & \{p_4, t_{11}, p_{16}, t_{12}, p_{17}, t_{13}\}. \end{array}$$

Here the number of tokens in each product cycle represents the number of available pallets for that product.

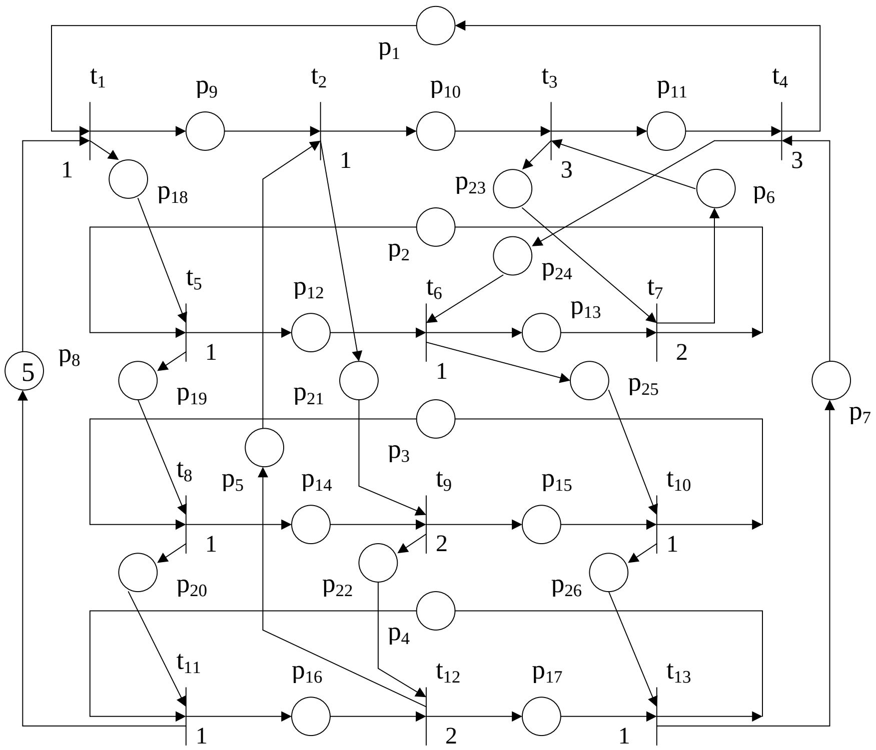

Figure 1 Event graph model of the job–shop.

The fixed sequencing of the part types on the machines and the corresponding cycles are:

$$\begin{array}{lll} \mathcal{M}_1: & (\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \mathcal{R}_3) & \{p_8, t_1, p_{18}, t_5, p_{19}, t_8, p_{20}, t_{11}\} \\ \mathcal{M}_2: & (\mathcal{R}_1, \mathcal{R}_3, \mathcal{R}_3) & \{p_5, t_2, p_{21}, t_9, p_{22}, t_{12}\} \\ \mathcal{M}_3: & (\mathcal{R}_1, \mathcal{R}_2) & \{p_6, t_3, p_{23}, t_7\} \\ \mathcal{M}_4: & (\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \mathcal{R}_3) & \{p_7, t_4, p_{24}, t_6, p_{25}, t_{10}, p_{26}, t_{13}\}. \end{array}$$

Here the number of tokens in each machine cycle represents the number of available servers for that machine.

The event graph representative of this system is sketched in figure 1. It is a strongly connected event graph with $n = |P| = 26$ and $m = |T| = 13$. We have also computed that there are 76 elementary cycles, thus the second proposed procedure better fits the solution of allocation problems of the form (2).

We assume that $P_a = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$ where the number of tokens in p_1, p_2, p_3 and p_4 represent the number of free pallets that must be optimally allocated; while the number of tokens in p_5, p_6 and p_7 represent the number of servers that has to be optimally distributed between machines $\mathcal{M}_2$, $\mathcal{M}_3$ and $\mathcal{M}_4$.

$P_r = P \setminus P_a = \{p_8, \cdots, p_{26}\}$ and we assume that: (a) the marking of place p_8 is 5 (i.e., $\mathcal{M}_1$ is the only machine that has a fixed number of servers, that is equal to 5); (b) the marking of all other places in P_r is zero (i.e., no part is initially being worked in the job-shop).

Holding times of transitions are reported in figure 1, as well as the marking of all places in P_r.

Now, let us consider the following optimization problem:

$$\begin{cases} \max J = \min\limits_{\gamma\in\Gamma} \dfrac{\boldsymbol{a}_\gamma \boldsymbol{M}}{\mu_\gamma} \\ s.t. \quad M_1 + M_2 + M_3 + M_4 \leq k_1 \\ \qquad M_5 + M_6 + M_7 \leq k_2 \\ \qquad M_8 = 5 \\ \qquad M_i = 0, \quad i = 9, \cdots, 26, \end{cases}$$

and let $k_1 = 100$ and $k_2 = 20$, i.e., we want to determine the optimal token allocation when the number of pallets is equal to 100, machine $\mathcal{M}_1$ has 5 servers and the global number of servers available to machines $\mathcal{M}_2$, $\mathcal{M}_3$ and $\mathcal{M}_4$ is equal to 20.

Now, problems of the form (5) and (11) can be immediately formulated. In accordance with the previous notation we have $r = 19$ and $s' = 2$. Thus, in the first case the number of variables is equal to 27 and the number of constraints is equal to $76 + 19 + 2 = 97$. In the second case, there are $13+26+1 = 40$ variables and $26+19+2 = 47$ constraints.

As expected, both procedures determine the same optimal firing rate $J^* = 1$, while the optimal allocations are different. The optimal token allocation computed using the first procedure is $M_1 = 6,\ M_2 = M_3 = M_4 = 10,\ M_5 = 5,\ M_6 = 9,\ M_7 = 6$, while the optimal token allocation computed using the second procedure is $M_1 = 6,\ M_2 = M_4 = 10,\ M_3 = 74,\ M_5 = 5,\ M_6 = 9,\ M_7 = 6$.

In this case, the second procedure provides a solution that allocates to machine $\mathcal{M}_3$ a number of pallets significantly greater than that computed with the first procedure. This is not a drawback of the second approach: it is due to the fact that when more than one optimal solution exists, the ILPP solver just stops when the first one is found.

4. CONCLUSIONS

In this paper we have dealt with deterministic timed cyclic event–graphs. We have discussed the problem of allocating a given number of tokens in a CTEG so as to maximize the firing rate of the net. We assumed that tokens can only be allocated within a given subset of places P_a, while the marking of all other places is assigned. Linear constraints on the marking of all places in P_a are also taken into account.

The novel contribution of this paper consists in the formulation of two mixed integer LPPs. The first one needs the knowledge of the elementary circuits, thus making it not useful for all classes of CTGEs. The second one overcome this difficulty and reveals to be efficient for analyzing complex manufacturing systems like job–shop systems.

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