Petri Net Monitor Design with Control and Observation Costs

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Abstract

The classical partition of the event set into controllable and uncontrollable events from supervisory control theory is replaced by introducing the concept of control and observation cost of an event. This leads naturally to consider an optimal control problem for a given logical control specification. Here the case of generalized mutual exclusion constraint is considered for a Petri net plant. It has been shown that a constraint of this kind may be enforced via a monitor place. In this paper we propose an integer programming approach to synthesize the optimal monitor so as to minimize a given cost.

1 Introduction

Supervisory control theory for discrete event systems (DESs) was initiated by Ramadge and Wonham [1]. In their seminal work they represent both the plant — i.e., the system to be controlled — and the desired closed-loop behaviour, by regular languages. The specific problem addressed was to synthesize a controller, called *supervisor*, to achieve the largest subset of the desired language, disabling or enabling controllable events. The unwanted sequences may be related, for example, to safety requirements. Although regular languages have been an useful framework to start such DES control theory, they are limited in representing systems consisting of numerous interacting subsystems. For this reason, a control theory for DES modeled by Petri Net (PN) has been developed, extending general PN models with the concept of *controllable* transitions.

In the supervisory control PN theory it is assumed that the set of transitions T of a net is partitioned into two disjoint subsets: T_{uc}, the set of uncontrollable transitions, and T_c, the set of controllable transitions. Similarly T may also be partitioned into the set T_{uo} of unobservable transitions, and the set T_o of observable transitions.

A controllable transition may be disabled by the supervisor, a controlling agent which ensures that the behaviour of the system be within a legal behaviour. When the controller is modeled by a PN structure, the disabling of transition t is possible if there is a pre-arc from a controller place to t. An uncontrollable transition represents an event which may not be prevented from occurring by a supervisor and thus we require that no arc goes from a controller place to it.

Dually, when the controller is modeled by a PN structure, the controller observes a transition t only if the firing of t changes the marking of a controller place p. This happens only if the number of pre-arcs from p to t is different from the number of post-arcs from t to p. To rule out this possibility we will require neither a pre-arc nor a post-arc may exist between a controller place and an unobservable transition (in the monitor control structure we consider self-loops are not allowed).

Here we consider the problem of forbidden state specification represented by *generalized mutual exclusion constraint* (GMEC) of the form $(\boldsymbol{l}, k)$. Such a constraint limits the weighted sum of tokens in a subset of places [3, 4, 5, 7]: the set of legal plant markings is $\{\boldsymbol{m} \mid \boldsymbol{l} \cdot \boldsymbol{m} \leq k\}$.

It was shown [3, 5] that it is possible to impose a GMEC by adding to a net a controller that takes the form of a single place called *monitor* with arcs going to and coming from the plant transitions. The monitor synthesis is very efficient from the computational point of view and it represents a compiled supervisor.

When the monitor has arcs going to uncontrollable (going to or coming from unobservable) transitions we say that the monitor and the corresponding GMEC are uncontrollable (unobservable).

It has been shown [6] that given a constraint $(\boldsymbol{l}, k)$, any constraint $(\boldsymbol{l}', k')$ where $\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l}$ — the elements of vector $\boldsymbol{r}_1$ and scalar r_2 are non negative integers — and k' is suitably chosen, is more restrictive than $(\boldsymbol{l}, k)$, i.e., $\{\boldsymbol{m} \mid \boldsymbol{l}' \cdot \boldsymbol{m} \leq k'\} \subseteq \{\boldsymbol{m} \mid \boldsymbol{l} \cdot \boldsymbol{m} \leq k\}$. Thus if $(\boldsymbol{l}, k)$ is not controllable (or not observable) we may look for a more restrictive but controllable and observable GMEC. Note that as the number of nonzero elements $\boldsymbol{r}_1$ increases the constraint becomes more restrictive.

We consider a generalization of this approach in which two function $\boldsymbol{z}_c : T \to \mathbb{R}^+$ and $\boldsymbol{z}_o : T \to \mathbb{R}^+$ associate to each transition t its control and observation cost, respectively. As a particular case, if the cost functions only take value in the binary set $\{0, \infty\}$ we go back to the controllable/uncontrollable and observable/unobservable case.

In this framework we consider different problems.

The first problem is the following: given a GMEC $(\boldsymbol{l}, k)$, we want to find, among all monitors that enforce the constraint the one that has minimal cost. The set of the all monitors that enforce this constraint is clearly the set of all monitors corresponding to GMECs that are more restrictive than $(\boldsymbol{l}, k)$, and that can be written using Moody's parameterization. The cost corresponding to a monitor p_s is given by the sum over t of $\boldsymbol{c}^-(p_s,t)\boldsymbol{z}_c(t)+\boldsymbol{c}^+(p_s,t)\boldsymbol{z}_o(t)$, where $\boldsymbol{c}^-(p_s,t)$ counts the arcs from p_s to t and $\boldsymbol{c}^+(p_s,t)$ counts the arcs from t to p_s. This problem can be easily framed as a integer-linear programming problem.

In this first case, the cost associated to the control and observation of a transition t depends on the number of arcs going to and coming from t. This make sense if the control and observation actions are associated with physical actions like a material flow or a power signal. In many cases, however, the control and observation actions only involve logical actions. In this case what we want to model with the control and observation cost of a transition is not the cost of the single action of enabling or detecting the event associated with the transitions — in this case it is negligible because it represents the cost of setting to one or zero a bit — but the cost of the device and its installation in order to perform these actions (sensor, network connection, etc.). To model this case we can use the same approach of before but with an objective function that is the sum over t of $\text{sign}(\boldsymbol{c}^-(p_s,t))\boldsymbol{z}_c(t) + \text{sign}(\boldsymbol{c}^+(p_s,t))\boldsymbol{z}_o(t)$. In this case the problem has a non linear objective function whose solution, however, can still be easily computed using standard optimization tool.

Finally, we also consider the possibility of imposing a trade-off between cost of the control and the restrictions imposed by the monitor. To do this we add to the objective function to be minimized a weighted sum of the elements of $\boldsymbol{r}_1$.

This paper is structured as follows. In section 2 we provide some technical background on Petri nets. Previous results on the monitor based controller synthesis are recalled in section 3. The optimal control problem for a GMEC in presence of control and observation cost is then presented in section 4 for three different objective functions and it is illustrated via two simple examples. Conclusions are drawn in section 5.

2 Background

A place/transition (P/T) net is a structure $N = \langle P, T, \mathbf{Pre}, \mathbf{Post}\rangle$ where: P is a set of m *places* represented by circles; T is a set of n *transitions* represented by bars; $P \cap T = \emptyset$, $P \cup T \neq \emptyset$; $\mathbf{Pre}$ ($\mathbf{Post}$) is the $\mid P \mid \times \mid T \mid$ sized, natural valued, pre-(post-)incidence matrix. For instance, $\mathbf{Pre}(p,t) = w$ ($\mathbf{Post}(p,t) = w$) means that there is an arc from $p(t)$ to $t(p)$ with weight w. The incidence matrix $\boldsymbol{C}$ of the net is defined as $\boldsymbol{C} = \mathbf{Post} - \mathbf{Pre}$. For pre- and post-sets we use the conventional dot notation, e.g. ${}^\bullet t = \{p \in P \mid \mathbf{Pre}(p,t) \neq 0\}$. A pair of a place p and a transition t is called a self-loop if p is both an input and output place of t. A *marking* is a $m \times 1$ vector $\boldsymbol{m} : P \to \mathbb{N}$ that assigns to each place of a P/T net a non-negative integer number of tokens. A P/T system or net system $\langle N, \boldsymbol{m}_0\rangle$ is a P/T net N with an initial marking $\boldsymbol{m}_0$. A transition $t \in T$ is enabled at a marking $\boldsymbol{m}$ iff $\boldsymbol{m} \geq \mathbf{Pre}(\cdot, t)$. If t is enabled, then it may fire yielding a new marking $\boldsymbol{m}' = \boldsymbol{m} + \mathbf{Post}(\cdot,t) - \mathbf{Pre}(\cdot,t) = \boldsymbol{m} + \mathbf{C}(\cdot,t)$. The notation $\boldsymbol{m}[t > \boldsymbol{m}'$ will mean that an enabled transition t may fire at $\boldsymbol{m}$ yielding $\boldsymbol{m}'$. A *firing sequence* from $\boldsymbol{m}_0$ is a (possibly empty) sequence of transitions $\sigma = t_1...t_k$ such that $\boldsymbol{m}_0[t_1 > \boldsymbol{m}_1[t_2 > \boldsymbol{m}_2..[t_k > \boldsymbol{m}_k$. A marking $\boldsymbol{m}$ is reachable in $\langle N, \boldsymbol{m}_0\rangle$ iff there exists a firing sequence σ such that $\boldsymbol{m}_0[\sigma > \boldsymbol{m}$. Given a net system $\langle N, \boldsymbol{m}_0\rangle$ the set of reachable markings is denoted $R(N, \boldsymbol{m}_0)$. The function $\boldsymbol{\sigma} : T \to \mathbb{N}$, where $\boldsymbol{\sigma}(t)$ represents the number of occurrences of t in σ, is called firing count vector of the fireable sequence σ. If $\boldsymbol{m}_0[\sigma > \boldsymbol{m}$, then we can write in vector form $\boldsymbol{m} = \boldsymbol{m}_0 + \boldsymbol{C}(\cdot,t) \cdot \boldsymbol{\sigma}$. This is known as the *state equation* of the system. Left annuller integer vectors of $\boldsymbol{C}$ are called P-flow, i.e. $\boldsymbol{x} : P \to \mathbb{Z}, \boldsymbol{x} \neq \mathbf{0} \mid \boldsymbol{x}^T\boldsymbol{C} = \mathbf{0}$.

Assume we are given a set of legal markings $\mathcal{L} \subseteq \mathbb{N}^m$, and consider the basic control problem of designing a supervisor that restricts the reachability set of plant in closed loop to $\mathcal{L} \cap R(N, \boldsymbol{m}_0)$. Of particular interest are those PN state-based control problems where the set of legal markings $\mathcal{L}$ is expressed by a set of n_c linear inequality constraints called Generalized Mutual Exclusion Constraint (GMEC). A single GMEC is a couple $(\boldsymbol{l}, k)$ where $\boldsymbol{l} : P \to \mathbb{Z}$ is a $1 \times m$ weight vector and $k \in \mathbb{Z}$. The support of $\boldsymbol{l}$ is the set $Q_l = \{p \in P \mid \boldsymbol{l}(p) \neq 0\}$. Given the net system $\langle N, \boldsymbol{m}_0\rangle$, a GMEC defines a set of markings that will be called *legal markings*: $\mathcal{M}(\boldsymbol{l}, \boldsymbol{k}) = \{\boldsymbol{m} \in \mathbb{N}^m \mid \boldsymbol{l}\boldsymbol{m} \leq k\}$. The markings that are not legal are called *forbidden markings*. A controlling agent, called supervisor, must ensure the forbidden markings will be not reached. So the set of legal markings under control is $\mathcal{M}_c(\boldsymbol{l}, k) = \mathcal{M}(\boldsymbol{l}, k) \cap R(N, \boldsymbol{m}_0)$.

3 Monitor approach

In the case all the transitions are controllable and observable, it has been shown [5] that the Petri net controller that enforces $(\boldsymbol{l}, k)$ has the incidence matrix $\boldsymbol{c}_c \in \mathbb{Z}^{1\times n}$ given by

$$\boldsymbol{c}_c = -\boldsymbol{l}\boldsymbol{C}_p \tag{1}$$

where $\boldsymbol{C}_p$ is the incidence matrix of the plant and the initial marking of the controller $m_{c0} \in \mathbb{N}$ is given by

$$m_{c0} = k - \boldsymbol{l}\boldsymbol{m}_{p0} \tag{2}$$

where $\boldsymbol{m}_{p0} \in \mathbb{N}^{m\times 1}$ is the initial marking of the plant. The controller exists iff the initial marking is a legal marking, i.e.

$$k - \boldsymbol{l}\boldsymbol{m}_{p0} \geq \boldsymbol{0}. \tag{3}$$

Note that when an element of $\boldsymbol{c}_c$ is zero, there are no arcs at all connecting the given place and transition, i.e. there are no cancelling self-loop in the net controller structure. Thus, if we decompose $\boldsymbol{c}_c$ as follows

$$\boldsymbol{c}_c = \boldsymbol{c}_c^+ - \boldsymbol{c}_c^- \tag{4}$$

where $\boldsymbol{c}_c^+$ is obtained from $\boldsymbol{c}_c$ replacing each negative element with zero, while $\boldsymbol{c}_c^-$ is obtained from $\boldsymbol{c}_c$ replacing each positive element with zero and each negative element with its absolute value, we can say that $\boldsymbol{c}_c^+$ ($\boldsymbol{c}_c^-$) is the post-(pre-)incidence matrix of the monitor based control net.

The controller so constructed is maximally permissive, i.e. it prevents only transitions firings that yield forbidden markings. The control net has only one control place; no transition is added. Such control place is called *monitor place*. It is connected to the plant transitions as specified by the incidence matrix $\boldsymbol{c}_c$.

It has been showed that it is possible to transform the control specification GMEC $(\boldsymbol{l}, k)$ into a more restrictive GMEC $(\boldsymbol{l}', k')$ as shown in the following proposition.

Proposition 1 (Moody, et al. [6]) *If we are able to find $\boldsymbol{r}_1 \in \mathbb{N}^{1\times m}$, $r_2 \in \mathbb{N}$ satisfying*

$$[\boldsymbol{r}_1 \quad r_2] \begin{bmatrix} \boldsymbol{m}_{p0} \\ \boldsymbol{l}\boldsymbol{m}_{p0} - (k+1) \end{bmatrix} \leq -1 \tag{5}$$

then the controller computed as

$$\boldsymbol{c}_c = -\boldsymbol{l}'\boldsymbol{C}_p \tag{6}$$

$$m_{c0} = k' - \boldsymbol{l}'\boldsymbol{m}_{p0} \tag{7}$$

where

$$\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l} \tag{8}$$

$$k' = r_2(k+1) - 1. \tag{9}$$

will be able to ensure that the closed-loop net system meet $\boldsymbol{l}\boldsymbol{m}_p \leq k$, and that the initial marking is a legal marking.

As consequence of proposition 1 we have that we can preserve the original constraint and a very efficient computation method for the controller (a simple matrices multiplication, as shown in (1)); at same time a number of freedom degrees represented by $\boldsymbol{r}_1$ and r_2 elements may be used to impose additional constraints. As shown in the following section here we want to use these freedom degrees to minimize the sum of the control and observation cost.

4 Optimal monitor design

We associate to each transition a positive real control cost by the vector $\boldsymbol{z}_c : T \rightarrow \mathbb{R}^+$ and an observation cost by the vector $\boldsymbol{z}_o : T \rightarrow \mathbb{R}^+$. Our problem consists in choosing among the set of all monitors corresponding to GMECs that are more restrictive than $(\boldsymbol{l}, k)$ and that can be written using Moody's parameterization, the one that minimizes an objective function representing the cost of the monitor based control net structure. This cost may be differently defined. Here we consider three different cases: they all lead to an integer programming problem formulation.

4.1 First case

Firstly, we suppose that the control and observation actions are associated with physical actions like a material flow or a power signal, even if it is not a very frequent situation. In this case if a monitor place p_c has an arc outgoing to a plant transition t with weight $\boldsymbol{c}_c^-(p_c, t)$, we define $\boldsymbol{c}_c^-(p_c, t)\boldsymbol{z}_c(t)$ the cost of disabling a firing of the transition; so, if a monitor has an input arc from a plant transition with weight $\boldsymbol{c}_c^+(p_c, t)$, we define $\boldsymbol{c}_c^+(p_c, t)\boldsymbol{z}_o(t)$ the cost of detecting a firing of this transition.

So the optimal monitor can be found by solving the following integer linear programming problem (ILP):

$$\begin{array}{ll} min\ \Delta = \boldsymbol{c}_c^-\boldsymbol{z}_c + \boldsymbol{c}_c^+\boldsymbol{z}_o & \\ s.t. & \\ (a) \quad \boldsymbol{r}_1\boldsymbol{C} + r_2\boldsymbol{l}\boldsymbol{C} = \boldsymbol{c}_c^- - \boldsymbol{c}_c^+ & \\ (b) \quad \boldsymbol{r}_1\boldsymbol{m}_{p0} + r_2(\boldsymbol{l}\boldsymbol{m}_{p0} - (k+1)) \leq -1 & \\ (c) \quad \boldsymbol{c}_c^- \geq \boldsymbol{0}_{1\times n} & \\ (d) \quad \boldsymbol{c}_c^+ \geq \boldsymbol{0}_{1\times n} & \\ (e) \quad \boldsymbol{r}_1 \geq \boldsymbol{0}_{1\times m} & \\ (f) \quad r_2 \geq 1 & \end{array} \tag{10}$$

with variables $\boldsymbol{r}_1 \in \mathbb{N}^{1\times m}$, $r_2 \in \mathbb{N}$, $\boldsymbol{c}_c^- \in \mathbb{N}^{1\times n}$, $\boldsymbol{c}_c^+ \in \mathbb{N}^{1\times n}$. The equations (10-a,c,d,e,f) imposes that the incidence matrix of the controller is obtained from a Moody's parameterization: $\boldsymbol{l}'\boldsymbol{C} = \boldsymbol{c}_c = \boldsymbol{c}_c^- - \boldsymbol{c}_c^+$, with $\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l}$. The equation (10-b) imposes the initial marking condition verification ($\boldsymbol{l}'\boldsymbol{m}_{p0} \leq k'$).

The ILP (10) can be written in a standard linear integer programming form as follows:

$$
\begin{aligned}
& min\ \Delta = \boldsymbol{z}^T \boldsymbol{y} \\
& s.t. \\
& (a) \quad [\,\boldsymbol{C}^T \quad \boldsymbol{C}^T\boldsymbol{l}^T \quad -\boldsymbol{I}_n \quad \boldsymbol{I}_n\,]\,\boldsymbol{y} = \mathbf{0}_{\mathbf{n}\times\mathbf{1}} \qquad (11) \\
& (b) \quad [\,\boldsymbol{m}_{p0}^T \quad (\boldsymbol{m}_{p0}^T\boldsymbol{l}^T - (k+1))^T \quad \mathbf{0}_{\mathbf{1}\times\mathbf{2n}}\,]\,\boldsymbol{y} \leq -1 \\
& (c) \quad \boldsymbol{I}_{m+1+2n}\,\boldsymbol{y} \leq [\,\mathbf{0}_{\mathbf{m}\times\mathbf{1}} \quad 1 \quad \mathbf{0}_{\mathbf{2n}\times\mathbf{1}}\,]^T
\end{aligned}
$$

where $\boldsymbol{z} = [\,\mathbf{0}_{\mathbf{m}\times\mathbf{1}} \quad 0 \quad \boldsymbol{z}_c \quad \boldsymbol{z}_o\,]^T$, $\boldsymbol{y} = [\,\boldsymbol{r}_1 \quad r_2 \quad \boldsymbol{c}_c^- \quad \boldsymbol{c}_c^+\,]^T$ and I_n denoting the identity matrix of dimension n. We denote as $\mathcal{F}(\boldsymbol{y})$ the set of the natural valued vectors that are solutions solutions of (11-a,b,c).

The following two properties characterize the optimal solution of ILP (10), but they are still true for the optimal monitor solutions obtained in the two next subsections by different objective functions.

Property 1 *The optimal monitor place obtained from solving ILP (10) verifies that* $\boldsymbol{c}_c^{-*}(j)\boldsymbol{c}_c^{+*}(j) = \mathbf{0}_{1\times m}, \forall j$, *i.e. a plant transition cannot be at same time input and output transition of the optimal monitor place.*

Proof: Suppose that $\exists j$, $\boldsymbol{c}_c^{-*}(j)\boldsymbol{c}_c^{+*}(j) \neq 0$ and without loss of generality that $\boldsymbol{c}_c^{-*}(j) \geq \boldsymbol{c}_c^{+*}(j)$. Now let us build a new solution $\boldsymbol{c}_c^{-\prime}(j) = \boldsymbol{c}_c^{-*}(j) - \boldsymbol{c}_c^{+*}(j)$, $\boldsymbol{c}_c^{+\prime}(j) = 0$. It is immediate to verify that the (10-a,b,c,d,e,f) are verified and that $\Delta' = \Delta^* - \boldsymbol{z}_c(j)\boldsymbol{c}_o^{-*}(j) - \boldsymbol{z}_o(j)\boldsymbol{c}_o^{-*}(j)$, and so Δ^* was not optimal. ■

Property 2 *The optimal controller cost to impose a given GMEC* $(\boldsymbol{l}, k)$ *is null, i.e. solving ILP (10) we obtain* $\Delta^* = 0$ *iff exists a P-flow of the plant net* $\boldsymbol{l}'$, *with* $\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l}$, $\boldsymbol{r}_1 \in \mathbb{N}^{1\times m}$, $r_2 \in \mathbb{N}$, *and* $\boldsymbol{l}'\boldsymbol{m}_{p0} \leq k'$, *where* $k' = r_2(k+1) - 1$.

Proof: (if) If $\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l}$ is a P-flow for the plant net then we have that $\boldsymbol{l}'\boldsymbol{C} = \mathbf{0}_{1\times n}$. Then from (10-a) it follows that $\boldsymbol{c}_c^- - \boldsymbol{c}_c^+ = \mathbf{0}_{1\times n}$, and also because of property 1 we have that $\boldsymbol{c}_c^- = \boldsymbol{c}_c^+ = \mathbf{0}_{1\times n} \rightarrow \Delta = 0$. The initial condition verification expressed by (10-b) and the (10-e,f) are met by hypothesis. Thus, $\Delta^* = 0$. (only if) $\Delta^* = 0$ implies $\boldsymbol{c}_c^- = \boldsymbol{c}_c^+ = \mathbf{0}_{1\times m}$ because of (10-c,d). From (10-a) we have that $\boldsymbol{l}'\boldsymbol{C} = \mathbf{0}_{1\times n}$, with $\boldsymbol{l}' = \boldsymbol{r}1 + \boldsymbol{r}2\boldsymbol{l}$ and so by definition of P-flow $\boldsymbol{l}'$ is a P-flow of the plant net. ■

4.2 Second case

As second case, we consider the frequent situation when no physical actions are involved in the control and observation actions, but only logical ones. In this case the weights of arcs have only a logical meaning, e.g. input arcs are associated with the logical condition verification $\boldsymbol{m} \geq \mathbf{Post}(\cdot, t), \forall p \in^{\bullet} t$ in order to enable a certain transition t. Thus, what we really are representing with the control cost of a given transition is the cost of the device to install in order to enable the action associated with the transition or in order to detect a transition firing, and not the cost to check the enabling condition for the transition that we consider negligible. This is why in this case if a monitor place p_c has an output arc directed to a plant transition t with weight $\boldsymbol{c}_c^-(p_c, t)$, we define $\boldsymbol{z}_c(t)$ the cost necessary to disable a firing of the transition; we proceed similarly with input transitions. Thus, in this case the optimal monitor can be found by solving the following integer non-linear programming problem:

$$
\begin{aligned}
& \Delta_{nl} = sign(\boldsymbol{c}_c^-)\boldsymbol{z}_c + sign(\boldsymbol{c}_c^+)\boldsymbol{z}_o \\
& s.t. \quad \boldsymbol{y} \in \mathcal{F}(\boldsymbol{y})
\end{aligned} \qquad (12)
$$

where where $\boldsymbol{y} = [\,\boldsymbol{r}_1 \quad r_2 \quad \boldsymbol{c}_c^- \quad \boldsymbol{c}_c^+\,]^T$, $sign(\boldsymbol{A}(i,j))$, with $\boldsymbol{A}$ $m \times n$ matrix of positive integers, is equal to 0 if $\boldsymbol{A}(i,j)$ is 0, else it is equal to one. The *sign* function let us to consider only the control or the observation cost without taking into account the weights of arcs from or to control places.

4.3 Third case

We also consider the possibility of imposing a trade-off between cost of the control and the restrictions imposed by the monitor. We are looking for a GMEC $(\boldsymbol{l}', k)$, according to Moody's parameterization that we are adopting here, such that $\boldsymbol{l}' = \boldsymbol{r}_1 + r_2\boldsymbol{l}$ and $k' = r_2(k+1) - 1$, with $\boldsymbol{r}_1 \in \mathbb{N}^{1\times m}$ and $r_2 \in \mathbb{N}$. Being $\boldsymbol{r}_1$ and r_2 natural valued, it is immediate to verify that $\mid \mathcal{M}(\boldsymbol{l}', k') \mid \leq \mid \mathcal{M}(\boldsymbol{l}, k) \mid$, and obviously $\mid \mathcal{M}_c(\boldsymbol{l}', k') \mid \leq \mid \mathcal{M}_c(\boldsymbol{l}, k) \mid$. Although this topic has to be further investigated, here, as first step in this research direction, we add a weighted sum of the elements of $\boldsymbol{r}_1$ in the objective function in order to minimize this restriction on the plant, without taking into account the effect of r_2 parameter.

In the case that the cost of controller is linearly dependent from its arc weights the optimal monitor will be found by solving

$$
\begin{aligned}
& \Delta_r = \boldsymbol{c}_c^-\boldsymbol{z}_c + \boldsymbol{c}_c^+\boldsymbol{z}_o + \boldsymbol{r}_1\boldsymbol{z}_r \\
& s.t. \quad \boldsymbol{y} \in \mathcal{F}(\boldsymbol{y})
\end{aligned} \qquad (13)
$$

where $\boldsymbol{y} = [\,\boldsymbol{r}_1 \quad r_2 \quad \boldsymbol{c}_c^- \quad \boldsymbol{c}_c^+\,]^T$, otherwise one has to consider the problem

$$\Delta_{nlr} = sign(\boldsymbol{c}_c^-)\boldsymbol{z}_c + sign(\boldsymbol{c}_c^+)\boldsymbol{z}_o + \boldsymbol{r}_1\boldsymbol{z}_r$$

$$\text{s.t.} \quad \boldsymbol{y} \in \mathcal{F}(\boldsymbol{y}). \tag{14}$$

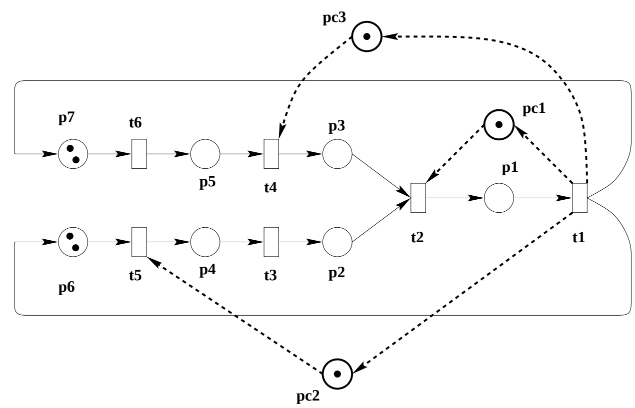

Figure 1: Net system in example 1.

Example 1 *Let us to consider the net system in fig. 1. We have that*

$$\boldsymbol{C}_p = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{m}_{p0} = [\,0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 2\,]$$

and consider the GMEC $(\boldsymbol{l}, k)$ *with*

$$\boldsymbol{l} = [\,1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\,]\,, \; k = 1$$

If we do not consider control and observation costs we obtain the monitor p_{c1} *applying (1) and (2).*

Now let us to introduce the control and observation costs:

$$\boldsymbol{z}_c = [\,1 \quad 6 \quad 5 \quad 3 \quad 2 \quad 3\,],$$

$$\boldsymbol{z}_o = [\,1 \quad 3 \quad 4 \quad 4 \quad 3 \quad 2\,].$$

If we adopt p_{c1} *we have* $\Delta = 7$*.*

From the ILP (10) it follows

$$\boldsymbol{l}'^{\,*} = [\,1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0\,]\,, \; k'^{\,*} = 1$$

and the relative monitor place p_{c2} *with incidence matrix and initial marking*

$$\boldsymbol{c}_{c1}^{*} = [\,1 \quad 0 \quad 0 \quad 0 \quad -1 \quad 0\,]\,, \; m_{c01} = 1$$

In this case $\Delta^* = 3$*.*

Introducing $\boldsymbol{z}_r = [\,2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2\,]$ *it follows by solving (13)*

$$\boldsymbol{l}''^{\,*} = [\,1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0\,]\,, \; k''^{\,*} = 1$$

and the relative monitor place p_{c3} *with incidence matrix and initial marking*

$$\boldsymbol{c}_{c2}^{*} = [\,1 \quad 0 \quad 0 \quad -1 \quad 0 \quad 0\,]\,, \; m_{c02} = 1$$

In this case $\Delta_r^* = 6$*.*

Example 2 *Consider the net system in fig. 2.*

$$\boldsymbol{C}_p = \begin{bmatrix} -1 & 0 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 1 \\ 2 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\boldsymbol{m}_{p0} = [\,1 \quad 4 \quad 0 \quad 0 \quad 0\,]$$

$$\boldsymbol{l} = [\,0 \quad 0 \quad 0 \quad 0 \quad 1\,]\,, \, k = 2.$$

Let

$$\boldsymbol{z}_c = [\,1 \quad 2 \quad 4 \quad 5 \quad 1\,]$$

$$\boldsymbol{z}_o = [\,3 \quad 2 \quad 2 \quad 2 \quad 1\,].$$

be the control and observation costs of the transitions. Applying the ILP (10) to this system, we obtain that the optimal transformed constraint is

$$\boldsymbol{l}'^{\,*} = [\,0 \quad 0 \quad 0 \quad 1 \quad 1\,]\,, \; k'^{\,*} = 2$$

and the optimal monitor, labeled p_{c1} *in the figure, has incidence matrix and initial marking*

$$\boldsymbol{c}_{c1}^{*} = [\,0 \quad 0 \quad -1 \quad 0 \quad 2\,]\,, \; m_{c01} = 2$$

The minimum value of the objective function Δ^* *in this case is* 6*.*

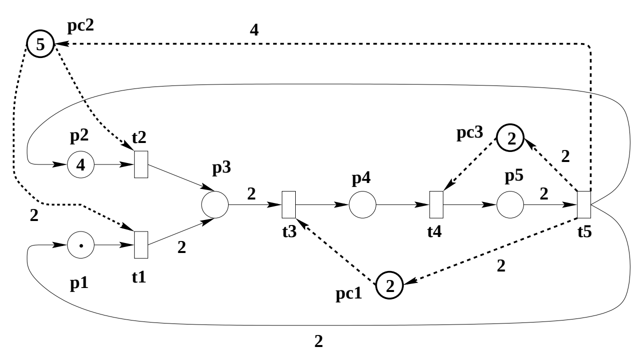

Figure 2: Net system in example 2.

While if we solve (12) we obtain

$$\boldsymbol{l}''^{\,*} = [\,0 \quad 0 \quad 1 \quad 2 \quad 2\,]\,, k''^{\,*} = 5$$

$$\boldsymbol{c}^*_{c2} = [\,-2 \quad -1 \quad 0 \quad 0 \quad 4\,],\ m_{c02} = 5.$$

The optimal monitor in this case is the one labeled p_{c2} in the figure, and $\Delta^*_{nl} = 4$.

Finally introducing $\boldsymbol{z}_r = [2 \quad 2 \quad 2 \quad 2 \quad 2]$ by solving (14) it follows that

$$\boldsymbol{l}'''^* = [0 \quad 0 \quad 0 \quad 0 \quad 1]\,, k'''^* = 2$$

$$\boldsymbol{c}^*_{c3} = [0 \quad 0 \quad 0 \quad -1 \quad 2]\,,\ m_{c03} = 2$$

In the this case the optimal monitor is the one labeled p_{c3} in the figure, and $\Delta^*_{nlr} = 6$.

All computations in the two previous examples have been performed by the optimization package LINGO.

Note 1 *We want to remark that the classical supervisory control problem is included in our third control problem formulation. Let us just consider, for sake of simplicity, the case of controllable/uncontrollable transitions. We give a positive small control cost ϵ to all controllable transitions and a much larger control cost ω to uncontrollable transitions. To avoid that a maximally permissive solution with, say, k arcs going to controllable transitions (whose control cost is $k\epsilon$) may be discarded, preferring to it a not maximally permissive solution with only $k' < k$ arcs going to controllable transitions (whose control cost is $k'\epsilon$), we also assign a value ρ, with $\epsilon \ll \rho \ll \omega$, to all elements of $\boldsymbol{z}_r$.*

In [8] it was shown that if the transitions of the PN plant model are not all controllable the class of controllable monitors, i.e. with no output arcs to uncontrollable transitions, that satisfies a given GMEC may not admit a unique supremal controllable element with respect to forbidden state problem: each solution is a suboptimal one. So a suboptimal criterion has to be introduced to choose the suboptimal one. Once that control and observation transition cost have been introduced, solving the problem 10 a suboptimal monitor may be selected, after that a proper control and observation cost to each transition has been assigned as said in the previous note.

5 Conclusions

In this paper we dealt with the control of Petri Net modeled plant. The concept of control and observation cost of a transition is introduced. We discussed the problem to enforce a mutual exclusion constraint so as to minimize the control and observation cost. A monitor based controller form is chosen, because of its simplicity.

The novel contribution of this paper is in the introduction of control and observation costs for transitions of a PN modeled plant. Although no timed event are considered and only event occurrence changes the state of our model, this makes possible to take into account the cost to detect or to enable an event occurrence that is not always negligible in the real plant. Two integer programming problems are formulated to synthesize the optimal monitor based controller: a linear one in the case that all arcs of controller have unitary weight and a non linear one in other cases. Finally, this approach was extended to also take into account a cost associated to the restriction imposed by the monitor (in term of places in the support of the GMEC).

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